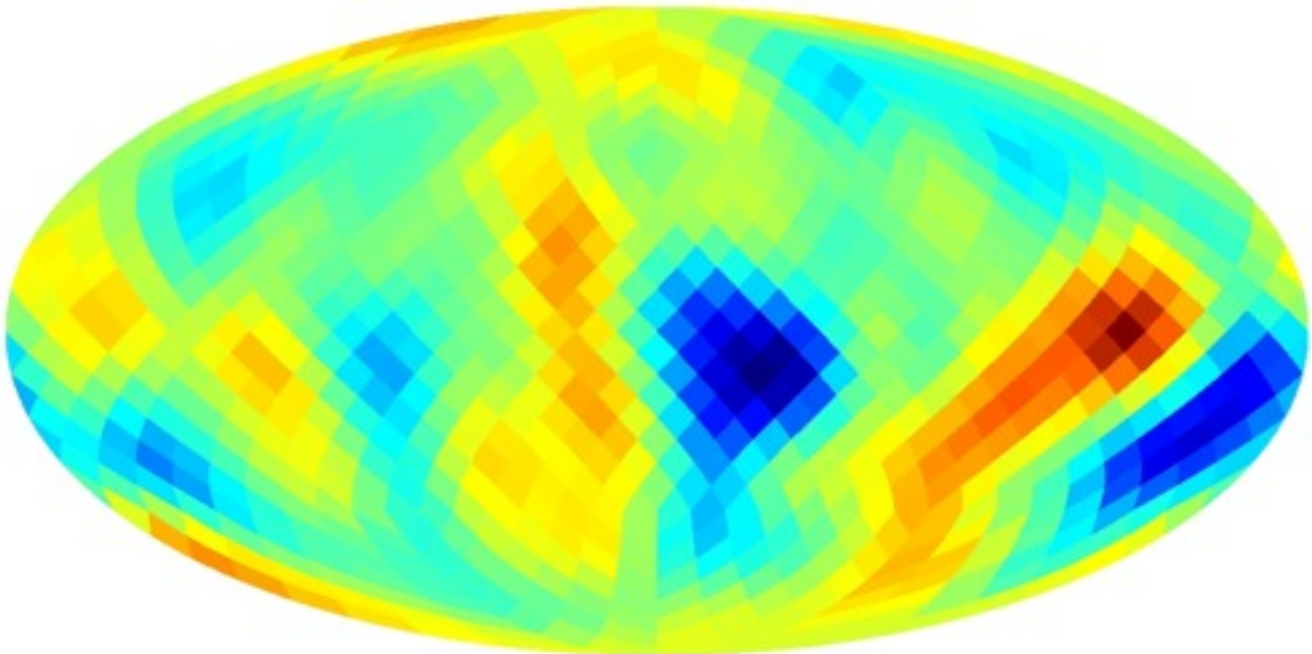
The axis of evil
- a polarization perspective -

Frommert & Enßlin. 2008, arXiv:0908.0453



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Outline

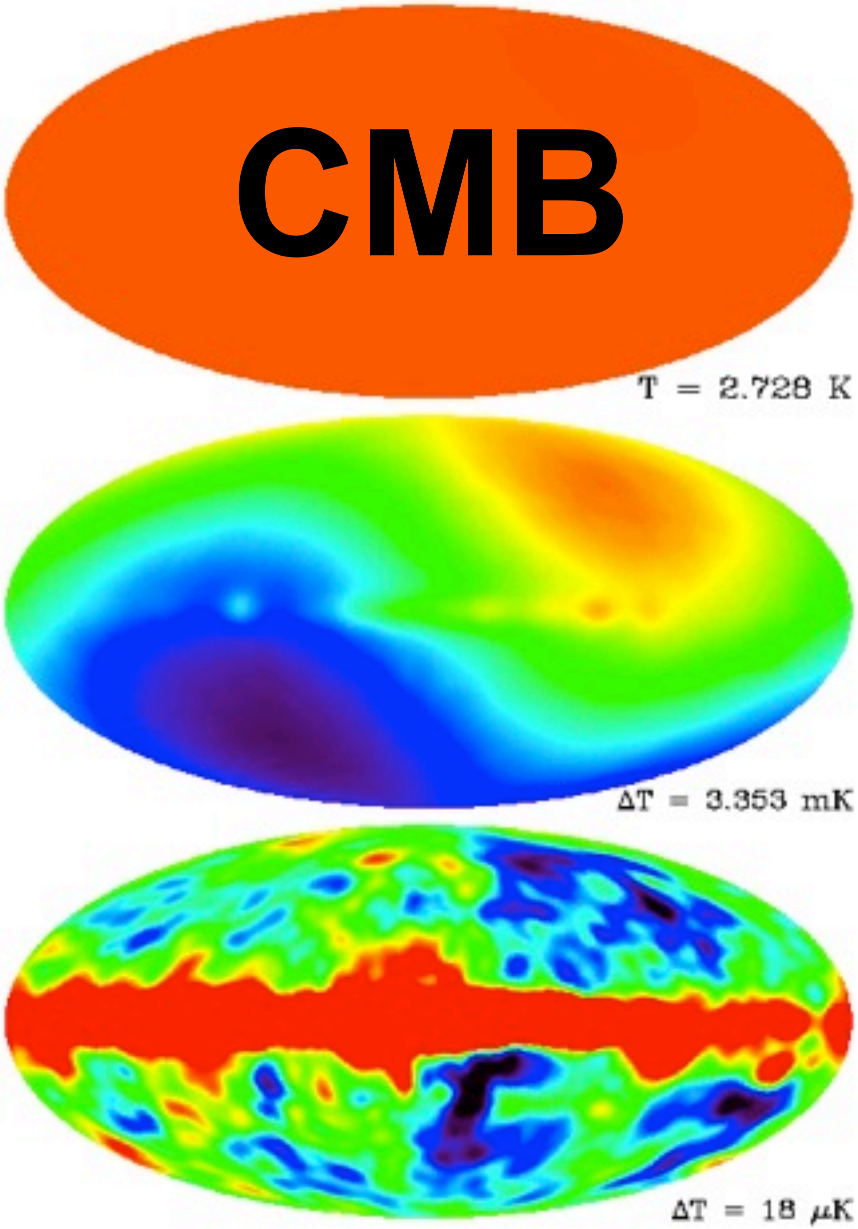
- Anomalies in the Cosmic Microwave Background (CMB)
  - The problem of a posteriori statistics
  - Going beyond
  - New problem: correlated datasets
- Find uncorrelated dataset
  - Apply to multipole alignment

Fundamental assumption of Cosmology:

Cosmological Principle

The Universe is homogeneous and isotropic

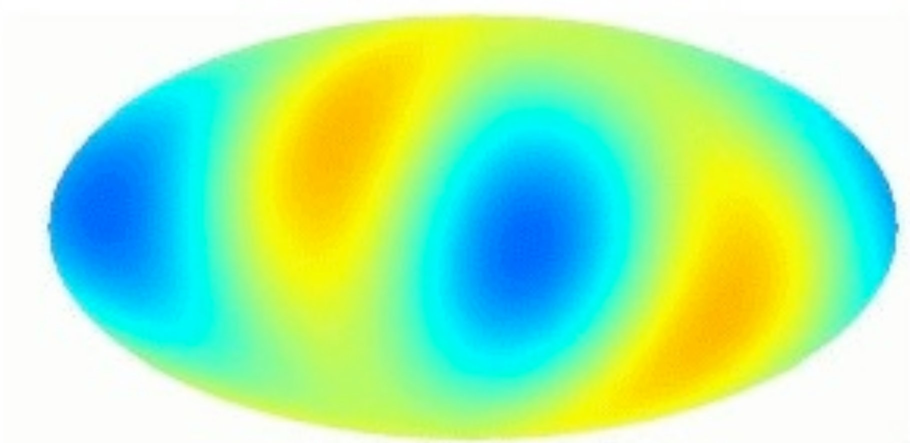
Observational evidence for Cosmological Principle:



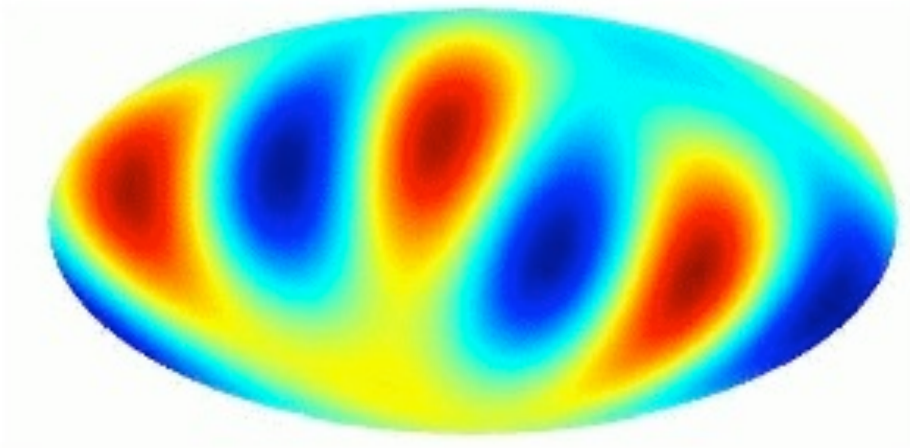
picture courtesy: COBE team

But: Anomalies on the large scales:

- Alignment of quadrupole and octopole *(Tegmark et al. 2003)*



Quadrupole

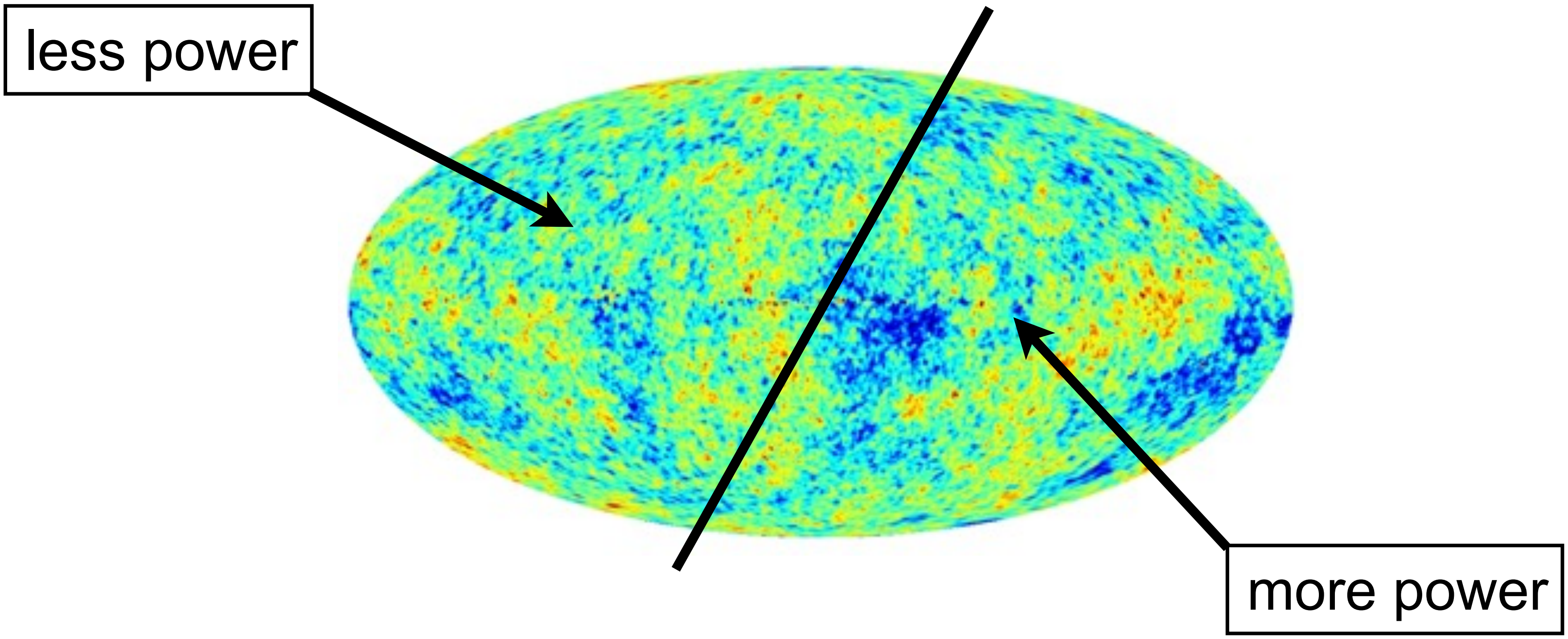


Octopole

picture courtesy: de Oliveira-Costa et al. 2004

But: Anomalies on the large scales:

- Alignment of quadrupole and octopole *(Tegmark et al. 2003)*
- Hemispherical power asymmetry *(Eriksen et al. 2004)*



But: Anomalies on the large scales:

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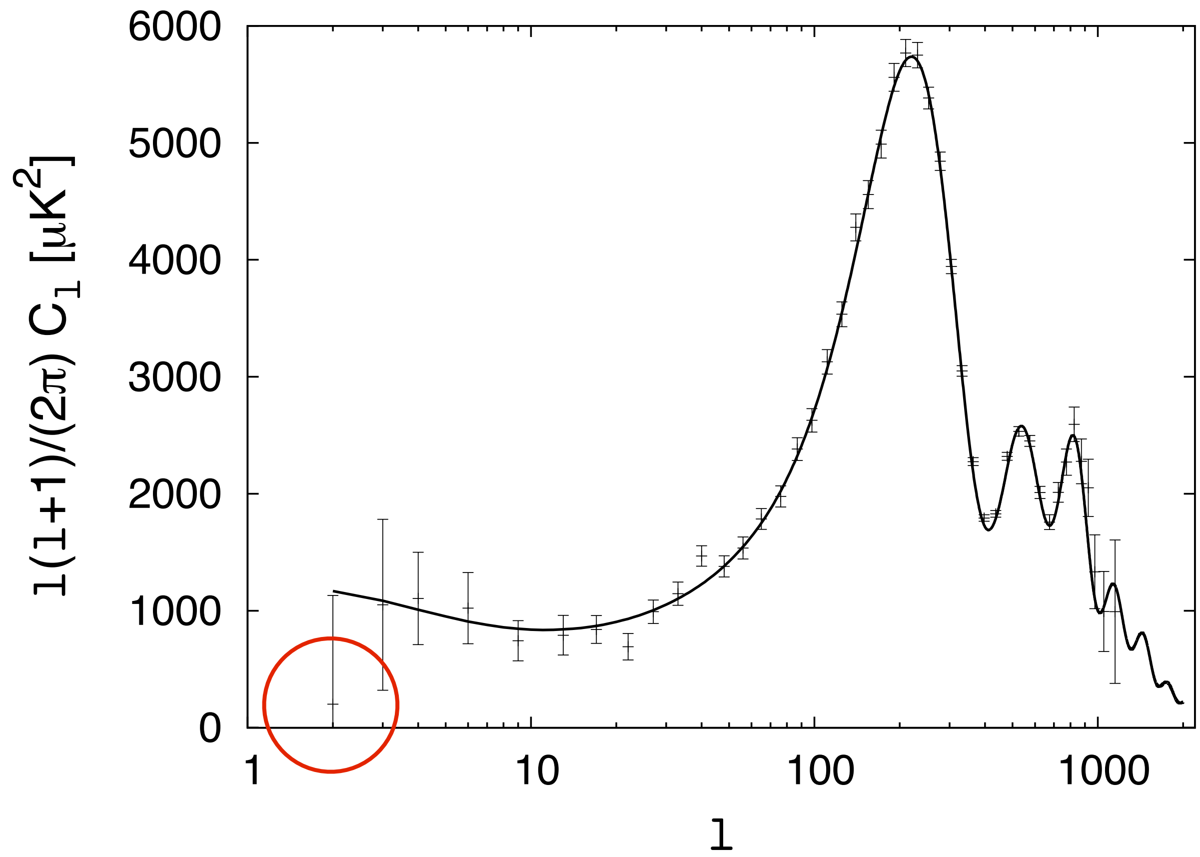
- Low quadrupole *(Bennett et al. 1992)*



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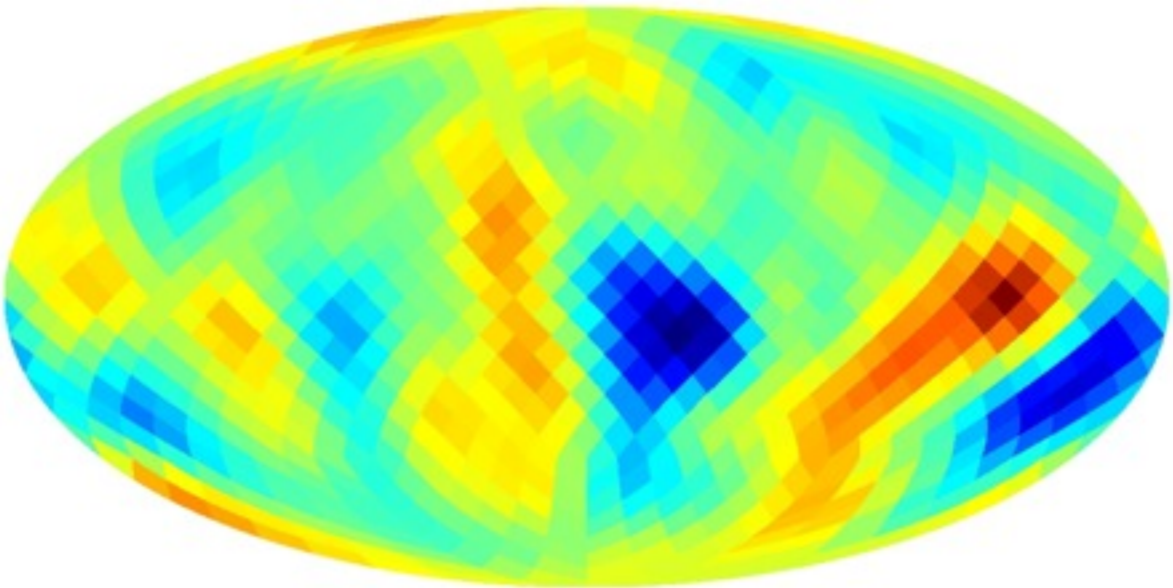
- Missing power on large scales *(Spergel et al. 2003, Copi et al. 2007)*



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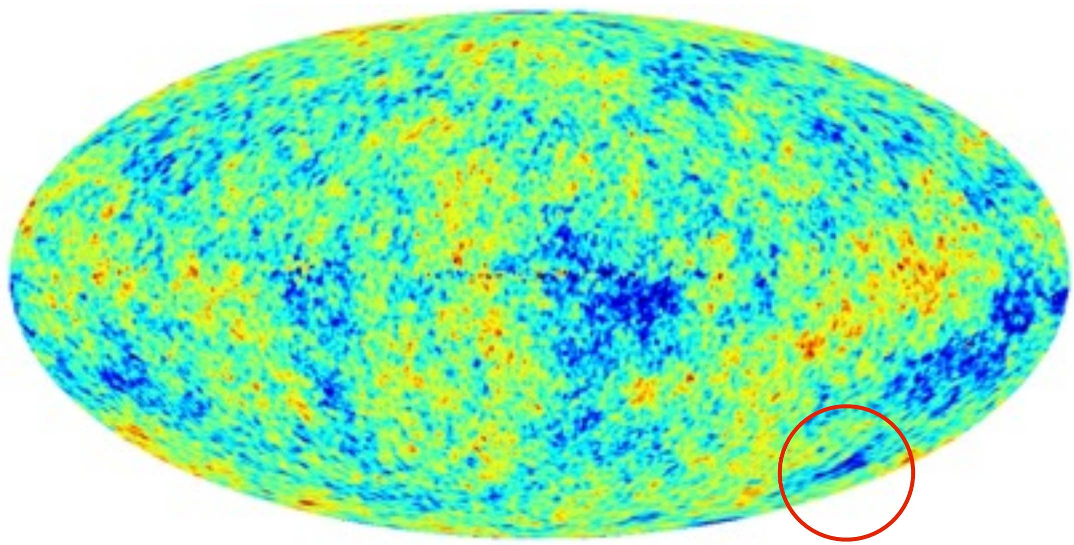
- Non-Gaussian Cold spot *(Vielva et al. 2004)*



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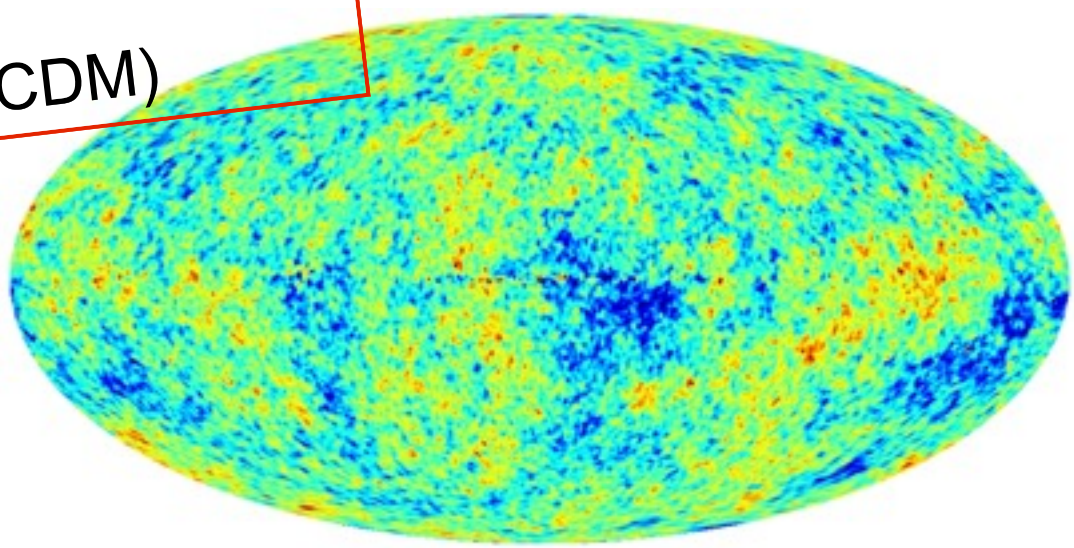
- Non-Gaussian Cold spot *(Vielva et al. 2004)*

Some of these anomalies challenge statistical isotropy!

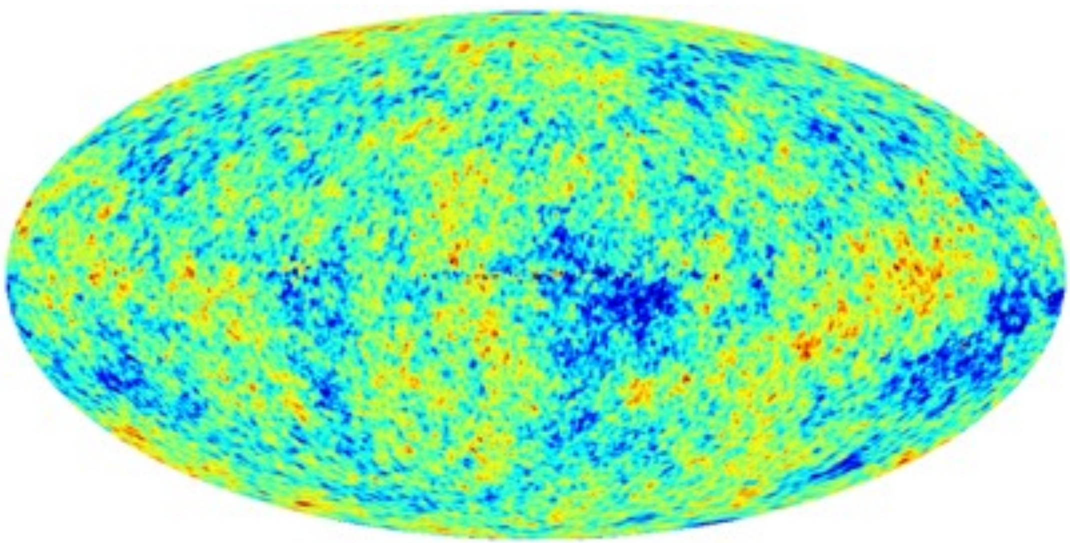
(All of them challenge ΛCDM)



Significances are ~1%

Where do these anomalies come from?

- Chance fluctuations in the temperature?
- Foregrounds?
- Systematics?
- Special direction intrinsic to our Universe / ΛCDM wrong?
- Are they so anomalous after all?



Problem of a posteriori choice of statistic

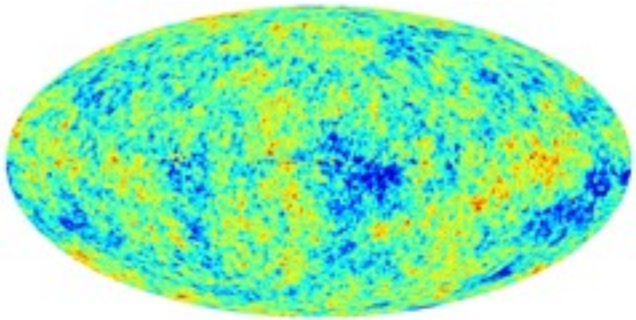
- We choose the statistic to quantify the anomaly AFTER looking at the map
- The number of statistics that one could choose is huge
- With most of them we would not see anomalies
- Difficult to account for the "volume of the space of all possible statistics"

→ We see what we want to see

WMAP 7 year results (Bennett et al. 2011)

- Multipole alignment: *?*
- Hemispherical power asymmetry: *due to a posteriori choice of statistic*
- Low quadrupole: *not anomalously low (Gibbs sampling including mask)*
- Missing power: *due to sky-cut + chance alignment of CMB with Galaxy + a posteriori statistic + method of computation (Efstathiou 2010)*
- Cold spot: *due to a posteriori choice of statistic?*

Most of these anomalies are due to an a posteriori choice of statistic



Going beyond a posteriori analysis

- Take a model that eases anomalies in the temperature map (e.g. anisotropic universe)
- Make predictions for other data (e.g. CMB polarization): get a similar anomaly for these data
- Test predictions
- *Dvorkin et al. 2008: Polarization predictions for certain models*

Going beyond a posteriori analysis

Problem: degeneracy

Anomaly due to preferred direction intrinsic to Universe

Anomaly is statistical fluke

Temperature and polarization are correlated!

Find similar anomaly in CMB polarization

Going beyond a posteriori analysis

<u>Problem: degeneracy</u>

Anomaly due to preferred direction intrinsic to Universe

Anomaly is statistical fluke

Need data that are statistically independent of CMB temperature

Find similar anomaly in CMB polarization

Statistically independent data:

Take sky map statistically independent of T_{CMB}:

See the anomaly in this map?

→ If yes: exclude statistical fluke

Statistically independent data:

Polarization is correlated with the temperature

Not independent

Statistically independent data:

Polarization is correlated with the temperature

➡ Not independent

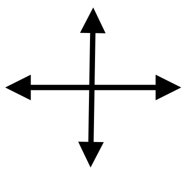
But can remove correlated part and use uncorrelated part

➡ Independent

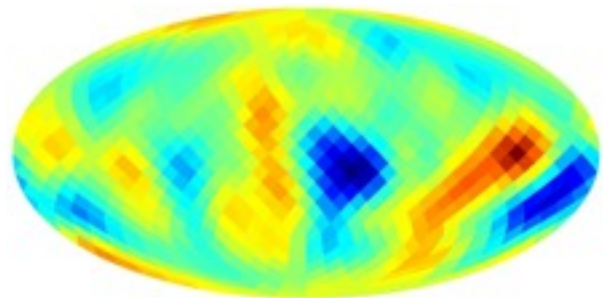
Frommert & Enßlin. 2008, arXiv:0908.0453

Polarization of the CMB

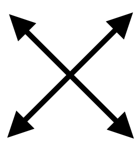
CMB linearly polarized: Stokes parameters:



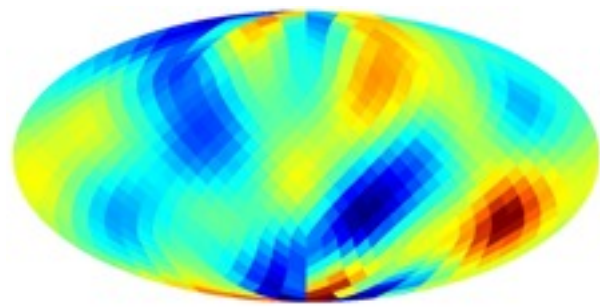
Stokes Q



Q map



Stokes U



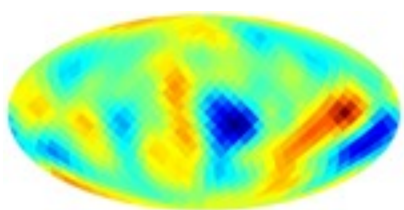
U map

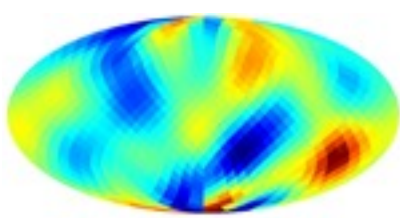
Polarization of the CMB

Spherical harmonics decomposition of temperature map T:

$$T(\hat{n}) = \sum_{\ell,m} a_{\ell m}^{T} Y_{\ell m}(\hat{n}) \qquad C_\ell^{TT} = \langle |a_{\ell m}^{T}|^2 \rangle$$

decompose polarization with spin-weighted spherical harmonics:





$$\left. \begin{array}{c} Q(\hat{n}) \\ \\ U(\hat{n}) \end{array} \right\} \boxed{a_{\ell m}^{E}}$$

$$C_\ell^{EE} = \langle |a_{\ell m}^{E}|^2 \rangle$$

$$C_\ell^{TE} = \langle a_{\ell m}^{T} a_{\ell m}^{E*} \rangle$$

Compute uncorrelated polarization map - idea

Frommert & Enßlin. 2008, arXiv:0908.0453

Translate temperature map into polarization map:

$$a_{\ell m}^{E,\mathrm{corr}} = \frac{C_\ell^{TE}}{C_\ell^{TT}} a_{\ell m}^{T}$$

Subtract this from polarization map:

$$a_{\ell m}^{E,\mathrm{uncorr}} = a_{\ell m}^{E} - a_{\ell m}^{E,\mathrm{corr}}$$

This is uncorrelated with temperature:

$$\langle a_{\ell m}^{E,\mathrm{uncorr}} a_{\ell m}^{T} \rangle = 0$$

Compute uncorrelated polarization map - idea

Proof:

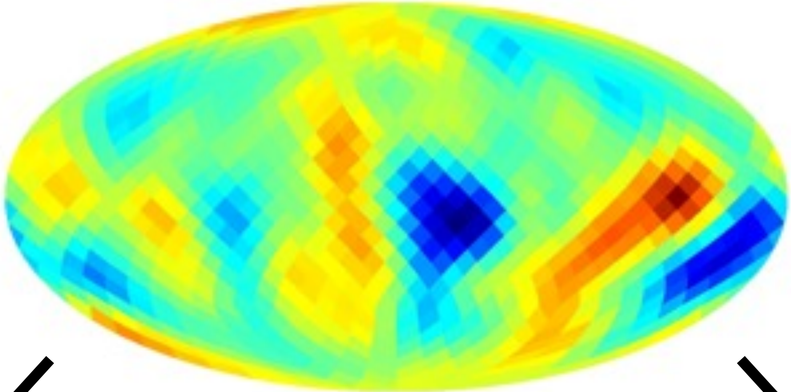
$$\langle a_{\ell m}^{E,\mathrm{uncorr}} a_{\ell m}^{T} \rangle = \langle a_{\ell m}^{E} a_{\ell m}^{T} \rangle - \frac{C_\ell^{TE}}{C_\ell^{TT}} \langle a_{\ell m}^{T} a_{\ell m}^{T} \rangle$$

$$= C_\ell^{TE} - \frac{C_\ell^{TE}}{C_\ell^{TT}} C_\ell^{TT}$$

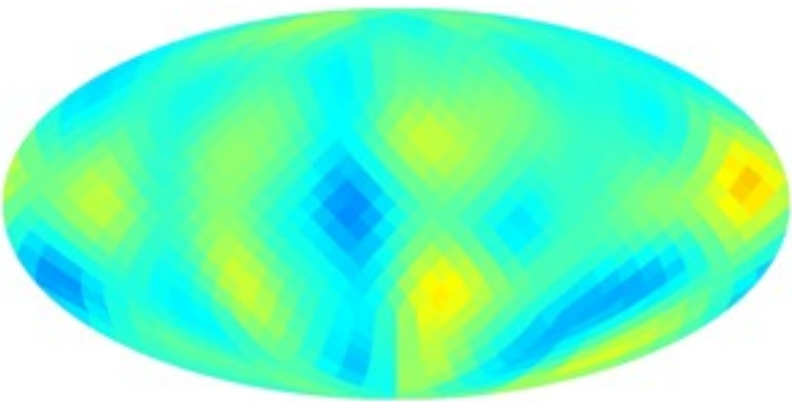
$$= 0$$

Compute uncorrelated polarization map - in practice

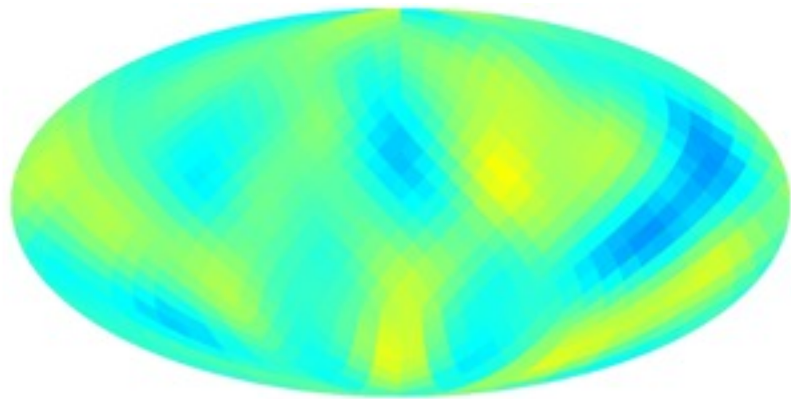
Correlated polarization maps: $a_{\ell m}^{E,\mathrm{corr}} = \frac{C_\ell^{TE}}{C_\ell^{TT}} a_{\ell m}^{T}$



Temperature



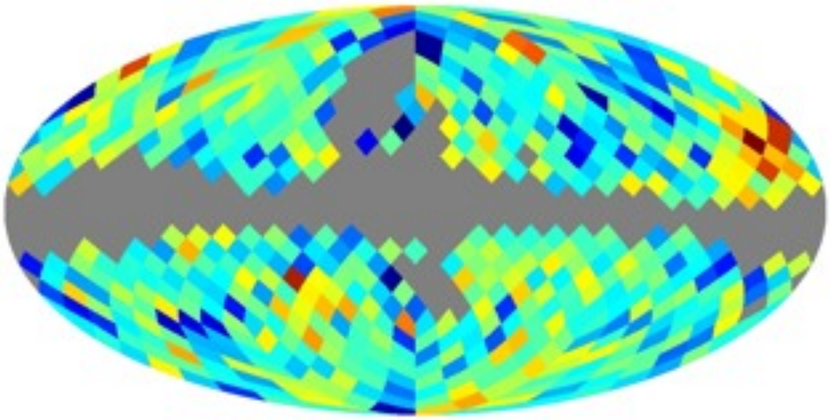
Q-map correlated



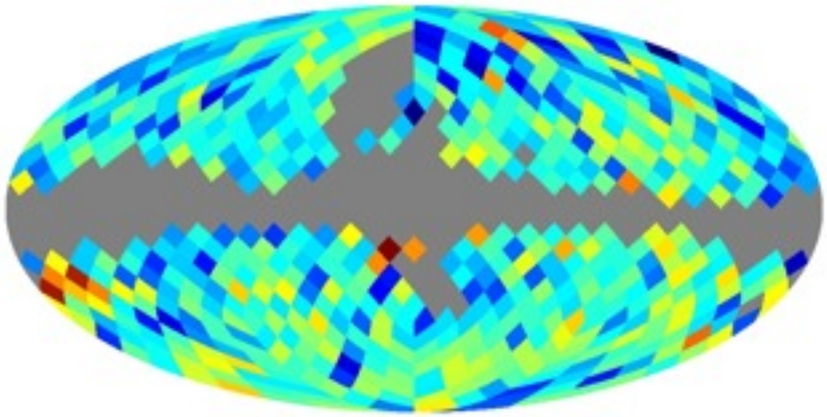
U-map correlated

Compute uncorrelated polarization map - in practice

Observed polarization maps:



Q-map



U-map

Noisy! → Reduce noise

Compute uncorrelated polarization map - in practice

Reduce noise by Wiener filtering:

$$d = Rs + n$$

d = data (observe), s = signal (want)
R = mask, n = noise

Assume Gaussian signal and Gaussian noise

⇒ Posterior distribution P(s | d) is Gaussian, too.

Posterior mean of signal:

$$\langle s \rangle = (S^{-1} + R^\dagger N^{-1} R)^{-1} R^\dagger N^{-1} d$$

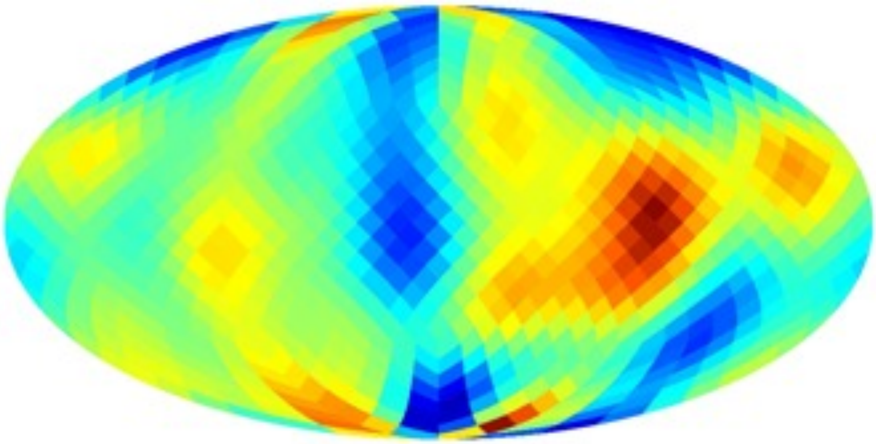
$$S = \langle s s^\dagger \rangle$$

$$N = \langle n n^\dagger \rangle$$

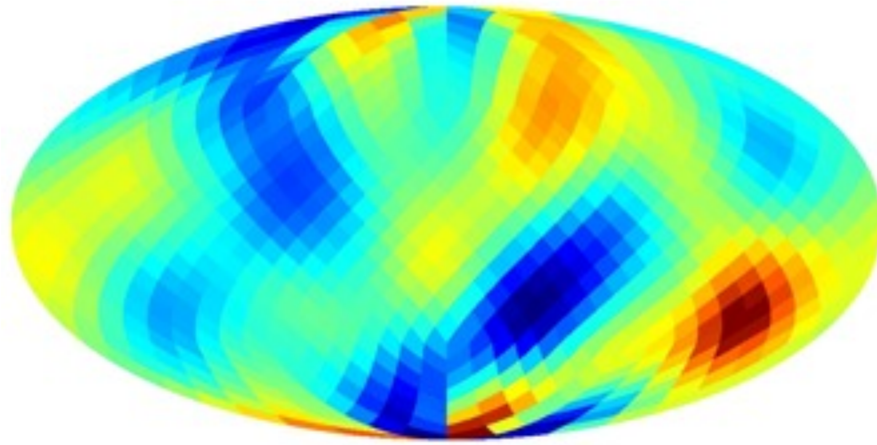
Wiener filter

Compute uncorrelated polarization map - in practice

Wiener filtered polarization maps



Q-map



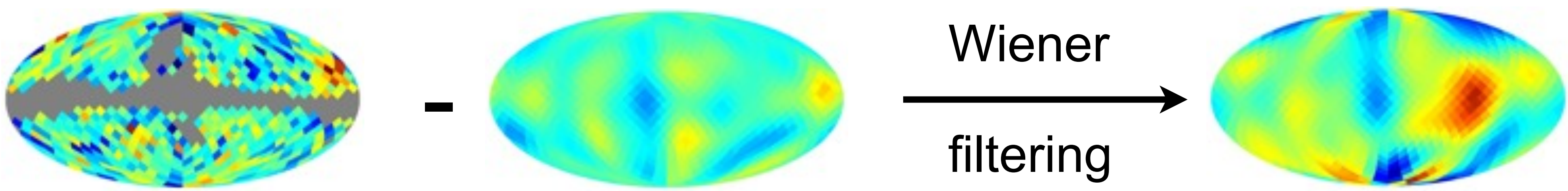
U-map

Note that the Wiener filter fills the masked regions

Subtracting corr. pol. map from these does NOT result in maps uncorrelated with the temperature

Compute uncorrelated polarization map - in practice

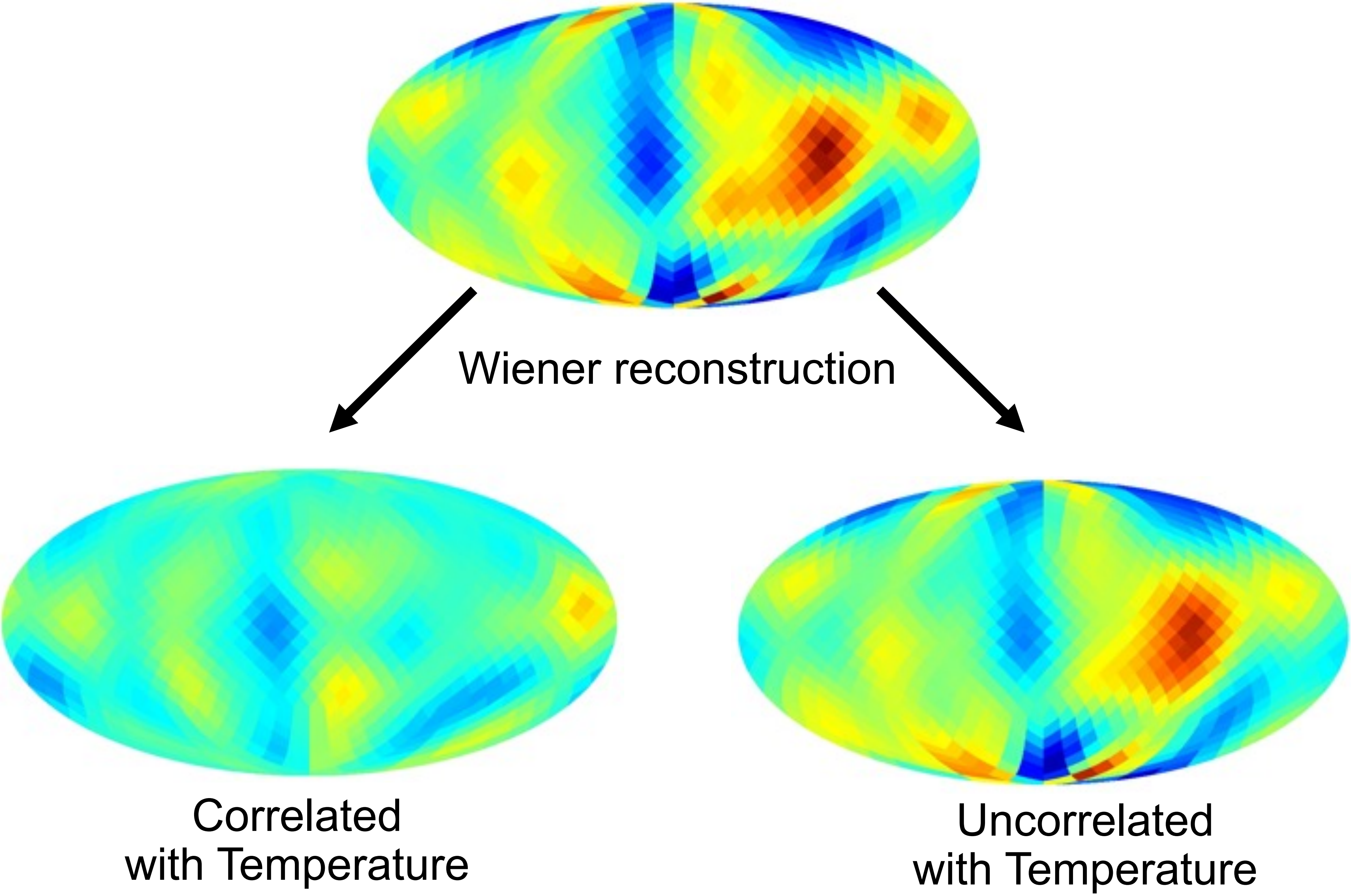
Subtract corr. map from raw maps and then do Wiener filtering:



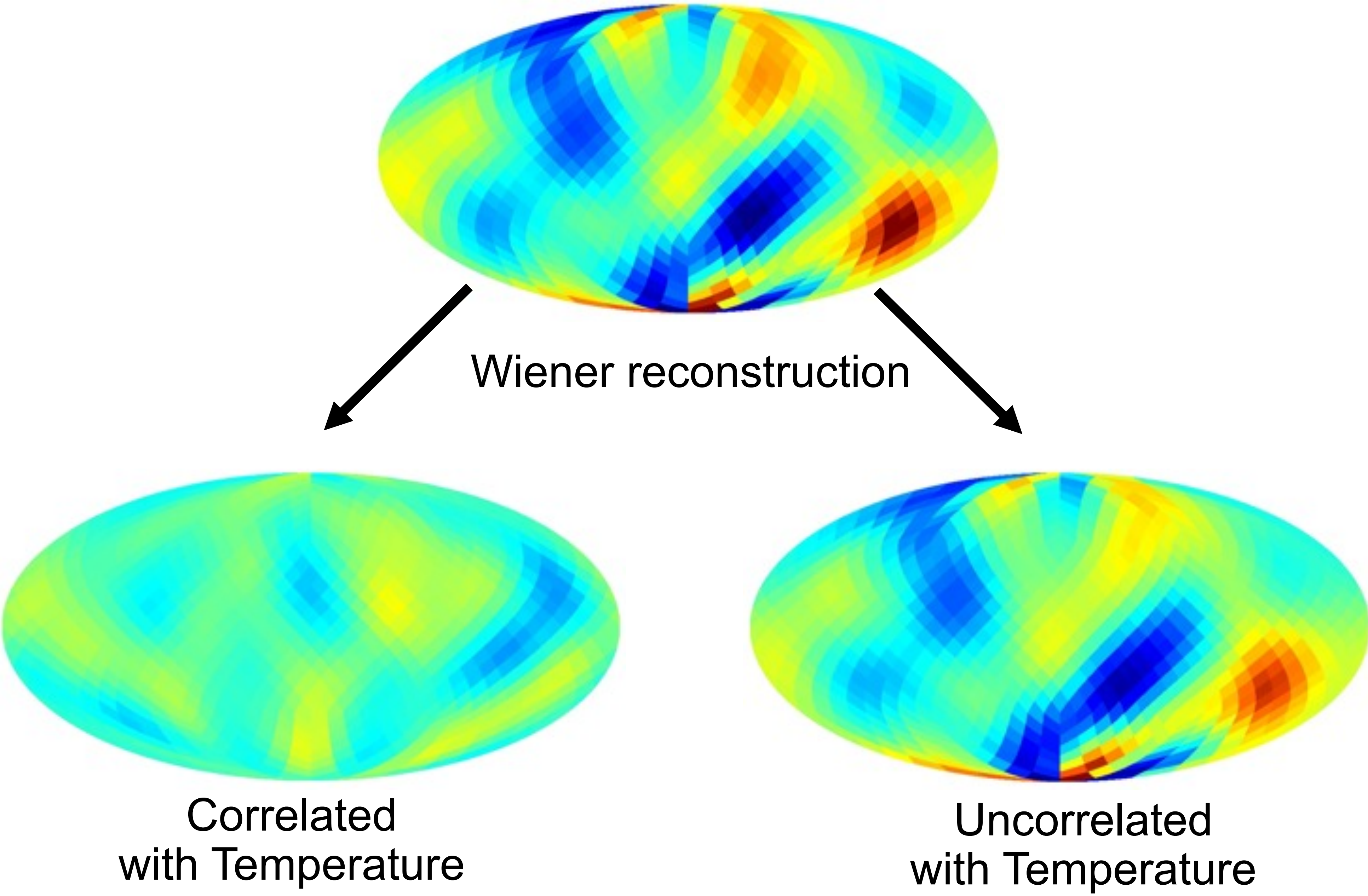
raw Q-map   corr. Q-map   uncorr. Q-map

Can prove that this is uncorrelated with temperature

Q-map splitting



U-map splitting

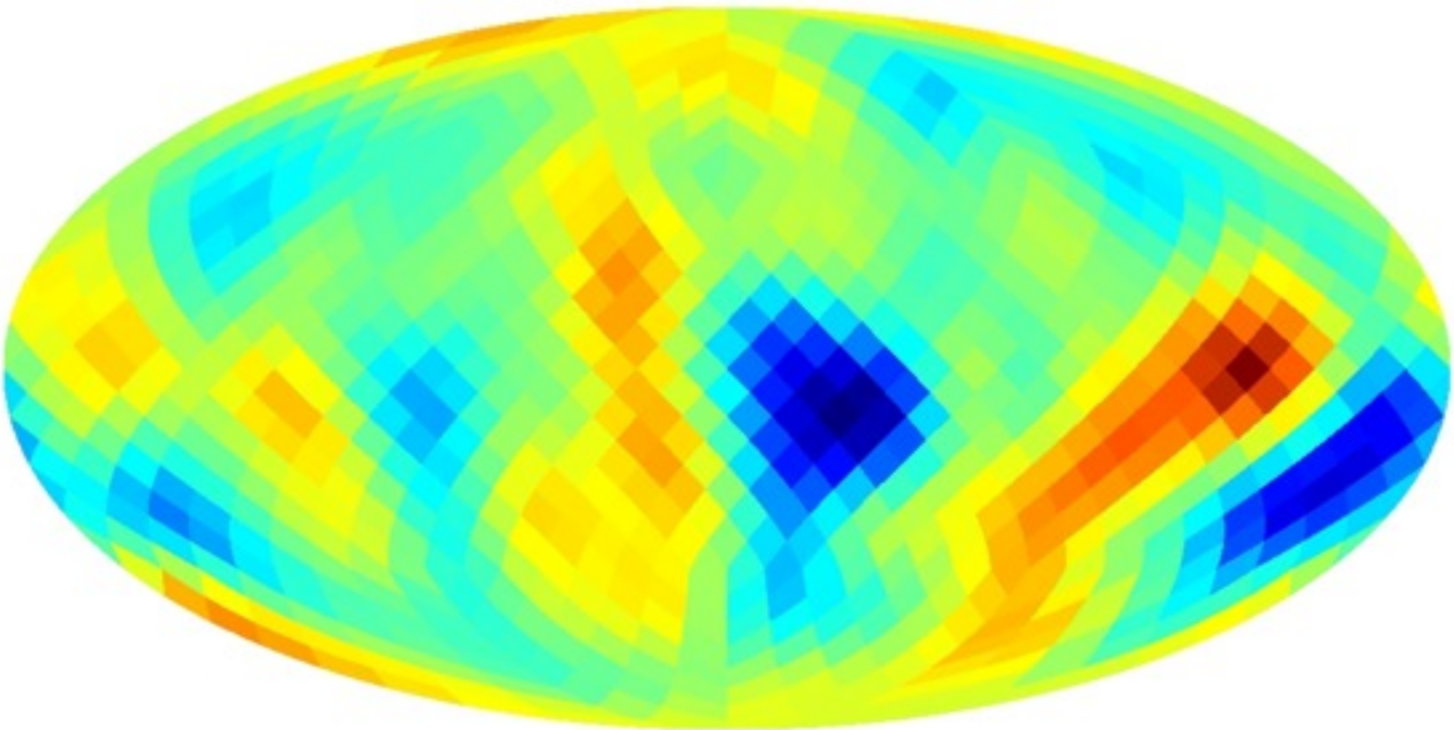


Can in principle use the uncorrelated polarization map to test any anomaly

We focus on the multipole alignment

Multipole alignment

Smoothed temperature map looks strange:

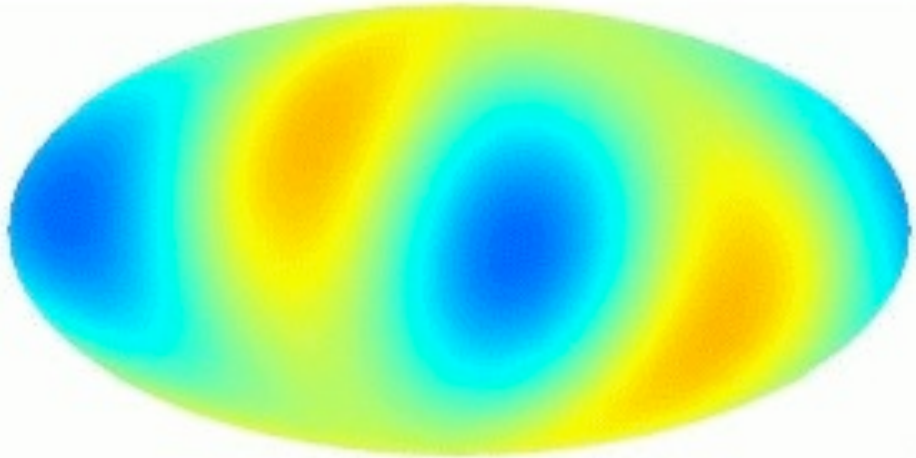


All blobs lie in one plane

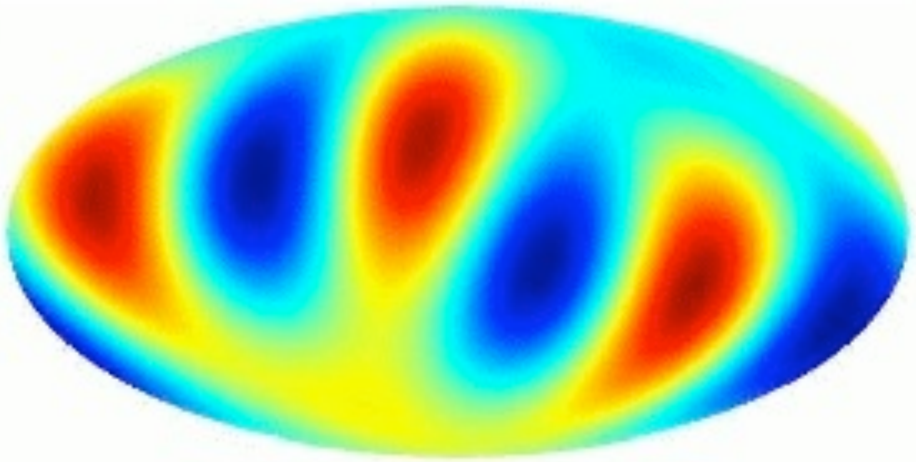
Multipole alignment

Tegmark et al. 2003, de Oliveira-Costa et al, 2004:

quadrupole:



octopole:



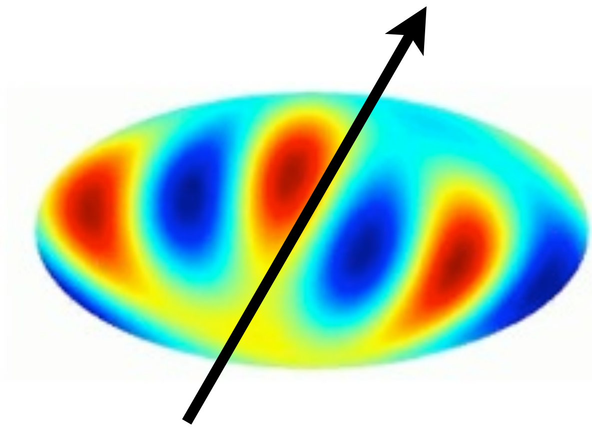
Is there a preferred axis?

Quantify the “preferred axis”:

de Oliveira-Costa et al, 2004:

Look for z-axis $\hat{n}$ around which $\sum_m m^2 |a^T_{\ell m}(\hat{n})|^2$

Is maximized (for given ℓ).



Planar multipoles have most power in high

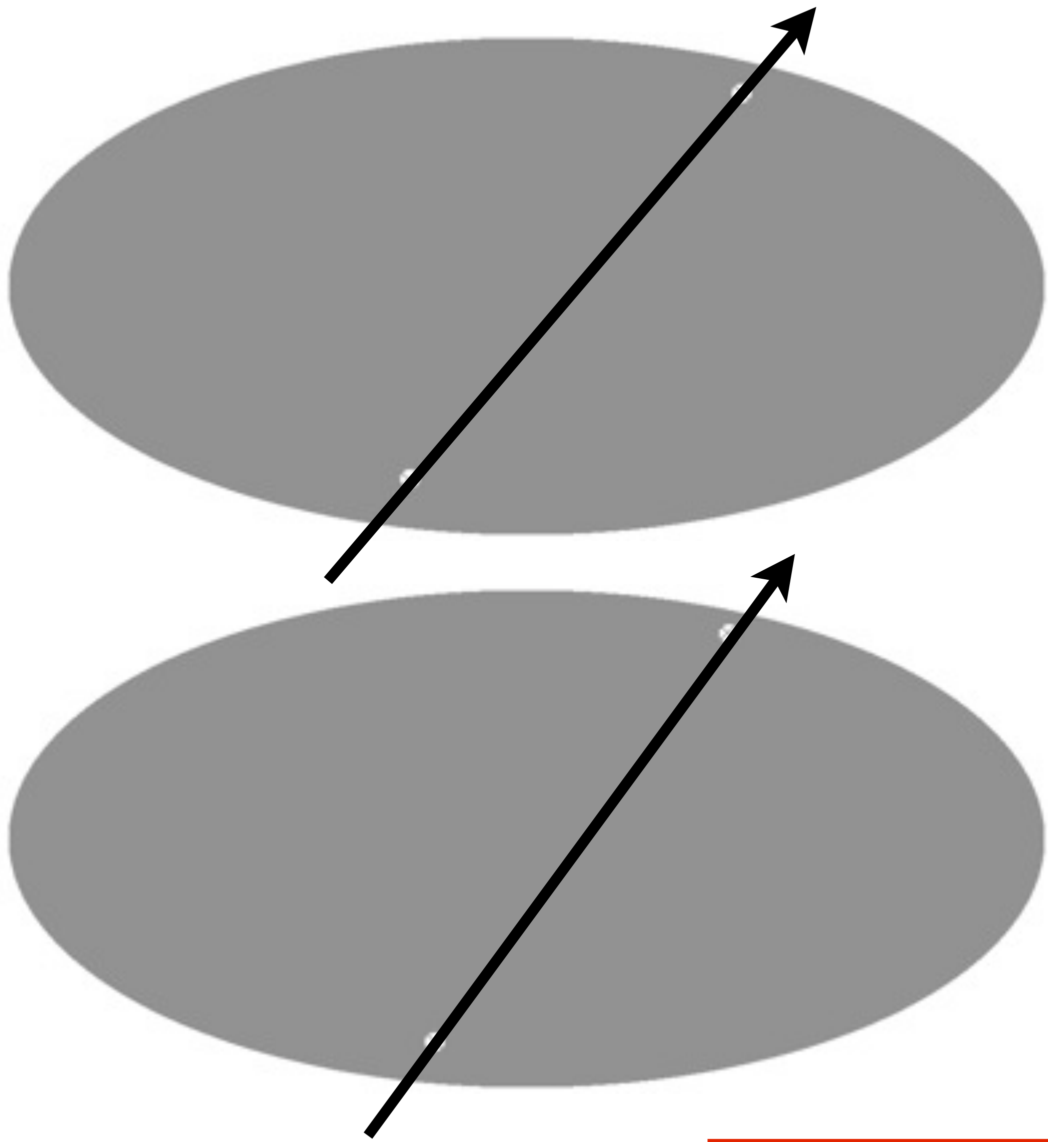
|m|, when the z-direction is ⊥ to plane

⇒ find this z-direction

For WMAP temperature map:

quadrupole:

octopole:

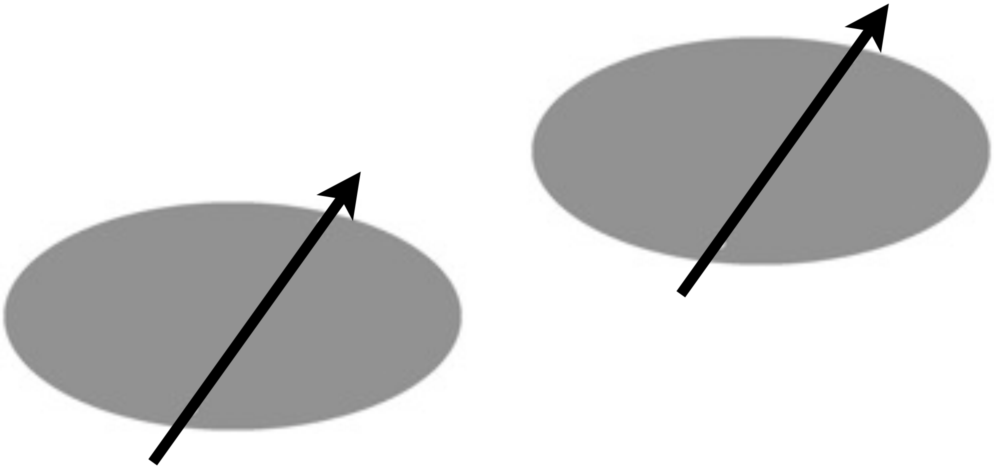


Strong alignment! → The "axis of evil"

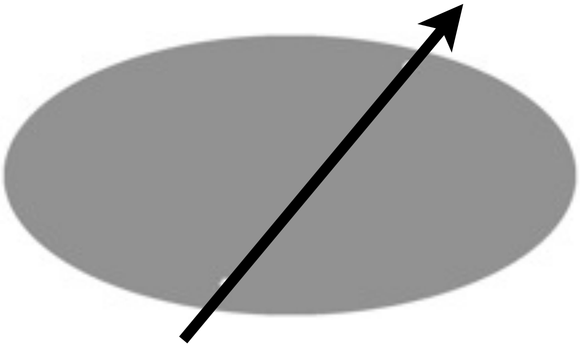
The “axis of evil”

de Oliveira-Costa et al, 2004:

The chance of such an alignment of the preferred axes of quadrupole and octopole in an isotropic universe is ~1%

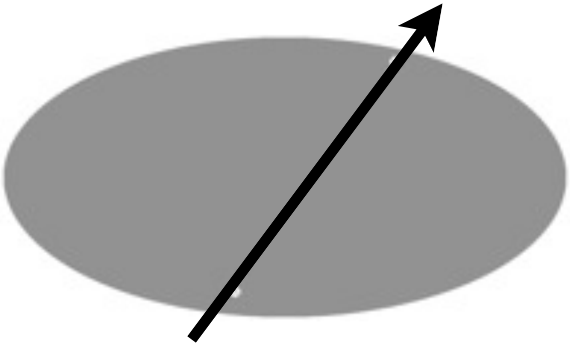


What is the reason for this alignment?



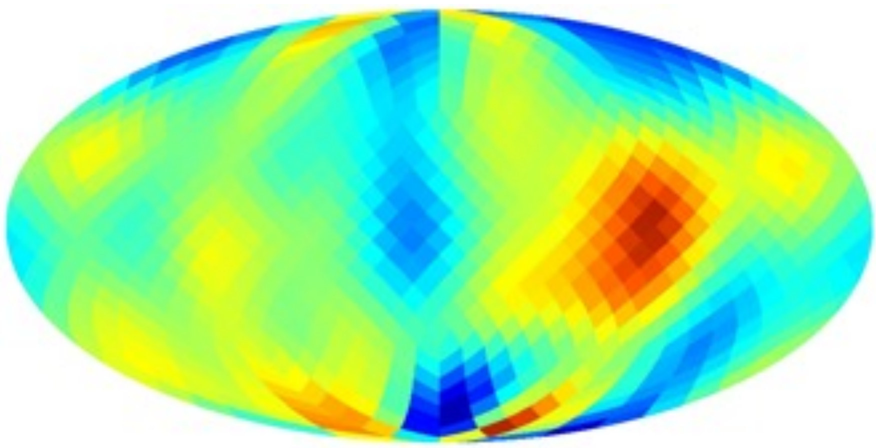
- Chance fluctuations in the temperature?
- Foregrounds?
- Systematics?
- Special direction intrinsic to our Universe?

Use uncorrelated polarization map to try and rule out the chance fluctuations!

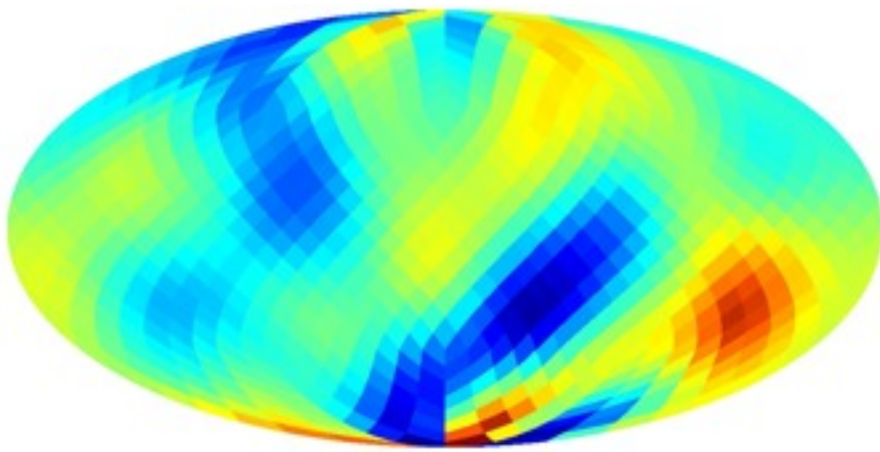


Axes for uncorr. pol. maps

Determine the axes for the uncorrelated polarization maps:



Q-map



U-map

Axes for uncorr. pol. maps

Use the same statistic to determine the multipole orientation:

maximize $\sum_m m^2 |a^E_{\ell m}(\hat{n})|^2$

But: noise, foregrounds and sky mask

⇒ uncertainty in the map

⇒ uncertainty in the axes

⇒ get error on axes from the posterior distribution of the real map around the Wiener estimate

Axes for uncorr. pol. maps

Monte Carlo (MC) simulations to get uncertainty:

Posterior distribution of real uncorr. pol. map around Wiener reconstruction:

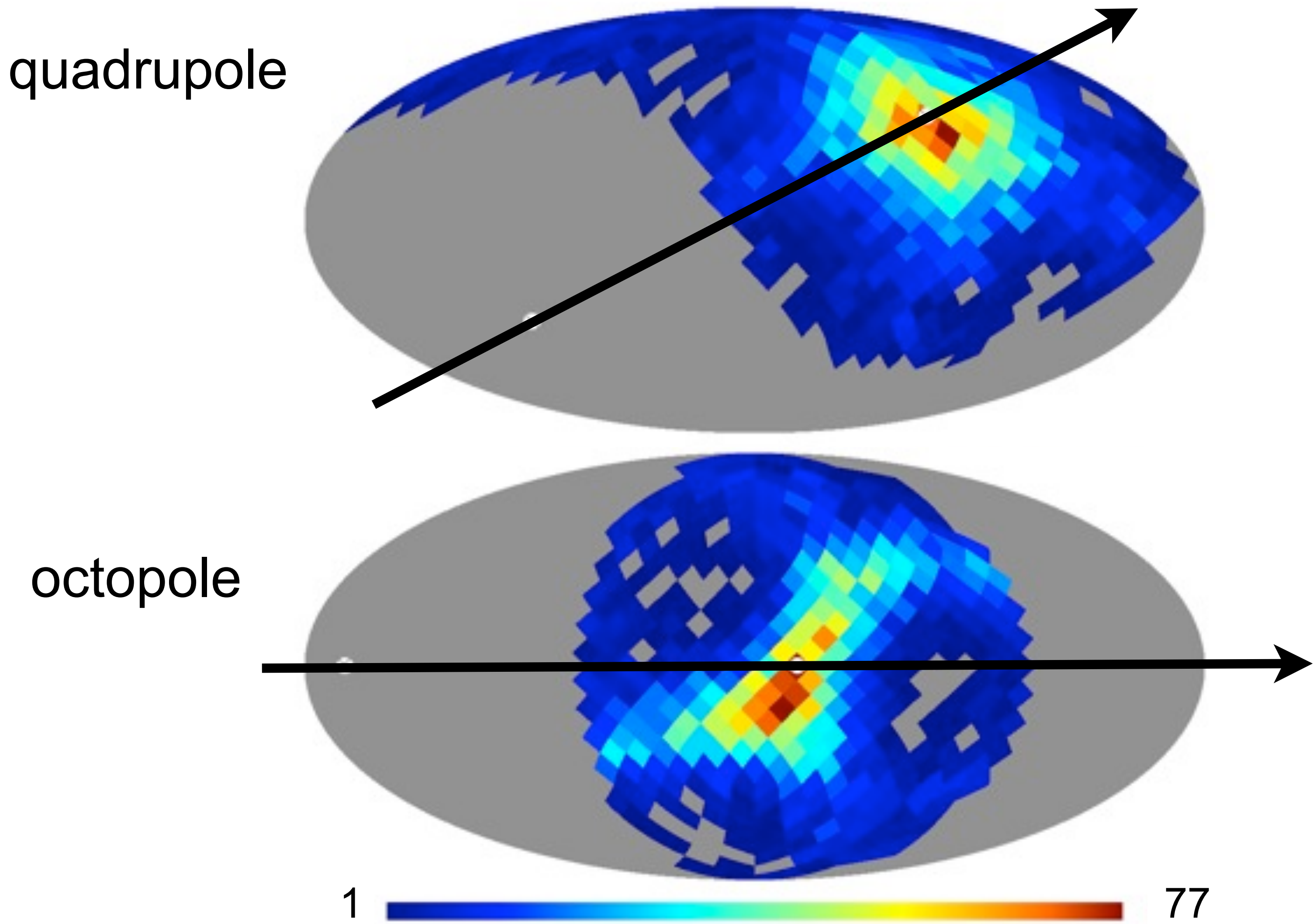
$$P\left(\begin{pmatrix} Q \\ U \end{pmatrix}^{\text{uncorr}} \middle| \begin{pmatrix} Q \\ U \end{pmatrix}^{\text{obs}}, T\right) =$$

$$= \frac{1}{\sqrt{|2\pi C|}} \exp\left\{\frac{1}{2}\left[\begin{pmatrix} Q \\ U \end{pmatrix}^{\text{uncorr}} - \left\langle \begin{pmatrix} Q \\ U \end{pmatrix} \right\rangle^{\text{uncorr}}\right]^{\dagger} C^{-1} \left[\begin{pmatrix} Q \\ U \end{pmatrix}^{\text{uncorr}} - \left\langle \begin{pmatrix} Q \\ U \end{pmatrix} \right\rangle^{\text{uncorr}}\right]\right\}$$

(Can compute C)

Draw samples from this distribution,
determine axis for each sample,
determine variance of “axes-distribution”

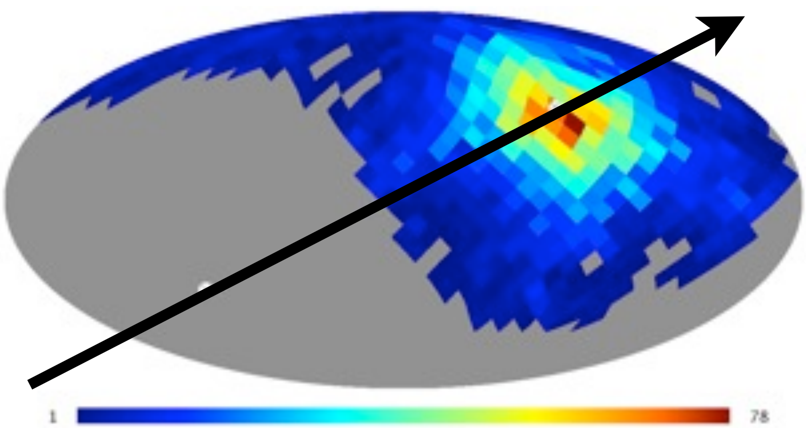
Axes for uncorr. pol. maps

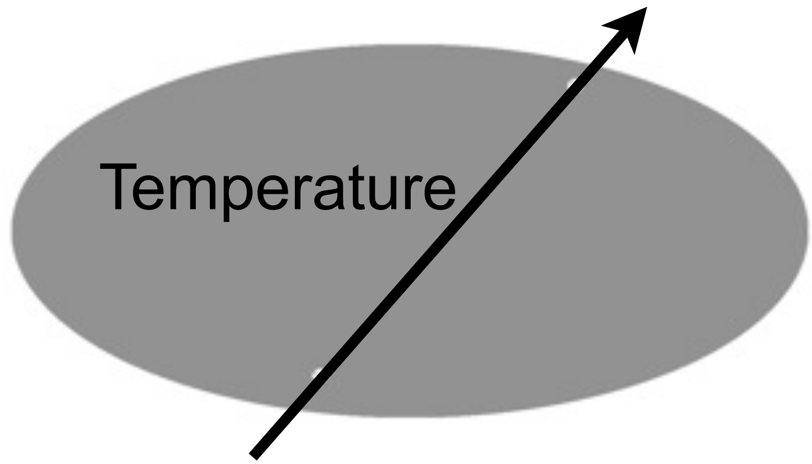


Colors count how many times the axis lies on a given pixel for 5000 MC samples

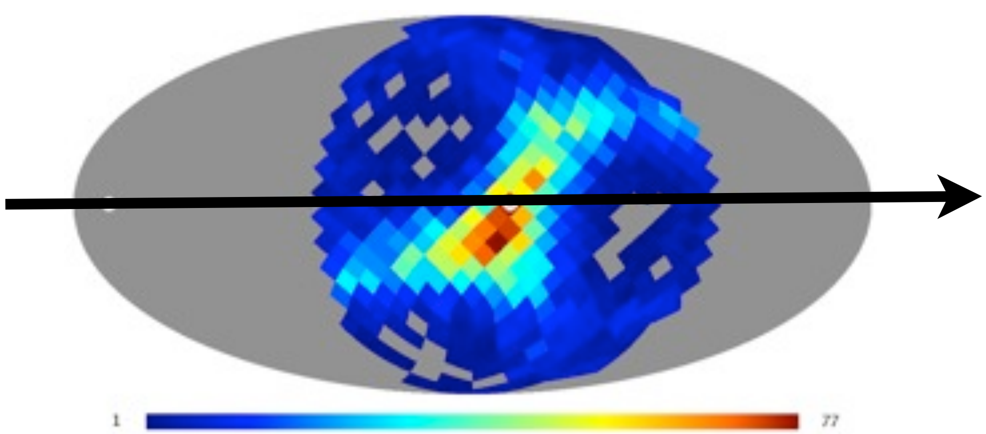
Compare with temperature map

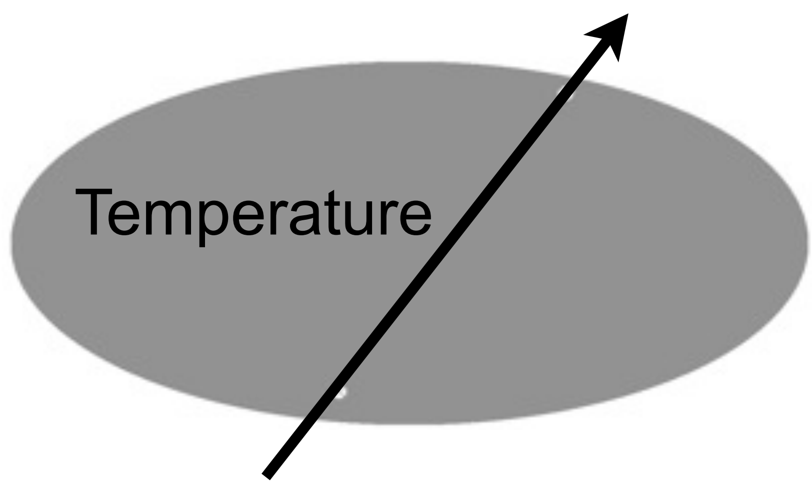
Quadrupole: aligned





Octopole: not aligned





Significance?

Uncertainty in our axes is 43^o !

➡ With WMAP data cannot say anything ☹

PLANCK: uncertainty is 7^o – 20^o (limited by foregrounds)

➡ Will obtain much stronger evidence!

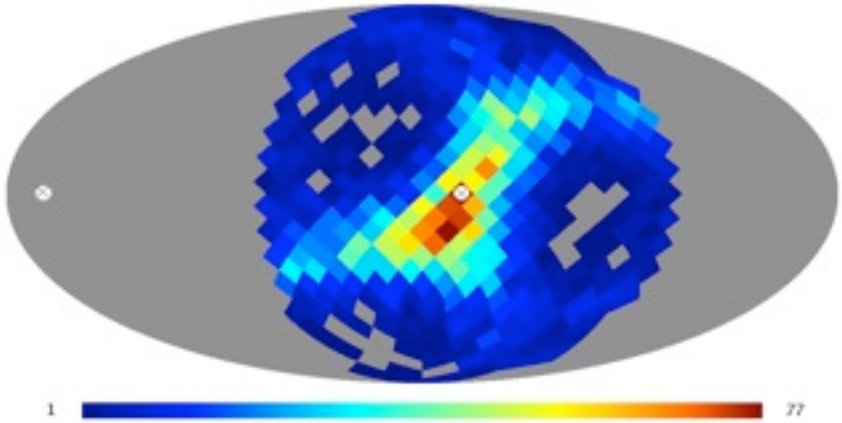
Possible improvements:

- We simply determined the direction of the multipole orientation and compared to the temperature map
- This can be used as a general assessment

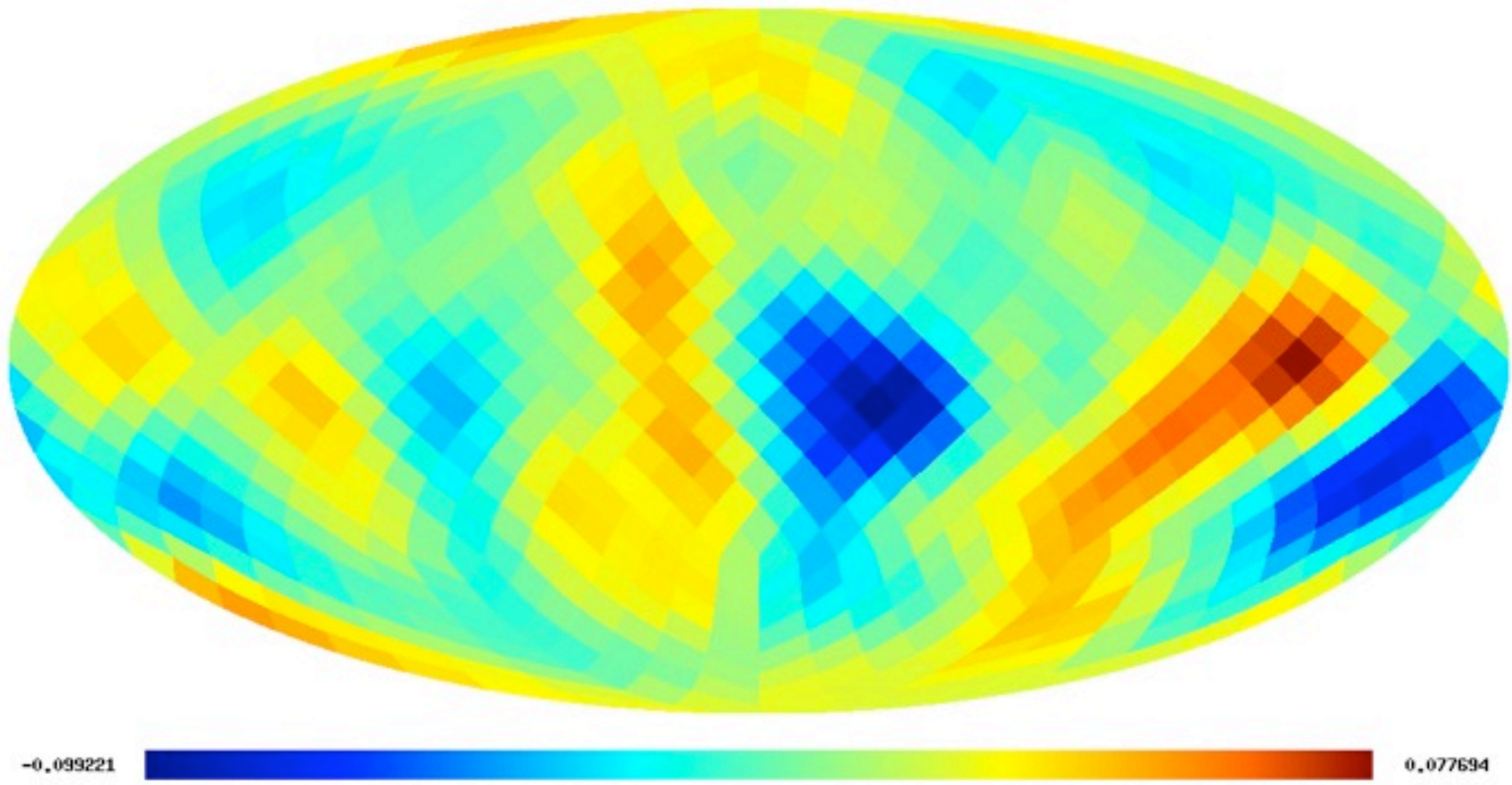
- But in principle one should make predictions for the uncorr. pol. map for a given model and test against observations

Conclusions

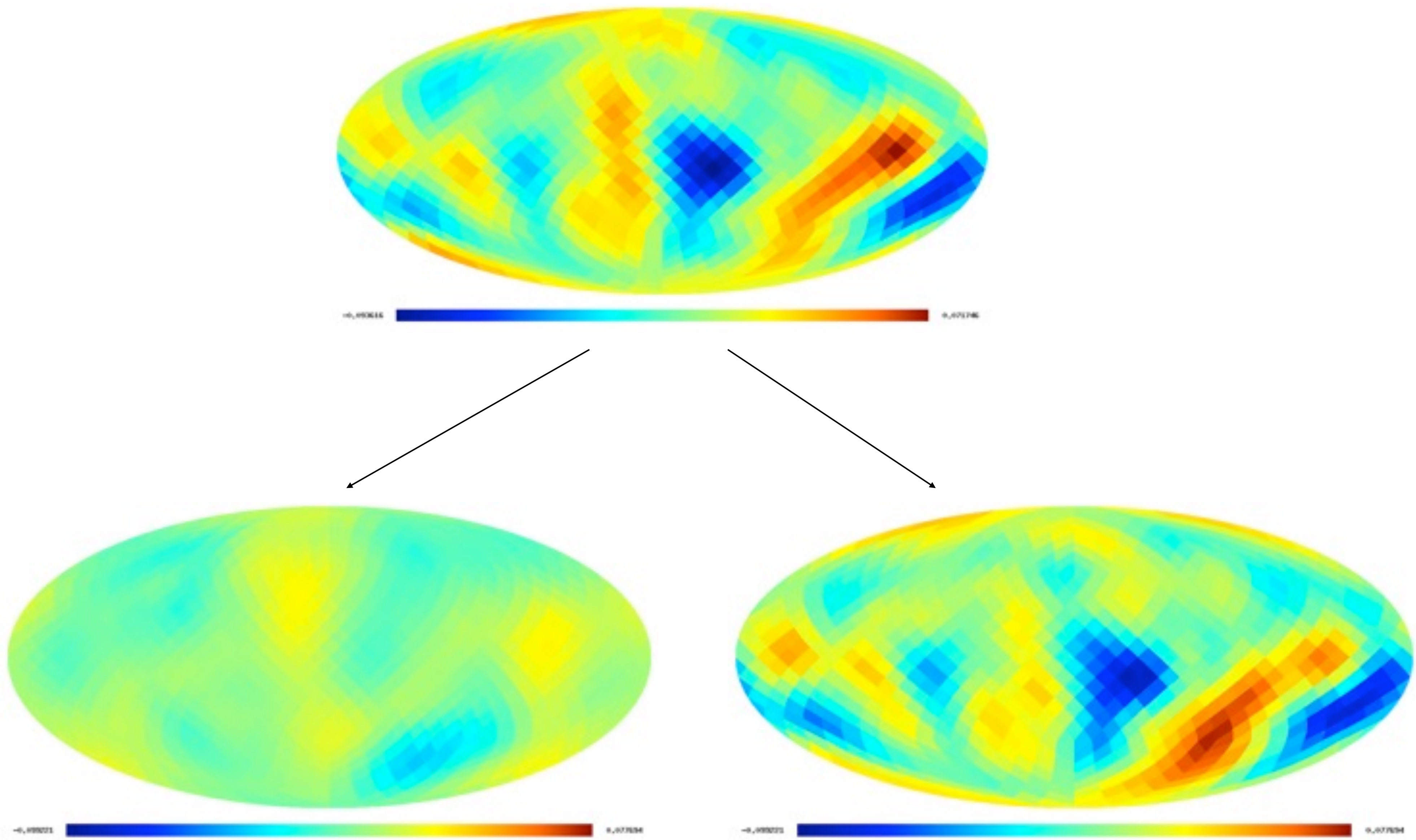
- We extracted a map from the WMAP polarization map, which is uncorrelated with the temperature
- Can use this map to exclude that the anomalies in the temperature are a statistical fluke
- Determined the multipole orientation in this map
- Quadrupole: aligned, octopole: not aligned
- But: WMAP data not good enough ⇒ Planck!
- This is going beyond the a posteriori analysis usually done for CMB anomalies



Thank you!!



Temperature map



Correlated with Q and U

Uncorr. With Q and U

Compute uncorrelated polarization map - in practice

Subtract corr. pol. map from raw polarization maps and then do Wiener filtering:

$$\left\langle \begin{pmatrix} Q \\ U \end{pmatrix} \right\rangle^{\text{uncorr}} = \left[\left(S_P - S_{PT} S_T^{-1} S_{TP} \right)^{-1} + R^\dagger N^{-1} R \right]^{-1} R^\dagger N^{-1} \left[\begin{pmatrix} Q \\ U \end{pmatrix}^{\text{obs}} - R \begin{pmatrix} Q \\ U \end{pmatrix}^{\text{corr}} \right]$$

Can prove that this is uncorrelated with temperature

$$S_P = \langle \begin{pmatrix} Q \\ U \end{pmatrix} \begin{pmatrix} Q \\ U \end{pmatrix}^\dagger \rangle$$

$$S_{PT} = \langle \begin{pmatrix} Q \\ U \end{pmatrix} T^\dagger \rangle$$

$$S_T = \langle T T^\dagger \rangle$$