Averaging the luminosity redshift relation: from theory to observations

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GR20/Amaldi10
Warsaw, Poland, 12 July 2013

Based on:
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I. Ben-Dayan, M. Gasperini, GM, F. Nugier, G. Veneziano, JCAP 04 (2012) 036;
I.Ben-Dayan, M.Gasperini, GM, F.Nugier, G.Veneziano, PRL 110, 021301 (2013);
I. Ben-Dayan, GM, F. Nugier, G. Veneziano, JCAP 11 (2012) 045;
I. Ben-Dayan, M. Gasperini, GM, F. Nugier, G. Veneziano, JCAP 06 (2013) 002.

The Problem

Evidence for a sizeable dark-energy component in the cosmic fluid comes from:

- CMB anisotropies.
- Models of large-scale-structure formation.
- Luminosity redshift relation of Type Ia supernovae (Nobel Prize 2011).

While the first two have to do, by definition, with the inhomogeneities present in our Universe, the third is based on an ideal homogeneous and isotropic Friedman-Lemaître-Robertson-Walker (FLRW) geometry.

It is clear that a better treatment of supernovae data should take inhomogeneities into account, at least in an average statistical sense.

Do inhomogeneities affect the measured fraction of dark energy from supernovae data? Can they play a role in the context of near-future precision cosmology?

Cosmological Backreaction

Inhomogeneities modify the null geodesics and affect photon propagation.

The way to interpretate the data on cosmological observables changes from a homogeneous and isotropic Universe to a inhomogeneous one.

The phenomenological reconstruction of the spacetime metric and of its dynamic evolution on a cosmological scale is necessarily based on past light-cone observations, since most of the relevant signals travel with the speed of light.

To describe the cosmological backreaction the averaging procedure should be so referred to a null hypersurface coinciding with the past light-cone of our observer.

Why do we average?

The distribution of the space-time inhomogeneities on large scale is known only from a statistical point of view (matter density power-spectrum).

The light-cone/ensemble average gives the mean effect of the large scale inhomogeneities on the observable considered.

The relative variance gives the dispersion that the observed data should have with respect to the average.

The final outcome of the procedure (light-cone/ensemble average + variance) should be in the end compared with the statistical distribution of the data to reveal if the effect of the inhomogeneities can explain or not the observations made.

Light-cone averaging: formalism

Let us start with a four-dimensional integral on a region bounded by two hypersurfaces, one spacelike and the other one null

$$I(S; -; A_0, V_0) = \int d^4x \sqrt{-g}\, \Theta(V_0 - V)\Theta(A - A_0)S(x),$$

where $V(x)$ is a scalar satisfying $\partial_\mu V \partial^\mu V = 0$ (with $V(x) = V_0$ the past light-cone of the observer) and $A(x)$ a timelike scalar.

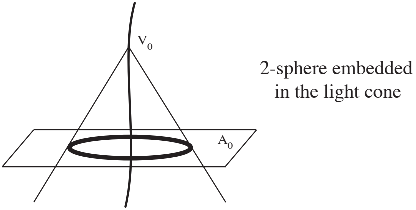
Starting with this hypervolume integral we can construct covariant and gauge invariant hypersurface and surface integrals considering the variation of the volume average along the flow lines n_μ normal to the reference hypersurface defined by $A(x)$ =constant.

Light-cone averaging: prescription

Considering the variation of the hypervolume integral both by shifting the light-cone $V = V_0$ and the hypersurface $A = A_0$ along the flow lines defined by n_μ we obtain

$$I(S; V_0, A_0; -) = \int d^4x \sqrt{-g}\, \delta(V_0 - V)\delta(A - A_0)|\partial_\mu V \partial^\mu A| S(x)$$

which gives the integral on the 2-sphere embedded in the past light-cone.



(c) $I(1; V_0, A_0; -)$

The averages of a scalar S is then defined by:
$$\langle S \rangle_{V_0, A_0} = \frac{I(S; V_0, A_0; -)}{I(1; V_0, A_0; -)}$$

Backreaction and Observables

Given the light-cone/ensemble average of a perturbed (inhomogeneous) observable S the average of a generic function of this observable differs, in general, from the function of its average, i.e. $\overline{\langle F(S)\rangle} \neq F(\overline{\langle S\rangle})$.

Expanding the observable to second order as $S = S_0 + S_1 + S_2$, one finds:

$$\overline{\langle F(S)\rangle} = F(S_0) + F'(S_0)\overline{\langle S - S_0\rangle} + F''(S_0)\overline{\langle S_1^2/2\rangle}.$$

Different functions of the luminosity distance $=$ different effects of the inhomogeneities.

To find the function that minimizes the backreaction means a better estimate of the cosmological parameters.

CLAIM! Such a privileged observable is given by the energy flux $\Phi = L/(4\pi d_L^2)$ received from a standard candle of luminosity L located on the observer's past light-cone.

Geodesic light-cone coordinates

An adapted light-cone coordinate system $x^\mu = (w, \tau, \tilde{\theta}^a)$, $a = 1, 2$ can be defined by the following metric:

$$ds^2 = \Upsilon^2 dw^2 - 2\Upsilon dw d\tau + \gamma_{ab}(d\tilde{\theta}^a - U^a dw)(d\tilde{\theta}^b - U^b dw) \; ; \qquad a, b = 1, 2 \; .$$

This metric depends on six arbitrary functions (Υ, the two-dimensional vector U^a and the symmetric tensor γ_{ab}) and corresponds to a complete gauge fixing.
As it is easy to check w is a null coordinate while $\partial_\mu \tau$ defines a geodesic flow.

Let us underline that such coordinates can be seen as a particular specification of the "observational coordinates" (Maartens (1980) and Ellis, Nel, Maartens, Stoeger, Whitman (1985)).

To understand the geometric meaning of GLC coordinates let us consider the limiting case of a spatially flat FLRW Universe

$$w = r + \eta, \qquad \tau = t, \qquad \Upsilon = a(t), \qquad U^a = 0,$$
$$\gamma_{ab} d\tilde{\theta}^a d\tilde{\theta}^b = a^2(t) r^2 (d\theta^2 + \sin^2\theta d\phi^2),$$

where η is the conformal time of the homogeneous metric: $d\eta = dt/a$.

Energy flux

The redshift and area distance are then given by

$$1+z = \frac{\Upsilon(w_0, \tau_0, \tilde{\theta}^a)}{\Upsilon(w_0, \tau, \tilde{\theta}^a)} \equiv \frac{\Upsilon_0}{\Upsilon} \qquad , \qquad d_A^2 \equiv \frac{dS}{d\Omega_o}$$

where $d\Omega_o$ is the infinitesimal solid angle at the observer, and dS is the cross-sectional area element perpendicular to the light ray at the source.

While for the $\Phi \sim d_L^{-2}$ we have the following exact result:

$$\langle d_L^{-2} \rangle(z, w_0) = \frac{4\pi(1+z)^{-4}}{\int d^2\tilde{\theta} \sqrt{\gamma(w_0, \tau(z, \tilde{\theta}^a) \tilde{\theta}^b)}},$$

The average flux, for a given z, is inversely proportional to the proper area of the surface lying on our past light cone at the given value of z.

Energy flux

To compute $\overline{\langle d_L^{-2} \rangle}$ we consider a spatially-flat ΛCDM model, perturbed by a stochastic background of inflation-generated inhomogeneities.

Using the Poisson gauge, the second order coordinate transformations between GLC and PG, and the stochastic Bardeen potential $\psi(\vec{x}, \eta)$, we can express our result in terms of the dimensionless power spectrum $\mathcal{P}_\psi(k, \eta)$ in the following way:

$$\overline{\langle d_L^{-2} \rangle} = (d_L^{FLRW})^{-2} \left[1 + f_\Phi(z)\right],$$

where, to leading order, we have

$$f_\Phi(z) = \left[\tilde{f}_{1,1}(z) + \tilde{f}_2(z)\right] \int_{\mathcal{H}_o}^{k_{UV}} \frac{dk}{k} \left(\frac{k}{\mathcal{H}_0}\right)^2 \mathcal{P}_\psi(k, \eta_o)$$

and $\tilde{f}_{1,1}(z)$, $\tilde{f}_2(z)$ are numerical function of the redshift z.

Luminosity distance and distance modulus

Using the previous results we can now evaluate the inhomogeneous corrections on the redshift to luminosity-distance relation d_L and on the distance modulus μ.

For the luminosity distance we obtain

$$\overline{\langle d_L \rangle}(z) = d_L^{FLRW} \left[1 + f_d(z)\right]$$

$$f_d = -(1/2) f_\Phi + (3/8) \overline{\langle (\Phi_1/\Phi_0)^2 \rangle}.$$

For the distance modulus ($\mu \sim -2.5 \log_{10} \Phi + \mathrm{const}$) we obtain

$$\overline{\langle \mu \rangle} - \mu^{FLRW} = -2.5 (\log_{10} e) \left[f_\Phi - \frac{1}{2} \overline{\langle (\Phi_1/\Phi_0)^2 \rangle} \pm \sqrt{\overline{\langle (\Phi_1/\Phi_0)^2 \rangle}} \right]$$

Luminosity distance and distance modulus

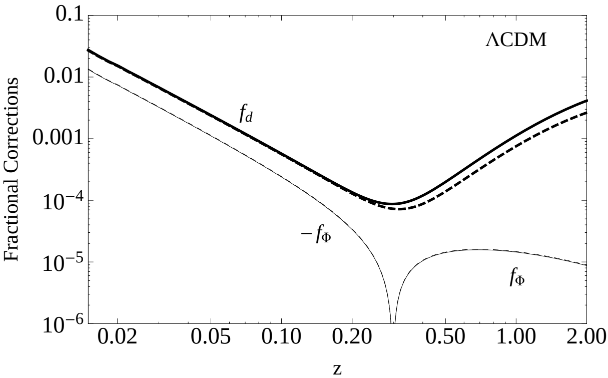
The corrections to the averaged values of d_L and μ, and of the dispersion, are qualitatively different from those of the flux (represented by f_Φ).

We have an extra contribution (inevitable for any non-linear function of the flux) proportional to the square of the first-order flux fluctuations.

While the averaged flux corrections have leading spectral contributions of the type $k^2\mathcal{P}_\psi(k)$, the new corrections to d_L and μ, on the contrary, are due to the so-called "lensing effect", dominate at large z, and have leading spectral contributions of the type $k^3\mathcal{P}_\psi(k)$.

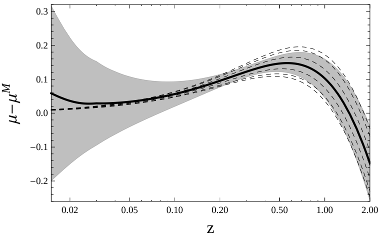
In the following we will present the explicit numerical integration of these backreaction effects, considering a non-linear power spectrum given by a HaloFit model (Smith et al. (2003) and Takahashi et al. (2012)).

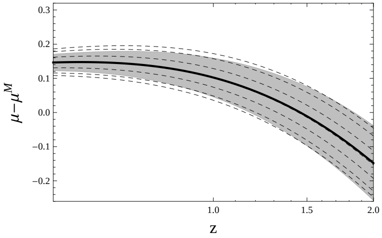
f_Φ vs f_d - Non-linear power spectrum



The fractional correction f_Φ (thin curves) and f_d (thick curves), for a perturbed ΛCDM model with $\Omega_{\Lambda 0} = 0.73$, are shown. We take into account the non-linear power spectrum given by the HaloFit model of Takahashi et al. (2012) (including baryons), and we have used the following cutoff values: $k_{UV} = 10h\,\mathrm{Mpc}^{-1}$ (dashed curves) and $k_{UV} = 30h\,\mathrm{Mpc}^{-1}$ (solid curves).

Distance Modulus - Non-linear power spectrum





The averaged distance modulus $\overline{\langle\mu\rangle} - \mu^M$ (thick solid curve), and its dispersion (shaded region), for a perturbed ΛCDM model with $\Omega_{\Lambda 0} = 0.73$. We take into account the non-linear power spectrum given by the HaloFit model of Takahashi et al. (2012) (including baryons), and used the cut-off $k_{UV} = 30h\,\mathrm{Mpc}^{-1}$. The averaged results are compared with the homogeneous values of μ predicted by unperturbed ΛCDM models with (from bottom to top) $\Omega_{\Lambda 0} = 0.68, 0.69\ 0.71, 0.73, 0.75, 0.77, 0.78$ (dashed curves). The right panel simply provides a zoom of the same curves, plotted in the smaller redshift range $0.5 \leq z \leq 2$.

Theory vs Observations, the intrinsic dispersion of the data

The total variance $\sigma_\mu^{\rm obs}$ associated with the observational data of the distance modulus μ can be written as

$$(\sigma_\mu^{\rm obs})^2 = (\sigma_\mu^{\rm fit})^2 + (\sigma_\mu^{z})^2 + (\sigma_\mu^{\rm int})^2 \quad .$$

where

$\sigma_\mu^{\rm fit}$ is the statistical uncertainty.

σ_μ^{z} represents the uncertainty in redshift from spectroscopic measurements and peculiar velocities of (and within) the host galaxy.

$\sigma_\mu^{\rm int}$ is an unknown phenomenological quantity, needed to account for the remaining dispersion of the data with respect to the chosen homogeneous model.

Theory vs Observations, the intrinsic dispersion of the data

We can then write $\sigma_\mu^{\rm int}$ as

$$(\sigma_\mu^{\rm int})^2 = (\widehat{\sigma_\mu^{\rm int}})^2 + (\sigma_\mu^{\rm lens})^2 \quad ,$$

where

$\sigma_\mu^{\rm lens}$ is the contribution of the lensing effect.

$\widehat{\sigma_\mu^{\rm int}}$ is the remaining source of intrinsic dispersion.

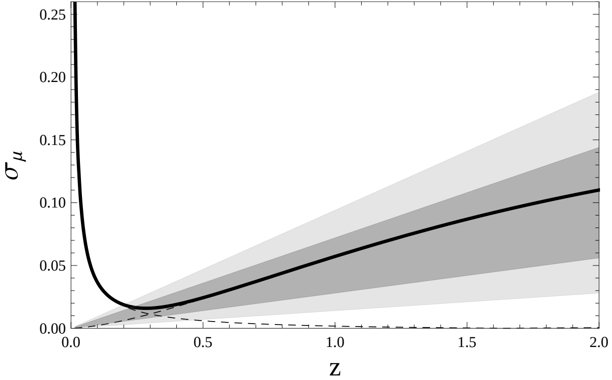
In the literature we have different experimental estimation of $(\sigma_\mu^{\rm lens})$.
In particular we have

$\sigma_\mu^{\rm lens} = (0.05 \pm 0.022)z$ Kronborg et al. (2010).

$\sigma_\mu^{\rm lens} = (0.055^{+0.039}_{-0.041})z$ Jonsson et al. (2010).

In the following figure we plot our dispersion σ_μ showing how this is in good agreement with the above observational estimates.

Dispersion σ_μ vs observational results



The dispersion σ_μ is illustrated by the thick solid curve, and it is separated into its “Doppler” part (dashed curve dominant at low z) and “lensing” part (dashed curve dominant at large z). The slope in the lensing-dominated regime is compared with the experimental estimates of Kronborg et al. (2010) (dark shaded area), and of Jönsson et al. (2010) (light shaded area).

Conclusions

- Dealing directly with the experimentally measured $d_L(z)$ within a gauge-independent approach leads to results which are automatically free from UV (and IR) divergences for *any* function of d_L.
- The corrections are typically too small to mimic dark energy. However, both their size and their z-dependence are strongly dependent on the particular function of d_L being averaged.
- The backreaction turns out to be minimal for the flux Φ, which therefore stands out as the safest observable for precision cosmology.
  For other observables the backreaction is larger at higher redshift and can be of the order of 10^{-3}.
- The dispersion due to stochastic fluctuations is much larger than the backreaction itself, implying an irreducible scatter of the data.
- σ_μ^{lens} at $z > 0.3$ is very well captured by a linear behaviour which can be roughly fitted by $\sigma_\mu^{\rm lens}(z) = 0.056z$. This is in agreement with previous observational estimates and stand out as a challenging prediction which could represent a further significative test of the concordance model.

THANKS FOR THE ATTENTION!