Consequences of consistency relations for non-local bias and modified gravity

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1311.0786 with Riotto, Kehagias and Noreña

Assumptions

- Perfect fluid
- Pressure-less
- Self-gravitating
- Newtonian approximation
- Conserved (DM)

Fluid equations

overdensity contrast

peculiar velocity field

Matter conservation $$\frac{\partial\delta(\vec{x},\tau)}{\partial\tau} + \vec{\nabla}\cdot[(1+\delta(\vec{x},\tau)\vec{v}(\vec{x},\tau)] = 0$$

Momentum conservation $$\frac{\partial\vec{v}(\vec{x},\tau)}{\partial\tau} + \mathcal{H}(\tau)\vec{v}(\vec{x},\tau) + [\vec{v}(\vec{x},\tau)\cdot\vec{\nabla}]\vec{v}(\vec{x},\tau) = -\vec{\nabla}\Phi(\vec{x},\tau)$$

Poisson $$\nabla^2\Phi(\vec{x},\tau) = \frac{3}{2}\Omega_{\rm m}\mathcal{H}^2(\tau)\delta(\vec{x},\tau)$$

gravitational potential

Fluid equations

also for galaxies!

$$\frac{\partial \delta_{\rm g}(\vec{x},\tau)}{\partial \tau} + \vec{\nabla}\cdot[(1+\delta_{\rm g}(\vec{x},\tau))\vec{v}_{\rm g}(\vec{x},\tau)] = A(\tau) j(\rho)$$

galaxy formation & merging

Chan, Scoccimarro, Sheth 1201.3614

$$\frac{\partial \vec{v}_{\rm g}(\vec{x},\tau)}{\partial \tau} + \mathcal{H}(\tau)\vec{v}_{\rm g}(\vec{x},\tau) + [\vec{v}_{\rm g}(\vec{x},\tau)\cdot\vec{\nabla}]\vec{v}_{\rm g}(\vec{x},\tau) = -\vec{\nabla}\Phi(\vec{x},\tau)$$

$$\nabla^2\Phi(\vec{x},\tau) = \frac{3}{2}\Omega_{\rm m}\mathcal{H}^2(\tau)\delta(\vec{x},\tau)$$

⇨ Bias

Symmetries

Equations of motion are invariant under the transformations

scalar $\quad \delta'_{\rm g}(\vec{x},\tau) = \delta_{\rm g}(\vec{x}',\tau'),$

non linear $\quad \vec{v}'_{\rm g}(\vec{x},\tau) = \vec{v}_{\rm g}(\vec{x}',\tau') - \dot{\vec{n}}(\tau),$

long wavelength velocity perturbation

scalar $\quad \delta'(\vec{x},\tau) = \delta(\vec{x}',\tau'),$

non linear $\quad \vec{v}'(\vec{x},\tau) = \vec{v}(\vec{x}',\tau') - \dot{\vec{n}}(\tau),$

non linear $\quad \Phi'(\vec{x},\tau) = \Phi(\vec{x}',\tau') - \left(\mathcal{H}\dot{\vec{n}}(\tau) + \ddot{\vec{n}}(\tau)\right)\cdot\vec{x}$

homogeneous gradient

$$\tau' = \tau, \qquad \vec{x}' = \vec{x} + \vec{n}(\tau), \qquad j(scalar)$$

Kehagias & Riotto 1302.0130

constant in space

Consistency relations

$$\vec{v}'(\vec{x},\tau) = \vec{v}(\vec{x}',\tau') - \dot{\vec{n}}(\tau)$$

$$\vec{x}' = \vec{x} + \vec{n}(\tau)$$

$$\vec{n}(\tau) = \int^{\tau} \mathrm{d}\eta\, \vec{v}_L(\vec{q},\eta) \qquad , \vec{q} \to 0$$

$$= i\frac{\vec{q}}{q^2}\delta_L(\vec{q},\tau)$$

$$= \frac{1}{6}\tau^2\vec{\nabla}\Phi_L \qquad \text{(MD)}$$

Dark Matter

$$\left\langle \delta(\vec{q},\tau)\delta(\vec{k}_1,\tau_1)\cdots\delta(\vec{k}_n,\tau_n)\right\rangle'_{q\to 0} = -P_{\delta_L}(q,\tau)\sum_{a=1}^{n}\frac{D(\tau_a)}{D(\tau)}\frac{\vec{q}\cdot\vec{k}_a}{q^2}\left\langle \delta(\vec{k}_1,\tau_1)\cdots\delta(\vec{k}_n,t_n)\right\rangle'$$

(n+1)-point function — n-point function

- This transformation is equivalent to going to the free-falling frame: Equivalence Principle
- Validity of the consistency relation does not depend on the fluid equations as the symmetry transformations are the non-relativistic limit of GR change of coordinates
- Vanish at equal time

Kehagias & Riotto 1302.0130, Peloso & Pietroni 1302.0223, GR version: Creminelli et al. 1309.3557

Consistency relations

Galaxies

$$\left\langle \delta_{\rm g}(\vec{q},\tau)\delta_{\rm g}(\vec{k}_1,\tau_1)\cdots\delta_{\rm g}(\vec{k}_n,\tau_n)\right\rangle'_{q\to 0} = -\left\langle \delta^L_{\rm g}(\vec{q},\tau)\delta_L(\vec{q},\tau)\right\rangle' \sum_{a=1}^{n}\frac{D(\tau_a)}{D(\tau)}\frac{\vec{q}\cdot\vec{k}_a}{q^2}\left\langle \delta_{\rm g}(\vec{k}_1,\tau_1)\cdots\delta_{\rm g}(\vec{k}_n,\tau_n)\right\rangle'$$

- Independent of the bias relation $\delta_{\rm g}(\vec{x},\tau) = F(\delta(\vec{x},\tau),\vec{v}(\vec{x},\tau),\Phi(\vec{x},\tau),...)$

- Tool to test theoretical models: e.g. the non-local bias model satisfies the consistency relation for bispectrum of the galaxies at tree-level and at one-loop level in PT

- One can write the relation in terms of observables: e.g. redshift space

- Measuring it directly is difficult but violations of it could be possible: e.g. modified gravity

Kehagias, Noreña, H.P. & Riotto 1311.0786, Creminelli et al. 1312.6074

Bias and symmetries

$$\delta_{\rm g}(\vec{x},\tau) = F(\delta(\vec{x},\tau), \vec{v}(\vec{x},\tau), \Phi(\vec{x},\tau), ...)$$

Galaxy overdensity is a scalar under the symmetries

⇨ it has to depend on scalars

$\delta(\vec{x},\tau)$

$$s_{ij}(\vec{x},\tau) = \partial_i\partial_j\Phi(\vec{x},\tau) - \frac{\delta_{ij}}{2}\Omega_{\rm m}\mathcal{H}^2\delta(\vec{x},\tau)$$

$$t_{ij}(\vec{x},\tau) = \partial_i v_j(\vec{x},\tau) - \frac{\delta_{ij}}{3}\theta(\vec{x},\tau) - \frac{2f}{3\Omega_{\rm m}\mathcal{H}}s_{ij}(\vec{x},\tau)$$

In general

$$\delta_{\rm g}(\vec{x},\tau) = b_1(\tau)\delta(\vec{x},\tau) + \frac{b_2(\tau)}{2}\delta^2(\vec{x},\tau) + c_{\nabla^2}(\tau)\nabla^2\delta(\vec{x},\tau) + c_{s^2}(\tau)s_{ij}(\vec{x},\tau)s^{ij}(\vec{x},\tau)$$
$$+ c_{s^2\nabla}\partial_k s_{ij}(\vec{x},\tau)\partial^k s^{ij}(\vec{x},\tau) + c_{s^2\nabla^2}\nabla^2(s_{ij}(\vec{x},\tau)s^{ij}(\vec{x},\tau)) + c_{s^2\nabla^4}(\tau)\nabla^2 s_{ij}(\vec{x},\tau)\nabla^2 s^{ij}(\vec{x},\tau)$$

- The so-called non-local bias satisfies this requirement
- Symmetries allow to compute the time dependence of the bias coefficients (MD)

Modified Gravity

- Symmetry transformation removes a constant gradient of the gravitational potential: this corresponds to going to the free-falling frame

$$\tau' = \tau, \quad \vec{x}' = \vec{x} + \int^{\tau} \mathrm{d}\eta\, \vec{v}_L(\eta) = \vec{x} + \frac{1}{6}\tau^2 \vec{\nabla}\Phi_L$$

- One expects a violation of the consistency relation if the EP is broken, e.g. modified gravity

- Modified gravity models all have a screening mechanism to satisfy

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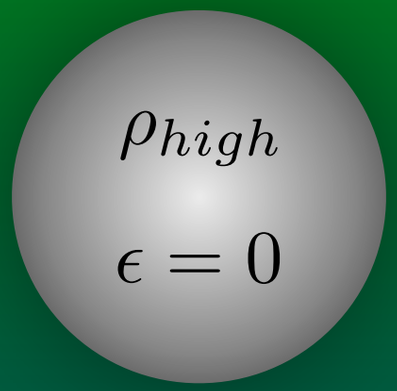
Modified Gravity Model

5^{th} force $\propto \epsilon$



ρ_{low}

$\epsilon = 1$

no unique free falling frame

free falling frame of screened objects $\quad \ddot{\vec{n}} + \mathcal{H}\dot{\vec{n}} + \vec{\nabla}\Phi = 0$

n is no longer unique but depends on the environment

$$\vec{n}^{<} = i\frac{\vec{q}}{q^2}\delta_L^{<}(\vec{q},\tau)$$

Screened

$$\vec{n}^{>} = i\frac{\vec{q}}{q^2}\frac{\delta_L^{>}(\vec{q},\tau)}{1+2\alpha^2\epsilon}$$

Unscreened

Consistency relations

bispectrum between one unscreened galaxy and two screened galaxies

$$\left\langle \delta_{\rm g}(\vec{q},\tau)\delta_{\rm g}(\vec{k}_1,\tau)\delta_{\rm g}(\vec{k}_2,\tau)\right\rangle'_{q\to 0} = \left[\frac{2\alpha^2\epsilon}{1+2\alpha^2\epsilon}\left\langle \delta^L_{\rm g}(\vec{q},\tau)\bar{\delta}_L(\vec{q},\tau)\right\rangle - \left\langle \delta^L_{\rm g}(\vec{q},\tau)\Delta\delta_L(\vec{q},\tau)\right\rangle\right]\frac{\vec{q}\cdot\vec{k}_1}{q^2}P_{\rm g}(\vec{k}_1,\tau)$$

$\bar{\delta}_L = \frac{\delta_L^< + \delta_L^>}{2}$ $\qquad$ $\Delta\delta_L = \frac{\delta_L^< - \delta_L^>}{2} \propto \alpha^2\epsilon$

- the consistency relation is non-zero at equal times
- the violation of the consistency relation is of order $\alpha^2\epsilon$

Conclusions

- consistency relations provide a tool to test models of LSS
- non-local bias model satisfies the basic symmetries of the system
- non-vanishing consistency relations at equal-time could be a sign of modification of gravity