Inflationary cosmological backreaction and test field observers

Giovanni Marozzi

University of Geneva, Switzerland

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GM, G. P. Vacca, Class. Quant. Grav. **29**, 115007 (2012);
GM, G. P. Vacca, R. H. Brandenberger, JCAP **1302**, 027 (2013);
GM, G. P. Vacca, arXiv:1304.2291 [gr-qc].

The Problem

The large scale properties of our Universe are usually described in the context of a homogeneous and isotropic FLRW space-time.

However:

The real Universe is not exactly homogeneous and isotropic

- neither in its present state (classic inhomogeneities)
- nor in its primordial state (quantum fluctuations)

Inflation can amplify scalar fluctuations up to be comparable with the background ($\langle \delta\phi^2 \rangle \sim \langle \phi^2 \rangle$), making their effects on the space-time dynamic non negligible.

$\Downarrow$

Inflationary backreaction

Gauge freedom in a FLRW universe

Let us consider a cosmological background sourced by a scalar field ϕ and described by the simple four-dimensional action

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi G} - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]$$

with spatially flat FLRW background geometry

$$ds^2 = -dt^2 + a(t)^2 \, \delta_{ij} \, dx^i dx^j$$

Gauge freedom in a FLRW universe

The background fields $\{\phi, g_{\mu\nu}\}$ can be expanded in non-homogeneous perturbations as follows:

$$\phi(t,\vec{x}) = \phi^{(0)}(t) + \delta\phi(t,\vec{x}),$$

$$g_{00} = -1 - 2\alpha, \qquad g_{i0} = -\frac{a}{2}\left(\beta_{,i} + B_i\right),$$

$$g_{ij} = a^2\left[\delta_{ij}\left(1 - 2\psi\right) + D_{ij}E + \frac{1}{2}\left(\chi_{i,j} + \chi_{j,i} + h_{ij}\right)\right],$$

where $D_{ij} = \partial_i\partial_j - \delta_{ij}(\nabla^2/3)$.

One obtains 11 degrees of freedom which are in part redundant.

To obtain a set of equations (Einstein equations + equation of motion of ϕ) well defined, order by order, we have, for example, to set to zero two scalar perturbations among $\delta\phi$, α, β, ψ and E, and one vector perturbation between B_i and χ_i.

Gauge freedom in a FLRW universe

The choice of such variables is called a choice of gauge.

For the scalar sector we may have:

$\psi = 0$, $E = 0$ Uniform Curvature Gauge
$\beta = 0$, $E = 0$ Longitudinal Gauge
$\alpha = 0$, $\beta = 0$ Synchronous Gauge
$\delta\phi = 0$, β or ψ or $E = 0$ Uniform Field Gauge
etc.

To connect different gauge we need an infinitesimal coordinates/gauge transformation.

Gauge Invariant variables

Request: Physics results should not depend on the gauge chosen to describe these.

Answer: Gauge Invariant (GI) formalism (Bardeen (1980), for a review see: Mukhanov, Feldman, Brandenberger(1992)).

Physically meaningful variable $\leftrightarrow$ GI variable.

A GI variable F is defined as a function of our perturbations which takes always the same value independently of the gauge chosen

$$F(\delta\phi, \alpha, \beta, ...) \rightarrow F(\delta\tilde{\phi}, \tilde{\alpha}, \tilde{\beta}, ...) = F(\delta\phi, \alpha, \beta, ...)$$

Gauge invariant quantum backreaction

One can construct a general observable by taking quantum averages of a scalar field $S(x)$ over a space-time hypersurface Σ_{A_0} where another scalar $A(x) = A_0$ (Gasperini, GM, Veneziano (2010))

$$\langle S \rangle_{A_0} = \frac{\langle \sqrt{|\overline{\gamma}(t_0, x)|} \; \overline{S}(t_0, x) \rangle}{\langle \sqrt{|\overline{\gamma}(t_0, x)|} \rangle}$$

with $\bar{x}^\mu$ coordinates where the scalar $A(x)$ is homogeneous, and $\sqrt{|\overline{\gamma}(t_0, \vec{x})|}$ determinant of the induced three dimensional metric on Σ_{A_0}.

The effective scale factor $a_{eff} = \langle \sqrt{|\bar{\gamma}|} \rangle^{1/3}$ describes then the dynamic of a perfect fluid-dominated early Universe and satisfies the GI effective cosmological equation (quantum generalization of Buchert (2000))

$$\left(\frac{1}{a_{eff}} \frac{\partial \, a_{eff}}{\partial A_0} \right)^2 = \frac{1}{9} \left\langle \frac{\Theta}{(-\partial^\mu A \partial_\mu A)^{1/2}} \right\rangle_{A_0}^2 ,$$

where $\Theta = \nabla_\mu n^\mu$ is the expansion scalar of the timelike congruence $n^\mu = -\frac{\partial^\mu A}{(-\partial^\nu A \partial_\nu A)^{1/2}}$.

Gauge invariant quantum backreaction

a_{eff} describes the expansion of the space as seen by the class of observers sitting on the hypersurface Σ_{A_0}.

To quadratic order (in the amplitude of the fluctuations), the inhomogeneities effect the effective expansion rate.

In the long wavelength (LW) limit and neglecting the tensor perturbations, we obtain

$$H_{eff}^2 \equiv \dot{A}^{(0)\,2} \left(\frac{1}{a_{eff}} \frac{\partial\, a_{eff}}{\partial A_0} \right)^2 = H^2 \left[1 + \frac{2}{H} \langle \bar{\psi} \dot{\bar{\psi}} \rangle - \frac{2}{H} \langle \dot{\bar{\psi}}^{(2)} \rangle \right] ,$$

which gives the quantum backreaction of cosmological fluctuations on the Hubble factor.

The observable H_{eff} is physical and gauge invariant.

On the other hand, different observers will measure different Hubble factors.

Geometrical observers

The observer/scalar $A(x)$ can be defined in a geometrical or dynamical way.

In general, we have a correspondence between a class of gauges and a class of observers with their physical properties.

In the long wavelength limit such physical properties are characterized by the time gauge condition on the vector generator $\epsilon^0_{(1)}$ and $\epsilon^0_{(2)}$ to go from a general gauge to the class of barred gauges chosen.

We can devide the geometrical observers in 3 different classes:

(a) The ones which correspond to gauges with $\psi = 0$.
These measure an unperturbed a_{eff} and identically zero backreaction.

(b) the ones which correspond to gauges with $\alpha = 0$ (or $\varphi = 0$ (UFG)), which are geodesic, or free falling, observers in the LW limit and to all order in slow-roll approximations.

(c) the ones which correspond to the gauges with $\beta = 0$ and $E = 0$ (longitudinal gauge), which have zero scalar and tensor shear.

Test field Observers

We consider a two field model with an inflaton ϕ and a second light field χ, which we study in the test field approximation

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi G} - \frac{1}{2} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - \frac{1}{2} m^2 \Phi^2 - \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \frac{1}{2} m_\chi^2 \chi^2 \right] .$$

We want evaluate the backreaction with respect to χ, namely using a clock which corresponds to a sub-dominant field.

As a consequence, we need to consider $\chi^{(0)}(t) \neq 0$, namely, a dynamical regime such that χ has non zero vacuum expectation value, a condensate.

Solving the background equations we find

$$\chi^{(0)}(t) = \chi^{(0)}(t_i) \left(\frac{H(t)}{H(t_i)} \right)^{\alpha} ,$$

with $\alpha = \frac{m_\chi^2}{m^2}$, and test field condition $\rho_\chi \ll \rho_\phi$.

Bounds

Test field condition for χ $(\rho_\chi \ll \rho_\Phi)$:

$$\chi^{(0)}(t_i)^2 \ll \left[1 + \frac{\alpha}{9}\frac{m^2}{H^2}\right]^{-1} \frac{1}{\alpha}\left(\frac{H}{H_i}\right)^{2-2\alpha} 6\frac{H_i^2}{m^2}M_{pl}^2 .$$

In particular, for the case $\alpha \ll 1$, we obtain the following limiting condition at the end of inflation ($H \simeq m$):

$$\chi^{(0)}(t_i)^2 \ll \frac{6}{\alpha}M_{pl}^2 .$$

Reliability of the perturbative approach:

$$\frac{\left(\langle\phi^{(1)2}\rangle\right)^{1/2}}{\phi^{(0)}} \ll 1 \quad , \quad \frac{\langle\phi^{(2)}\rangle}{(\langle\phi^{(1)2}\rangle)^{1/2}} \ll 1 ,$$

$$\frac{\langle\phi^{(1)}\chi^{(1)}\rangle}{\phi^{(0)}\chi^{(0)}} \ll 1 ,$$

$$\frac{\left(\langle\chi^{(1)2}\rangle\right)^{1/2}}{\chi^{(0)}} \ll 1 \quad , \quad \frac{\langle\chi^{(2)}\rangle}{(\langle\chi^{(1)2}\rangle)^{1/2}} \ll 1 .$$

Quantum backreaction

Under such conditions, we can write the cosmological backreaction for the expansion rate as seen by an observer comoving with the light test field χ in the following way

$$H^2_{eff} \simeq H^2 \left[1 + \frac{6}{H}\langle\bar{\psi}\dot{\bar{\psi}}\rangle + \frac{2}{\dot{\chi}^{(0)}}\left(2\frac{\dot{H}}{H} + \frac{\ddot{H}}{\dot{H}}\right)\langle\chi^{(2)}\rangle\right] .$$

In the limit $H \ll H_i$ we then have

$$\frac{2}{\dot{\chi}^{(0)}}\left(2\frac{\dot{H}}{H} + \frac{\ddot{H}}{\dot{H}}\right)\langle\chi^{(2)}\rangle \simeq -\frac{1}{12\pi^2}\left(1 - \frac{\alpha}{2}\right)\frac{H_i^6}{M_{pl}^2 H^4}$$

and

$$\frac{6}{H}\langle\bar{\psi}\dot{\bar{\psi}}\rangle \simeq -\frac{1}{\chi^{(0)}(t_i)^2}\frac{27}{2\pi^2}\frac{1}{\alpha^2(2-\alpha)}\frac{H^2 H_i^4}{m^4} .$$

Both contribution are negative at the end of inflation and go in the direction of of decreasing the measured effective expansion rate of the Universe.

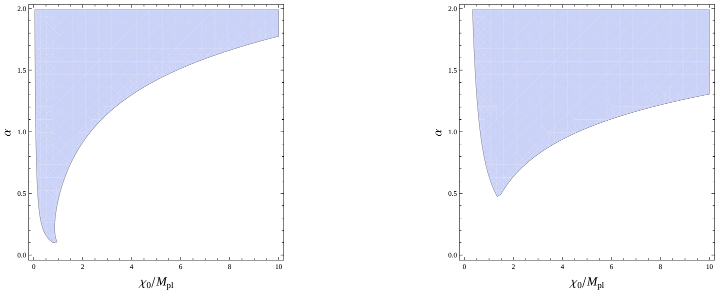
Quantum backreaction

For typical value of our parameter (as, for example, $H_i = \mathcal{O}(10)m$ with $m = 10^{-5}M_{pl}$) the second contribution will be the leading one at the end of inflation ($H \sim m$).

Furthermore, for a very light test field, the contribution of $\sim \frac{6}{H}\langle\bar{\psi}\dot{\bar{\psi}}\rangle$ may result (since it contains a $1/\dot{\chi}^{(0)\,2}$ enhancement factor) in a value which is even greater in magnitude than 1. This may happen even if the perturbative expansion used to study the dynamics is still valid.

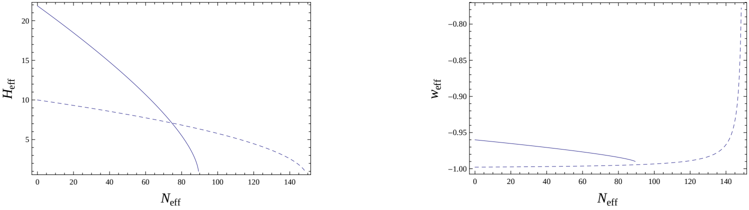
Such non-perturbative effect corresponds to a non-infinitesimal gauge trasformation to go from a general gauge to the barred gauge (with a big time shift).

Bounds and magnitude backreaction



We plot the region where the conditions validity of the perturbation theory, test field approximation and backreaction to be at most 1% are satisfied, for the values $H_i = 10m$ (left plot) and $H_i = 20m$ (right plot). The horizontal axis denotes $\chi^{(0)}(t_i)/M_{pl}$ and the vertical axis α. We chose the value of m given by $M_{pl} = 10^5 m$ to be consistent with the observed value of cosmological perturbations.

Non-perturbative backreaction effects



The effective expansion rate (left plot) and the equation of state w_{eff} (right plot), which both include the quantum backreaction, measured by an observer comoving with the test field χ, as a function of the associated effective number of e-folds of inflation $N_{eff} = \int dtH_{eff}$ (continuous lines) are shown together with the corresponding background values versus the standard number of e-folds (dashed lines). We have chosen $H_i = 10m$, $\alpha = 0.0085$ and $\chi_0 = 0.5M_{pl}$ ($M_{pl} = 10^5 m$).

Conclusions

- We have defined and applied a gauge-invariant observer-dependent approach to the evaluation of backreaction effects induced by long wavelength scalar fluctuations generated by an inflationary era in the early universe.

- Different observables and the associated measurement can probe different backreaction effects.

- Not Wrong! The observables are observers dependent!!
  Different observers $\Longleftrightarrow$ different features of the Universe dynamics.

- For the case of a test field observer, in a large region of the parameter space, the observed inflation stops much earlier with respect to the background one.
  Not safe choice! The backreaction might have important perturbative contributions at higher order!!
  But useful to gain some qualitative indication of the effect.

Geodesic Observers

The dynamic of a free falling observer is determined by the equation $t_\mu = n^\nu \nabla_\nu n_\mu = 0$ for its velocity n_μ.

The scalar field $A(x)$ associated with this observer is, for example, the one homogeneous in the SG (see G.M. (2011) for details).

In this case and for any potential we have (GM, Vacca (2013))

$$H_{eff}^{Free\ Falling} \equiv \dot{A}^{(0)\,2} \left(\frac{1}{a_{eff}} \frac{\partial\, a_{eff}}{\partial A_0} \right) = H\,.$$

The geodetic observers do not see on the effective Hubble rate any quantum backreaction effect, in the LW limit, for any potential $V(\phi)$ and to all orders in slow-roll parameters.

The UFG observers, which always sees as homogeneous the inflaton, are physically equivalent to the free falling ones and experience the same backreaction.

Isotropic Observers

The isotropic observer is defined to first order by

$$A(x) = A^{(0)} + \dot{A}^{(0)} \left[\frac{a}{2}\beta + \frac{a^2}{2}\dot{E} \right]$$

The relatives scalar σ and tensor $\sigma_{\mu\nu}$ shear (neglecting tensor perturbations) are identically zero.

As a consequence $\rightarrow \Theta_{\mu\nu} = \frac{1}{3} h_{\mu\nu}\Theta$, and the expansion is seen as isotropic from all the observers associated with the longitudinal gauge!

Performing the calculation for a chaotic model $V(\phi) = \frac{m^2}{2}\phi^2$, in the UCG and in LW limit at leading order in slow-roll approximation, we have

$$\left(\frac{1}{a_{eff}} \frac{\partial\, a_{eff}}{\partial A_0} \right)^2 = H^2 \left[1 + \frac{3}{5} \frac{\dot{H}}{H^2} \frac{\langle \delta\phi^2 \rangle}{M_{pl}^2} + \mathcal{O}\left(\frac{\dot{H}^2}{H^4} \right) \frac{\langle \delta\phi^2 \rangle}{M_{pl}^2} \right] .$$

The observers associated to the longitudinal gauge foliation, which are not free-falling, but see an inhomogeneous isotropic space, experiences a backreaction such that $H_{eff}^2 < H^2$.

Is Quantum BR important for the isotropic observer?

For a massive chaotic model in the LW limit and $H_i = H(t_i) \gg H$, one has

$$\frac{\langle \delta\phi^2 \rangle}{M_{pl}^2} \simeq -\frac{1}{24\pi^2} \frac{H_i^6}{M_{pl}^2 H^2 \dot{H}} \sim \frac{H_i^4}{H^2 M_{pl}^2} \ln a$$

If the coefficient of $\langle \delta\phi^2 \rangle$ is not zero (as for the isotropic observers), quantum backreaction appears with a secular term related to the infrared growth of inflaton fluctuations. On the other hand such a growth gives a negligible effect whenever $\frac{\langle \delta\phi^2 \rangle}{M_{pl}^2} \ll \epsilon^{-1}$.

In general non negligible effects could appear at the end of inflation ($H \sim m$) only for $H(t_i) \sim (m^2 M_{pl})^{1/3}$. Such values give a typical number of e-folds of the order of $\mathcal{O}(10^4)$, for $M_{pl} = 10^5 m$, and correspond to the case where non-linear corrections become really important (Finelli, GM, Starobinsky, Vacca, Venturi (2009), Finelli, GM, Vacca, Venturi (2006)).