General Relativistic N-body Simulations for Cosmic Large Scale Structure

based on arXiv:1308.6524

and work in progress with C. Clarkson, E. DiDio, R. Durrer, and M. Kunz

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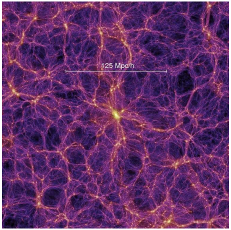
A Case for GR
Newton vs. Einstein



Millennium simulation,
Springel *et al.* 2005

In order to study the regime of nonlinear structure formation, large N-body simulations are the method of choice.

N-body simulations use
Newton's law of gravity

Works well for *nonrelativistic matter* (CDM), because of
see, e.g., Green & Wald 2012

- Exact correspondence between Newtonian gravity and GR on the background solution (FRW)
- Exact correspondence also on the level of linear (scalar) perturbations
- Nonlinear scale $\ll$ Hubble scale

A Case for GR

Newton vs. Einstein (cont.)

The Newtonian picture has several drawbacks, though

- Strong assumtion about material content of the Universe
- Misses some degrees of freedom (gravity waves!)
- Gauge issues are not apparent
- Trivial propagation of light beams (relativistic effects have to be put back "by hand")

A unified relativistic treatment of structure formation would automatically solve these issues. When constructing observables (galaxy catalogs, lensing maps etc.), all geometric effects and gauge issues would be treated in a transparent way.

A Case for GR
The Issue of Backreaction

Long standing question: how important is nonlinear evolution of structure for understanding & interpreting the observed “average” cosmological evolution?

References include Ellis 1984, Buchert 2000 & 2008, Schwarz 2002 & 2012, Wetterich 2003, Kolb, Matarrese, Notari & Riotto 2005, Buchert & Ellis 2005, Räsänen 2011

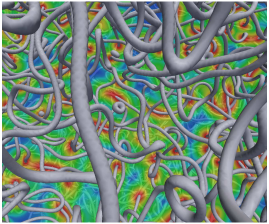
This issue has many facets. Some can be addressed in the Newtonian picture, others require a relativistic treatment ($\rightarrow$ perturbation theory, exact solutions . . .).
A unified relativistic treatment of structure formation would be the logical framework to address the issue in full generality.

A Case for GR

Relativistic Sources of Stress-Energy

GR effects are expected to be important for intrinsically relativistic entities

- Cosmic strings
- Dynamical Dark Energy
- Relativistic particles (neutrinos?)
- ...



credit: Daverio *et al.*

In order to test some of the proposed alternatives/extensions to ΛCDM, general relativistic simulations may be necessary in order to obtain percent accuracy required by future observations (e.g. Euclid)

Choice of Variables

Metric of perturbed FRW in “longitudinal gauge”

$$ds^2 = -(1+2\Psi)\,dt^2 + a^2(t)\left[(1-2\Phi)\,\delta_{ij} + h_{ij}\right] dx^i dx^j - 2B_i dx^i dt$$

Gauge condition: $\nabla^i B_i = \nabla^i h_{ij} = \delta^{ij} h_{ij} = 0$

Stress-energy tensor

$$T^\mu_\nu = \bar{T}^\mu_\nu + \delta T^\mu_\nu\,,\ \bar{T}^\mu_\nu = \mathrm{diag}(-\bar{\rho}, \bar{P}, \bar{P}, \bar{P})\,,\ \bar{P} = w\bar{\rho}$$

Fix “background” equation of state for each constituent $\rightarrow$ background scale factor a solves Friedmann’s equations for $\bar{T}^\mu_\nu$

Weak Field Approximation

Perturbative approach:

- Metric perturbations Ψ, Φ, ... remain small in cosmological context ($\sim 10^{-5}$) $\rightarrow$ keep only to first order
- Spatial derivatives $\Psi_{,i}$, ... are $\sim v$ ($\sim 10^{-3}$) $\rightarrow$ keep to quadratic order
- Second spatial derivatives $\Delta\Psi$, ... are $\sim \delta$ and therefore non-perturbative

See again Green & Wald 2012

System of Equations

"$G^0_0 = 8\pi G T^0_0$":

$$\frac{1}{a^2}\left(1+4\Phi\right)\Delta\Phi - 3H\dot{\Phi} - 3H^2\Psi + \frac{3}{2a^2}\left(\nabla\Phi\right)^2 = -4\pi G\delta T^0_0$$

"$G^i_i - 3G^0_0 - \frac{1}{H}\dot{G}^0_0 = 8\pi G(T^i_i - 3T^0_0 - \frac{1}{H}\dot{T}^0_0)$":

$$\begin{gathered}\left(1+2\Phi-2\Psi\right)\Delta\Psi - \left(\nabla\Psi\right)^2 - \nabla\Psi\nabla\Phi + \\ \frac{1}{H}\partial_t\left[\Delta\Phi + 4\Phi\Delta\Phi + \frac{3}{2}\left(\nabla\Phi\right)^2\right] = \\ 4\pi G\frac{a}{H}\left[\delta T^i_{0,i} - \delta T^i_0\left(3\Phi_{,i} - \Psi_{,i} + a\dot{B}_i\right)\right. \\ \left. -a\dot{\Phi}\left(3\delta T^0_0 - \delta T^i_i\right) - \frac{a}{2}\delta^{ik}\dot{h}_{jk}\delta T^j_i\right]\end{gathered}$$

The Framework

System of Equations (cont.)

"$G^0_i = 8\pi G T^0_i$":

$$-\frac{4}{a^2}\Delta B_i - \frac{1}{a}\dot{\Phi}_{,i} - \frac{H}{a}\Psi_{,i} = 4\pi G \delta T^0_i$$

"$G^i_j - \frac{1}{3}\delta^i_j G^k_k = 8\pi G (T^i_j - \frac{1}{3}\delta^i_j T^k_k)$":

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{1}{a^2}\Delta h_{ij} + \frac{1}{a}\left(\dot{B}_{(i,j)} + 2HB_{(i,j)}\right)$$
$$+\frac{1}{a}\text{ "traceless part } \left[(1+4\Phi)\,\Phi_{,ij} - (1+2\Phi-2\Psi)\,\Psi_{,ij}\right.$$
$$\left.+\Psi_{,i}\Psi_{,j} - 2\Phi_{(,i}\Psi_{,j)} + 3\Phi_{,i}\Phi_{,j}\right]\text{"} = 8\pi G\left(\delta_{ik}\delta T^k_j - \frac{1}{3}\delta_{ij}\delta T^k_k\right)$$

System of Equations (cont.)

In order to close the system of equations, one needs evolution equations for all sources of stress-energy.

Geodesic equation for nonrelativistic massive particles

$$\dot{v}^i + H v^i + \delta^{ij} \left(\frac{1}{a} \Psi_{,j} - \dot{B}_j - H B_j + \frac{2}{a} B_{[j,k]} v^k \right) = 0$$

determines the evolution of the particle ensemble and therefore the evolution of the full T^μ_ν of CDM.

Algorithmic Solutions

Φ: parabolic equation (diffusion type)

- Explicit scheme too inefficient (Courant condition!)
- First-order (in time) implicit scheme shows excellent performance in 1D tests
- ADI *(Alternating Direction Implicit)* scheme in 3D should perform well (easy to implement & parallelizable)

Ψ: elliptic equation

- Nonlinear Gauß-Seidel / Multigrid solver shows excellent performance in 1D tests
- Same class of solvers is already used for the Poisson equation in modern Newtonian codes

The Framework

Algorithmic Solutions (cont.)

B_i: linear elliptic operator

Two possibilities:

- Solve in Fourier space (transverse component can easily be extracted, but incompatible with AMR)
- Use Multigrid solver (gauge condition more difficult to implement)

h_{ij}: linear hyperbolic equation (wave equation)

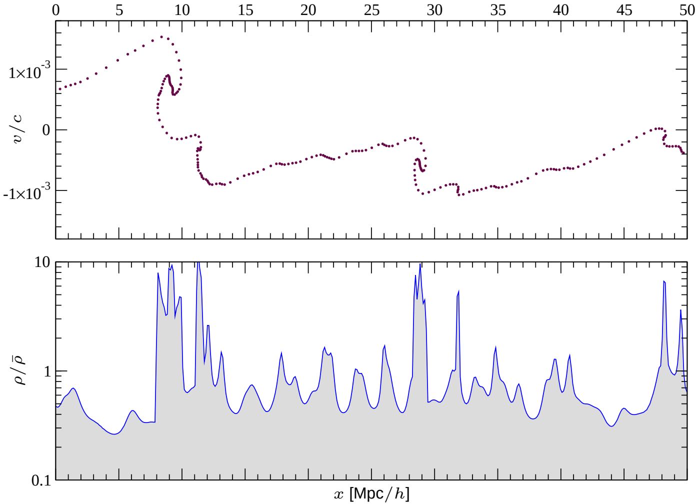
- No conceptual problem, but can be expensive (depending on relevant range of scales)
- h_{ij} does not enter the geodesic equation for massive particles (at our approximation order) $\rightarrow$ expendable, leave for future work

A Plane-Symmetric Setup

- Restriction to plane-symmetric configuration ($y - z$-plane) trivializes two dimensions $\rightarrow$ high resolution possible with cheap computational requirements (no parallelization)
- No vector & tensor perturbations (by construction)
- 32768 particles, 4096 grid points
- Initial conditions: Gaussian random field obtained from semi-realistic initial power spectrum
- Initialized at $z > 1000$ using linear theory (Zel'dovich approximation)

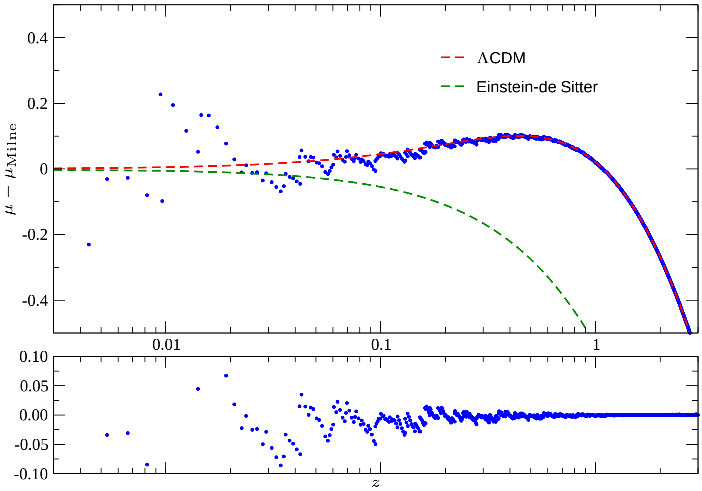
Numerical Results

A Plane-Symmetric Setup



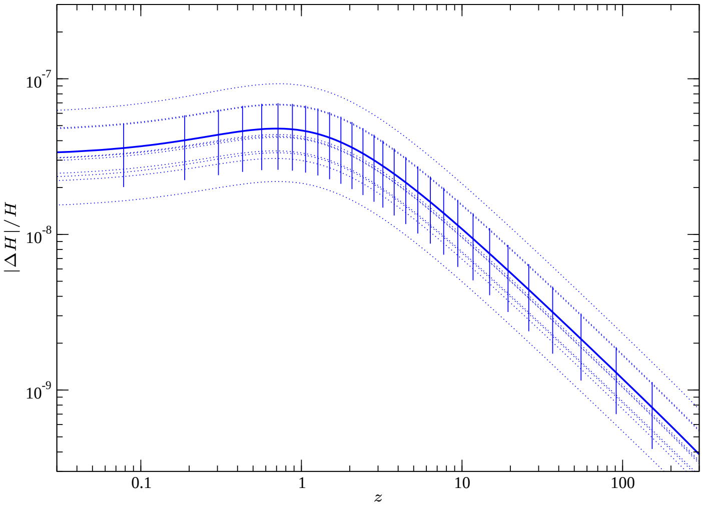
$z = 0.0$

Numerical Results
Luminosity Distance



Numerical Results

Newtonian vs. GR Simulation



Summary

- Cosmological simulations within a **GR framework** are feasible
- A unified relativistic treatment is a clear, logical and transparent way to address the **most general observables** with minimal assumptions about the cosmological model
- Technology is useful for simulations with **relativistic sources** (dynamical DE, cosmic strings, neutrinos) – feasibility depends on ability to model the sources accurately
- For CDM simulations, modifications are computationally relatively inexpensive (but may be unnecessary)
- The issue of **backreaction** can be addressed quantitatively within the non-perturbative regime