

The measured galaxy correlation function

Ruth Durrer
Université de Genève
Département de Physique Théorique et Center for Astroparticle Physics



**UNIVERSITÉ
DE GENÈVE**



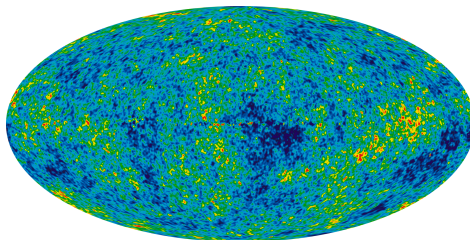
Center for Astroparticle Physics
GENEVA

Benasque Modern Cosmology 2012

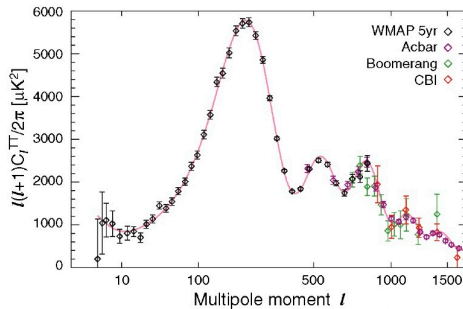
- 1 Introduction
- 2 What are very large scale galaxy catalogs really measuring?
 - Matter fluctuations per redshift bin, volume perturbations
- 3 The angular power spectrum and the correlation function of galaxy density fluctuations
 - The transversal power spectrum
 - The radial power spectrum
- 4 Alcock-Paczyński test
- 5 Conclusions

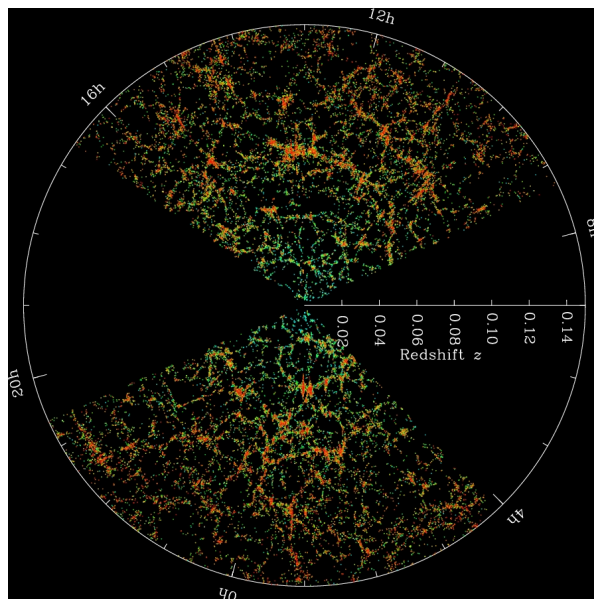
The CMB

WMAP 7 year CMB sky



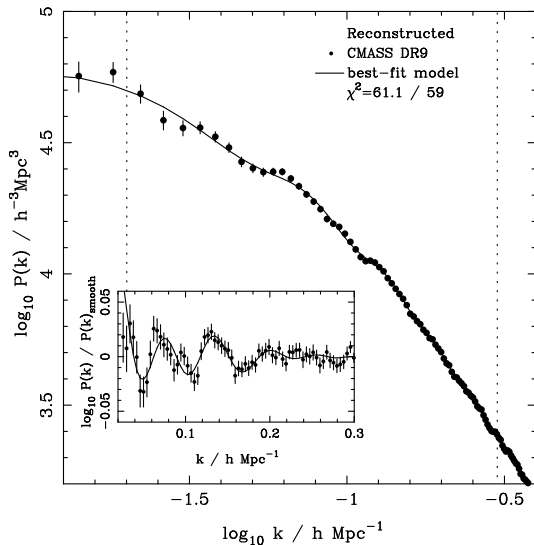
The WMAP Team





M. Blanton and the Sloan Digital Sky Survey Team.

Galaxy power spectrum from the Sloan Digital Sky Survey (BOSS)

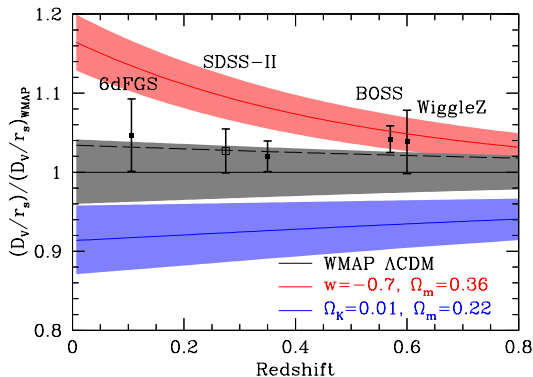


from [Anderson et al. '12](#)

SDSS-III (BOSS)
power spectrum.

Galaxy surveys \simeq
matter density fluctuations,
biasing and redshift space
distortions.

The observed Universe is well approximated by a Λ CDM model,
 $\Omega_\Lambda \simeq 0.72$, $\Omega_m = \Omega_{\text{cdm}} + \Omega_b \simeq 0.28$, $\Omega_b \simeq 0.04$.



from [Anderson et al. '12](#)

- We have to take fully into account that all observations are made on our **past lightcone**, for example, we see density fluctuations which are further away from us, further in the past;

- We have to take fully into account that all observations are made on our **past lightcone**, for example, we see density fluctuations which are further away from us, further in the past;
- the **measured redshift** is not only perturbed by velocities but also by the gravitational potential;

- We have to take fully into account that all observations are made on our **past lightcone**, for example, we see density fluctuations which are further away from us, further in the past;
- the **measured redshift** is not only perturbed by velocities but also by the gravitational potential;
- not only the number of galaxies but also the **volume** is distorted;

- We have to take fully into account that all observations are made on our **past lightcone**, for example, we see density fluctuations which are further away from us, further in the past;
- the **measured redshift** is not only perturbed by velocities but also by the gravitational potential;
- not only the number of galaxies but also the **volume** is distorted;
- the **angles** we are looking into are not the ones into which the photons from a given galaxy arriving at our position have been emitted.

- We have to take fully into account that all observations are made on our **past lightcone**, for example, we see density fluctuations which are further away from us, further in the past;
- the **measured redshift** is not only perturbed by velocities but also by the gravitational potential;
- not only the number of galaxies but also the **volume** is distorted;
- the **angles** we are looking into are not the ones into which the photons from a given galaxy arriving at our position have been emitted.
- For small galaxy catalogs, these effects are not very important, but when we go out to **$z \sim 1$ or more**, they become relevant. Already for SDSS which goes out to $z \simeq 0.2$ (main catalog) or even $z \simeq 0.6$ (BOSS).

- We have to take fully into account that all observations are made on our **past lightcone**, for example, we see density fluctuations which are further away from us, further in the past;
- the **measured redshift** is not only perturbed by velocities but also by the gravitational potential;
- not only the number of galaxies but also the **volume** is distorted;
- the **angles** we are looking into are not the ones into which the photons from a given galaxy arriving at our position have been emitted.
- For small galaxy catalogs, these effects are not very important, but when we go out to **$z \sim 1$ or more**, they become relevant. Already for SDSS which goes out to $z \simeq 0.2$ (main catalog) or even $z \simeq 0.6$ (BOSS).
- But of course much more for **future surveys like DES, bigBOSS and Euclid**.

- We have to take fully into account that all observations are made on our **past lightcone**, for example, we see density fluctuations which are further away from us, further in the past;
- the **measured redshift** is not only perturbed by velocities but also by the gravitational potential;
- not only the number of galaxies but also the **volume** is distorted;
- the **angles** we are looking into are not the ones into which the photons from a given galaxy arriving at our position have been emitted.
- For small galaxy catalogs, these effects are not very important, but when we go out to **$z \sim 1$ or more**, they become relevant. Already for SDSS which goes out to $z \simeq 0.2$ (main catalog) or even $z \simeq 0.6$ (BOSS).
- But of course much more for **future surveys like DES, bigBOSS and Euclid**.
- **Whenever we convert a measured redshift or angle into a length scale, we make an assumption about the underlying cosmology.**

What are very large scale galaxy catalogs really measuring?

Following [C. Bonvin & RD \[arXiv:1105.5080\]](#); [F. Montanari & RD \[arXiv:1206.3545\]](#)
see also [Challinor & Lewis, \[arXiv:1105:5092\]](#).

Relativistic corrections to galaxy surveys are also discussed in:
[J. Yoo et al. 2009](#); [J. Yoo 2010](#)

What are very large scale galaxy catalogs really measuring?

Following [C. Bonvin & RD \[arXiv:1105.5080\]](#); [F. Montanari & RD \[arXiv:1206.3545\]](#)
see also [Challinor & Lewis, \[arXiv:1105:5092\]](#).

Relativistic corrections to galaxy surveys are also discussed in:
[J. Yoo et al. 2009](#); [J. Yoo 2010](#)

For each galaxy in a catalog we measure

$$(z, \theta, \phi) = (z, \mathbf{n}) \quad + \text{info about mass, spectral type...}$$

What are very large scale galaxy catalogs really measuring?

Following [C. Bonvin & RD \[arXiv:1105.5080\]](#); [F. Montanari & RD \[arXiv:1206.3545\]](#)
see also [Challinor & Lewis, \[arXiv:1105:5092\]](#).

Relativistic corrections to galaxy surveys are also discussed in:
[J. Yoo et al. 2009](#); [J. Yoo 2010](#)

For each galaxy in a catalog we measure

$$(z, \theta, \phi) = (z, \mathbf{n}) \quad + \text{info about mass, spectral type...}$$

We can count the galaxies inside a redshift bin and small solid angle, $N(z, \mathbf{n})$ and measure the fluctuation of this count:

$$\Delta(z, \mathbf{n}) = \frac{N(z, \mathbf{n}) - \bar{N}(z)}{\bar{N}(z)}.$$

What are very large scale galaxy catalogs really measuring?

Following [C. Bonvin & RD \[arXiv:1105.5080\]](#); [F. Montanari & RD \[arXiv:1206.3545\]](#)
see also [Challinor & Lewis, \[arXiv:1105:5092\]](#).

Relativistic corrections to galaxy surveys are also discussed in:
[J. Yoo et al. 2009](#); [J. Yoo 2010](#)

For each galaxy in a catalog we measure

$$(z, \theta, \phi) = (z, \mathbf{n}) \quad + \text{info about mass, spectral type...}$$

We can count the galaxies inside a redshift bin and small solid angle, $N(z, \mathbf{n})$ and measure the fluctuation of this count:

$$\Delta(z, \mathbf{n}) = \frac{N(z, \mathbf{n}) - \bar{N}(z)}{\bar{N}(z)}.$$

This quantity is directly measurable \Rightarrow gauge invariant.

What are very large scale galaxy catalogs really measuring?

Density fluctuation per redshift bin dz and per solid angle $d\Omega$ as $\delta_z(\mathbf{n}, z)$.

$$\delta_z(\mathbf{n}, z) = \frac{\rho(\mathbf{n}, z) - \bar{\rho}(z)}{\bar{\rho}(z)} = \frac{\frac{N(\mathbf{n}, z)}{V(\mathbf{n}, z)} - \frac{\bar{N}(z)}{V(z)}}{\frac{\bar{N}(z)}{V(z)}}$$

What are very large scale galaxy catalogs really measuring?

Density fluctuation per redshift bin dz and per solid angle $d\Omega$ as $\delta_z(\mathbf{n}, z)$.

$$\delta_z(\mathbf{n}, z) = \frac{\rho(\mathbf{n}, z) - \bar{\rho}(z)}{\bar{\rho}(z)} = \frac{\frac{N(\mathbf{n}, z)}{V(\mathbf{n}, z)} - \frac{\bar{N}(z)}{V(z)}}{\frac{\bar{N}(z)}{V(z)}}$$

This together with the volume fluctuations, results in the directly observed number fluctuations

$$\Delta(\mathbf{n}, z) = \delta_z(\mathbf{n}, z) + \frac{\delta V(\mathbf{n}, z)}{V(z)}$$

What are very large scale galaxy catalogs really measuring?

Density fluctuation per redshift bin dz and per solid angle $d\Omega$ as $\delta_z(\mathbf{n}, z)$.

$$\delta_z(\mathbf{n}, z) = \frac{\rho(\mathbf{n}, z) - \bar{\rho}(z)}{\bar{\rho}(z)} = \frac{\frac{N(\mathbf{n}, z)}{V(\mathbf{n}, z)} - \frac{\bar{N}(z)}{V(z)}}{\frac{\bar{N}(z)}{V(z)}}$$

This together with the volume fluctuations, results in the directly observed number fluctuations

$$\Delta(\mathbf{n}, z) = \delta_z(\mathbf{n}, z) + \frac{\delta V(\mathbf{n}, z)}{V(z)}$$

Both these terms are in principle measurable and therefore gauge invariant. We want to express them in terms of standard gauge invariant perturbation variables.

We consider a photon emitted from a galaxy (S), moving in direction \mathbf{n} into our telescope. The observer (O) receives the photon redshifted by a factor

$$1 + z = \frac{(\mathbf{n} \cdot \mathbf{u})_S}{(\mathbf{n} \cdot \mathbf{u})_O} .$$

We consider a photon emitted from a galaxy (S), moving in direction \mathbf{n} into our telescope. The observer (O) receives the photon redshifted by a factor

$$1 + z = \frac{(n \cdot u)_S}{(n \cdot u)_O}.$$

To first order in perturbation theory one finds (in longitudinal gauge)

$$\frac{\delta z}{(1+z)} = - \left[(\mathbf{n} \cdot \mathbf{V} + \Psi)(\mathbf{n}, z) + \int_0^{\chi(z)} (\Phi + \dot{\Psi}) d\chi \right]$$

We consider a photon emitted from a galaxy (S), moving in direction \mathbf{n} into our telescope. The observer (O) receives the photon redshifted by a factor

$$1 + z = \frac{(\mathbf{n} \cdot \mathbf{u})_S}{(\mathbf{n} \cdot \mathbf{u})_O}.$$

To first order in perturbation theory one finds (in longitudinal gauge)

$$\frac{\delta z}{(1+z)} = - \left[(\mathbf{n} \cdot \mathbf{V} + \Psi)(\mathbf{n}, z) + \int_0^{\chi(z)} (\dot{\Phi} + \dot{\Psi}) d\chi \right]$$

With this, the density fluctuation in redshift space becomes

$$\delta_z(\mathbf{n}, z) = D_g(\mathbf{n}, z) + 3(\mathbf{V} \cdot \mathbf{n})(\mathbf{n}, z) + 3(\Psi + \Phi)(\mathbf{n}, z) + 3 \int_0^{\chi_S} (\dot{\Psi} + \dot{\Phi})(\mathbf{n}, z(\chi)) d\chi$$

This quantity is gauge invariant and therefore, in principle, measurable. But when we count numbers of galaxies per solid bangle and per redshift bin, we also have to consider volume perturbations.

The total galaxy density fluctuation per redshift bin, per solid angle

Putting the density and volume fluctuations together one obtains the galaxy number density fluctuations

$$\begin{aligned}\Delta(\mathbf{n}, z) = & D_g + \Phi + \Psi + \frac{1}{\mathcal{H}} \left[\dot{\Phi} + \partial_\chi(\mathbf{V} \cdot \mathbf{n}) \right] \\ & + \left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{2}{\chi(z)\mathcal{H}} \right) \left(\Psi + \mathbf{V} \cdot \mathbf{n} + \int_0^{\chi(z)} d\chi (\dot{\Phi} + \dot{\Psi}) \right) \\ & + \frac{1}{\chi(z)} \int_0^{\chi(z)} d\chi \left[2 - \frac{\chi(z) - \chi}{\chi} \Delta_\Omega \right] (\Phi + \Psi).\end{aligned}$$

(C. Bonvin & RD '11)

The total galaxy density fluctuation per redshift bin, per solid angle

Putting the density and volume fluctuations together one obtains the galaxy number density fluctuations

$$\begin{aligned}\Delta(\mathbf{n}, z) = & D_g + \Phi + \Psi + \frac{1}{\mathcal{H}} \left[\dot{\Phi} + \partial_\chi(\mathbf{V} \cdot \mathbf{n}) \right] \\ & + \left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{2}{\chi(z)\mathcal{H}} \right) \left(\Psi + \mathbf{V} \cdot \mathbf{n} + \int_0^{\chi(z)} d\chi (\dot{\Phi} + \dot{\Psi}) \right) \\ & + \frac{1}{\chi_S} \int_0^{\chi(z)} d\chi \left[2(\Phi + \Psi) - \frac{\chi(z) - \chi}{\chi} \Delta_\Omega(\Phi + \Psi) \right].\end{aligned}$$

(C. Bonvin & RD '11)

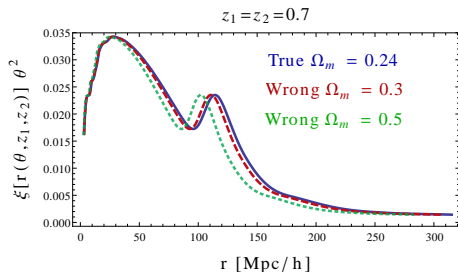
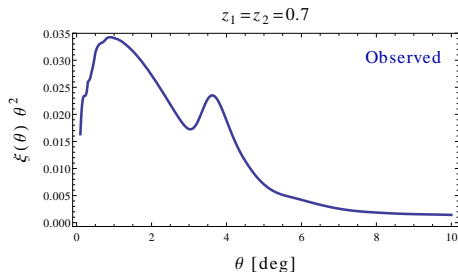
For fixed z , we can expand $\Delta(\mathbf{n}, z)$ in spherical harmonics,

$$\Delta(\mathbf{n}, z) = \sum_{\ell m} a_{\ell m}(z) Y_{\ell m}(\mathbf{n}), \quad C_{\ell}(z, z') = \langle a_{\ell m}(z) a_{\ell m}^*(z') \rangle.$$

$$\xi(\theta, z, z') = \langle \Delta(\mathbf{n}, z) \Delta(\mathbf{n}', z') \rangle = \frac{1}{4\pi} \sum_{\ell} (2\ell + 1) C_{\ell}(z, z') P_{\ell}(\cos \theta)$$

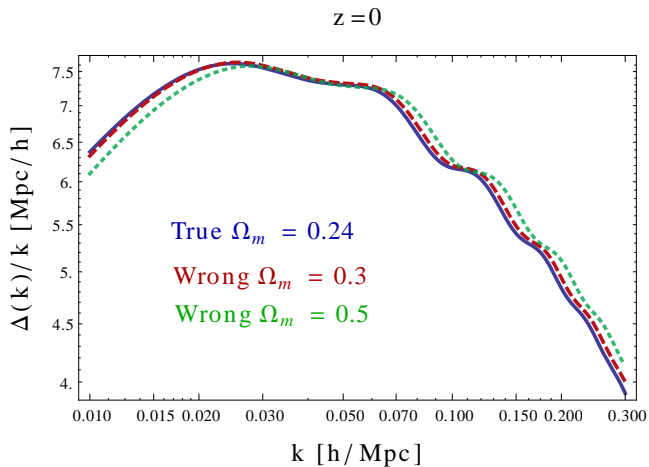
What are very large scale galaxy catalogs really measuring?

If we convert the measured $\xi(\theta, z, z')$ to a power spectrum, we have to introduce a cosmology, to convert angles and redshifts into length scales. $r(z, z', \theta) = \sqrt{\chi^2(z) + \chi^2(z') - 2\chi(z)\chi(z') \cos \theta}$.



(Figure by F. Montanari)

What are very large scale galaxy catalogs really measuring?

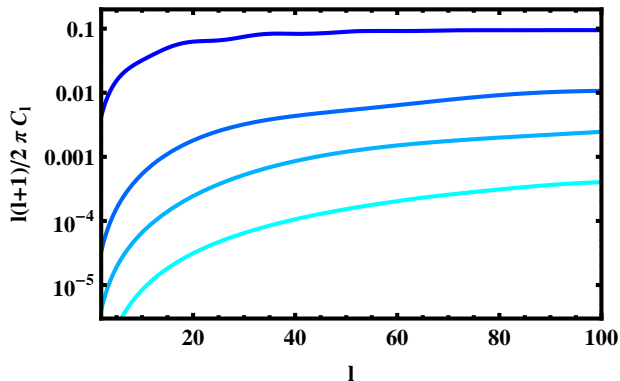


(Figure by F. Montanari)

$$\Delta(k)/k = k^2 P(k)$$

The transversal power spectrum

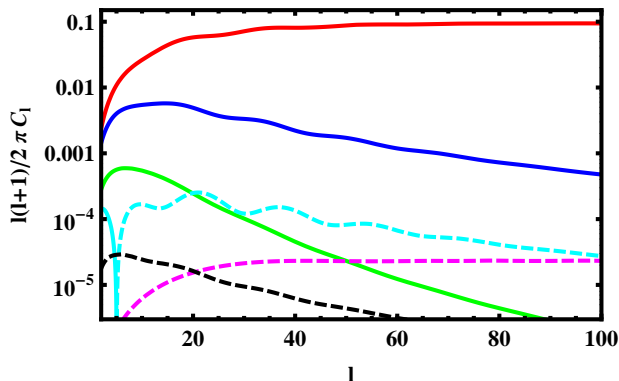
The transverse power spectrum, $z' = z$ (from [Bonvin & RD '11](#))



From top to bottom $z = 0.1$, $z = 0.5$, $z = 1$ and $z = 3$.

The transversal power spectrum

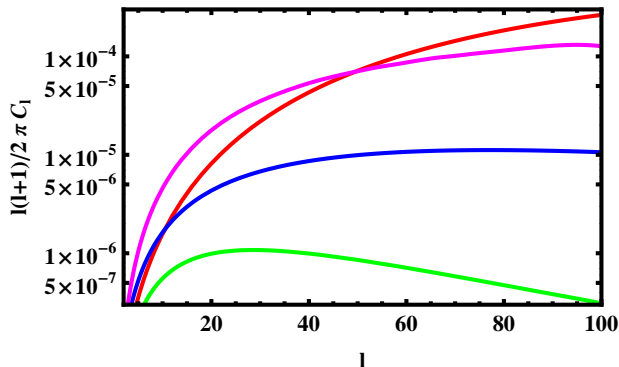
Contributions to the transverse power spectrum at redshift $z = 0.1$, $\Delta z = 0.01$
(from Bonvin & RD '11)



C_ℓ^{DD} (red), C_ℓ^{zz} (green), $2C_\ell^{Dz}$ (blue), $C_\ell^{Doppler}$ (cyan), $C_\ell^{lensing}$ (magenta), C_ℓ^{grav} (black).

The transversal power spectrum

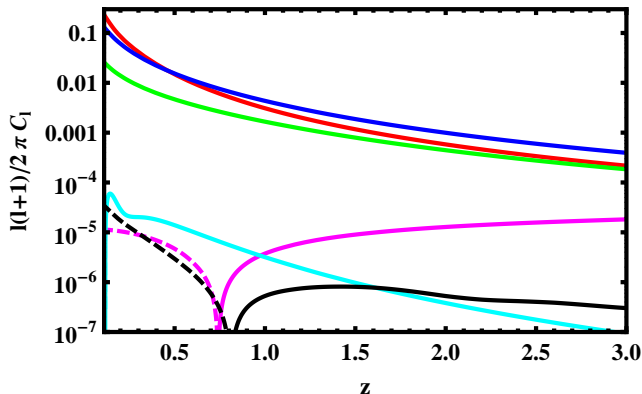
Contributions to the transverse power spectrum at redshift $z = 3$, $\Delta z = 0.3$
(from [Bonvin & RD '11](#))



C_l^{DD} (red), C_l^{ZZ} (green), $2C_l^{Dz}$ (blue), C_l^{lensing} (magenta).

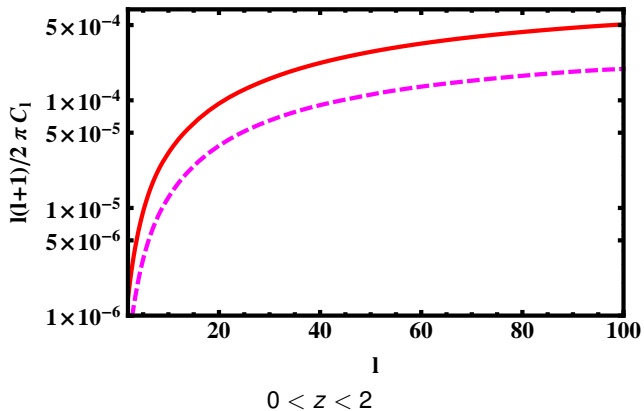
The transversal power spectrum

Contributions to the transversal power spectrum as function of the redshift, $\ell = 20$, $\Delta z = 0$ (from Bonvin & RD '11)



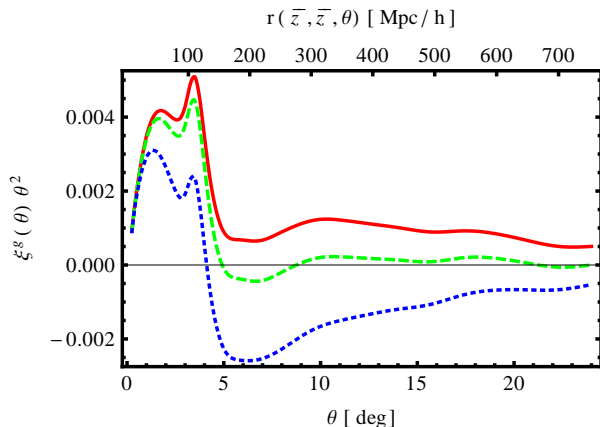
C_ℓ^{DD} (red), C_ℓ^{ZZ} (green), $2C_\ell^{Dz}$ (blue), C_ℓ^{lensing} (magenta), C_ℓ^{Doppler} (cyan),
 C_ℓ^{grav} (black).

The transversal power spectrum



C_l^{DD} (red), $C_l^{lensing}$ (magenta).

The transversal correlation function



(from
Montanari & RD '12)

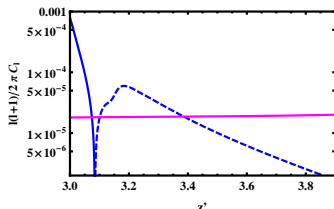
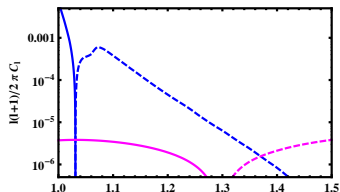
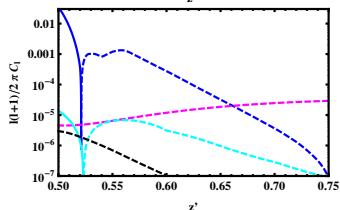
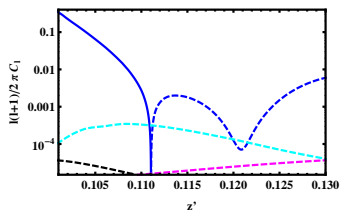
$\theta^2 \xi(\theta, z, z)$

blue C_ℓ^{DD} (real space),

green flat space approximation for redshift space distortions,

red C_ℓ^{DD} , C_ℓ^{ZZ} and $2C_\ell^{DZ}$ (fully positive!).

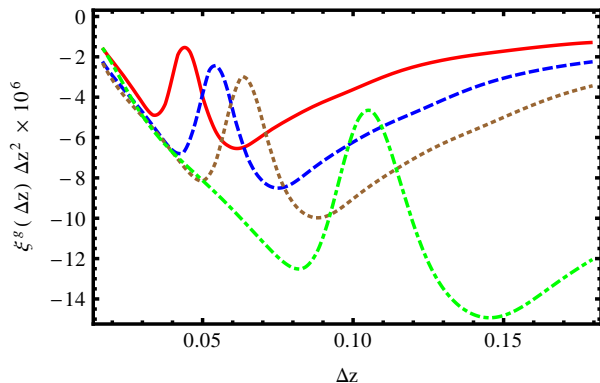
The radial power spectrum



The radial power spectrum $C_\ell(z, z')$ for $\ell = 20$
 Left, top to bottom: $z = 0.1, 0.5, 1$,
 top right: $z = 3$

Standard terms (blue), $C_\ell^{lensing}$ (magenta),
 $C_\ell^{Doppler}$ (cyan), C_ℓ^{grav} (black),

The radial correlation function



$$(\Delta z)^2 \xi(\theta, z - \Delta z/2, z + \Delta z/2)$$

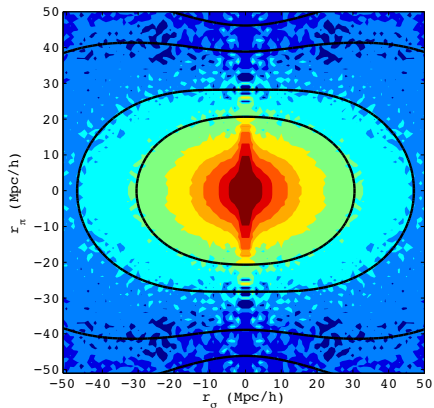
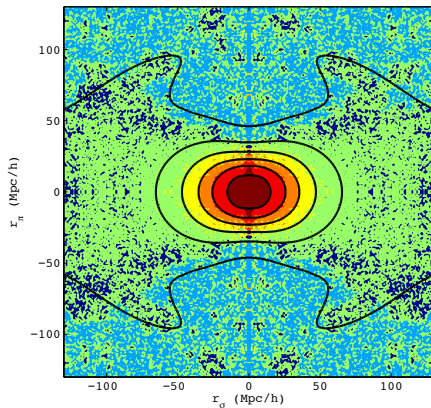
Purely negative for $\Delta z \gtrsim 0.01$.

(from
Montanari & RD '12)

$z = 2,$
 $z = 1,$
 $z = 0.7,$
 $z = 0.3.$

Anisotropic clustering as seen in the BOSS survey

(from Reid et al. '12)



Example: Alcock-Paczyński test

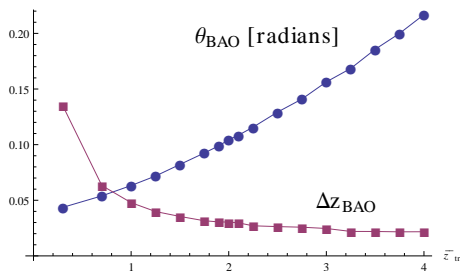
(Alcock & Paczyński '79)

Consider a comoving scale L in the sky.

Horizontally it is projected to the angle $\theta_L = \frac{L}{(1+z)D_A(z)}$.

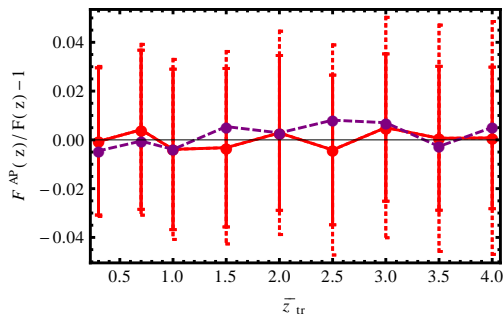
Radially its ends are at a slightly different redshifts, $\Delta z_L = LH(z)$.

$$\frac{\Delta z_L}{\theta_L} = (1+z)D_A(z)H(z) = F(z) \equiv \int_0^z \frac{H(z)}{H(z')} dz'$$



Example: Alcock-Paczyński test

$F(z)^{AP} \equiv \Delta z_L / \theta_L$ measured from the theoretical power spectrum (with Euclid-like redshift accuracies) $F(z) \equiv \int_0^z \frac{H(z')}{H(z)} dz'$.



solid errors:
angular resolution 0.02°
dashed errors:
angular resolution 0.05°

(from [Montanari & RD '12](#))

- So far cosmological LSS data mainly determined $\xi(r)$, or equivalently $P(k)$. These 1d functions are easier to measure (less noisy) but they require an input cosmology converting redshift and angles to length scales,

$$r = \sqrt{\chi(z)^2 + \chi(z')^2 - 2\chi(z)\chi(z') \cos \theta} .$$

- So far cosmological LSS data mainly determined $\xi(r)$, or equivalently $P(k)$. These 1d functions are easier to measure (less noisy) but they require an input cosmology converting redshift and angles to length scales,

$$r = \sqrt{\chi(z)^2 + \chi(z')^2 - 2\chi(z)\chi(z') \cos \theta} .$$

- But future large & precise 3d galaxy catalogs like **Euclid** will be able to determine directly the measured 3d correlation function $\xi(\theta, z, z')$ and $C_\ell(z, z')$ from the data.

- So far cosmological LSS data mainly determined $\xi(r)$, or equivalently $P(k)$. These 1d functions are easier to measure (less noisy) but they require an input cosmology converting redshift and angles to length scales,

$$r = \sqrt{\chi(z)^2 + \chi(z')^2 - 2\chi(z)\chi(z') \cos \theta} .$$

- But future large & precise 3d galaxy catalogs like **Euclid** will be able to determine directly the measured 3d correlation function $\xi(\theta, z, z')$ and $C_\ell(z, z')$ from the data.
- These 3d quantities will of course be more noisy, but they also contain more information.

- So far cosmological LSS data mainly determined $\xi(r)$, or equivalently $P(k)$. These 1d functions are easier to measure (less noisy) but they require an input cosmology converting redshift and angles to length scales,

$$r = \sqrt{\chi(z)^2 + \chi(z')^2 - 2\chi(z)\chi(z') \cos \theta} .$$

- But future large & precise 3d galaxy catalogs like **Euclid** will be able to determine directly the measured 3d correlation function $\xi(\theta, z, z')$ and $C_\ell(z, z')$ from the data.
- These 3d quantities will of course be more noisy, but they also contain more information.
- These spectra are not only sensitive to the matter distribution (**density**) but also to the velocity via (**redshift space distortions**) and to the perturbations of spacetime geometry (**lensing**) .

- So far cosmological LSS data mainly determined $\xi(r)$, or equivalently $P(k)$. These 1d functions are easier to measure (less noisy) but they require an input cosmology converting redshift and angles to length scales,

$$r = \sqrt{\chi(z)^2 + \chi(z')^2 - 2\chi(z)\chi(z') \cos \theta} .$$

- But future large & precise 3d galaxy catalogs like **Euclid** will be able to determine directly the measured 3d correlation function $\xi(\theta, z, z')$ and $C_\ell(z, z')$ from the data.
- These 3d quantities will of course be more noisy, but they also contain more information.
- These spectra are not only sensitive to the matter distribution (**density**) but also to the velocity via (**redshift space distortions**) and to the perturbations of spacetime geometry (**lensing**) .
- The spectra depend sensitively and in several different ways on dark energy (growth factor, distance redshift relation), on the matter and baryon densities, bias, etc. Their measurements provide a new route to estimate cosmological parameters.

- So far cosmological LSS data mainly determined $\xi(r)$, or equivalently $P(k)$. These 1d functions are easier to measure (less noisy) but they require an input cosmology converting redshift and angles to length scales,

$$r = \sqrt{\chi(z)^2 + \chi(z')^2 - 2\chi(z)\chi(z') \cos \theta} .$$

- But future large & precise 3d galaxy catalogs like **Euclid** will be able to determine directly the measured 3d correlation function $\xi(\theta, z, z')$ and $C_\ell(z, z')$ from the data.
- These 3d quantities will of course be more noisy, but they also contain more information.
- These spectra are not only sensitive to the matter distribution (**density**) but also to the velocity via (**redshift space distortions**) and to the perturbations of spacetime geometry (**lensing**) .
- The spectra depend sensitively and in several different ways on dark energy (growth factor, distance redshift relation), on the matter and baryon densities, bias, etc. Their measurements provide a new route to estimate cosmological parameters.
- An example is the Alcock-Paczyński test.