Cosmological Parameter Estimation with Large Scale Structure Observations

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Outline

Introduction

Galaxy number counts

2D redshift dependent power spectra

Fisher matrix analysis

Lensing Potential

CLASSgal

Standard Cosmological Model



 $G_{\mu\nu} = \kappa T_{\mu\nu}$

CMB



Planck Collaboration





Large Scale Structures





Angular position n Redshift z

 $N(\mathbf{n},z) d\Omega_{\mathbf{n}} dz$



Angular position nRedshiftz

 $N(\mathbf{n},z) d\Omega_{\mathbf{n}} dz$



Angular position n Redshift 2

 $N(\mathbf{n},z) d\Omega_{\mathbf{n}} dz$

In 3D analysis we need to convert a redshift and angles into length scales

 $r(z_1, z_2, \theta) = \sqrt{\chi^2(z_1) + \chi^2(z_2) - 2\chi(z_1)\chi(z_2)\cos\theta}$

We need to assume a cosmology!



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We need to assume a cosmology!



To compute $\Delta(\mathbf{n}, z) \equiv \frac{N(\mathbf{n}, z) - \langle N \rangle(z)}{\langle N \rangle(z)}$ we have to consider:

- observation on the past lightcone
- redshift perturbed by peculiar velocity
- light deflection
- volume distortion

$$\begin{aligned} \Delta(\mathbf{n},z) &= D_g + \Phi + \Psi + \frac{1}{\mathcal{H}} \left[\Phi' + \partial_r (\mathbf{V} \cdot \mathbf{n}) \right] \\ &+ \left(\frac{\mathcal{H}'}{\mathcal{H}^2} + \frac{2}{r_S \mathcal{H}} \right) \left(\Psi + \mathbf{V} \cdot \mathbf{n} + \int_0^{r_S} dr (\Phi' + \Psi') \right) \\ &+ \frac{1}{r_S} \int_0^{r_S} dr \left[2 - \frac{r_S - r}{r} \Delta_\Omega \right] (\Phi + \Psi) \end{aligned}$$

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Additional effects

galaxy bias

magnification bias

galaxy evolution

$$\Delta^{(N)}(\mathbf{n}, z, m_*) = bD_g(L > \bar{L}_*) + (1 + 5s)\Phi + \Psi + \frac{1}{\mathcal{H}} \left[\Phi' + \partial_r (\mathbf{V} \cdot \mathbf{n}) \right] \\ + \left(\frac{\mathcal{H}'}{\mathcal{H}^2} + \frac{2 - 5s}{r_S \mathcal{H}} + 5s - f_{\text{evo}}^N \right) \left(\Psi + \mathbf{V} \cdot \mathbf{n} + \int_0^{r_S} dr (\Phi' + \Psi') \right) \\ + \frac{2 - 5s}{2r_S} \int_0^{r_S} dr \left[2 - \frac{r_S - r}{r} \Delta_\Omega \right] (\Phi + \Psi)$$

Additional effects



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b

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$$\Delta^{(N)}(\mathbf{n}, z, m_*) = bD_g(L > \bar{L}_*) + (1 + 5s)\Phi + \Psi + \frac{1}{\mathcal{H}} [\Phi' + \partial_r (\mathbf{V} \cdot \mathbf{n})] \\ + \left(\frac{\mathcal{H}'}{\mathcal{H}^2} + \frac{2 - 5s}{r_S \mathcal{H}} + 5s - f_{\text{evo}}^N\right) \left(\Psi + \mathbf{V} \cdot \mathbf{n} + \int_0^{r_S} dr(\Phi' + \Psi')\right) \\ + \frac{2 - 5s}{2r_S} \int_0^{r_S} dr \left[2 - \frac{r_S - r}{r}\Delta_\Omega\right] (\Phi + \Psi)$$

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$$s(z, m_*) \equiv \frac{\partial \log_{10} \bar{N}(z, m < m_*)}{\partial m_*},$$
$$\bar{N}(z, L_S < \bar{L}_*) = \int_{F_*}^{\infty} \bar{n}_S(z, \ln F) d\ln F$$

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$$f_{\rm evo}^N = \frac{\partial \ln(a^3 \bar{N}(z, L > \bar{L}_*))}{\mathcal{H} \partial \tau_S}$$

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z-dependent angular power spectrum

Multipole expansion

$$\Delta(\mathbf{n}, z) = \sum_{\ell m} a_{\ell m}(z) Y_{\ell m}(\mathbf{n})$$

Power spectrum

 $c_{\ell}(z, z') = \langle a_{\ell m}(z) a_{\ell m}^{*}(z') \rangle$ $c_{\ell}(z_{1}, z_{2}) = \langle a_{\ell m}(z_{1}) a_{\ell m}(z_{2}) \rangle = 4\pi \int \frac{dk}{k} \mathcal{P}(k) \Delta_{\ell}(z_{1}, k) \Delta_{\ell}(z_{2}, k)$

Correlation function

 $\xi(z, z', \theta) = \langle \Delta(\mathbf{n}, z) \Delta(\mathbf{n}', z') \rangle = \sum_{\ell} \frac{2\ell + 1}{4\pi} c_{\ell}(z, z') P_{\ell}(\cos \theta)$

z-dependent angular power spectrum

Power spectrum

 $c_{\ell}(z_1, z_2) = \langle a_{lm}(z_1) a_{lm}(z_2) \rangle = 4\pi \int \frac{dk}{k} \mathcal{P}(k) \Delta_{\ell}(z_1, k) \Delta_{\ell}(z_2, k)$



z-dependent angular power spectrumWindow Function $\Delta_{\ell}^{i}(k) = \int dz \frac{dN}{dz} W_{i}(z) \Delta_{\ell}(z,k)$ $c_{\ell}^{ij} = 4\pi \int \frac{dk}{k} \mathcal{P}(k) \Delta_{\ell}^{i}(k) \Delta_{\ell}^{j}(k)$ i = jAuto-Correlationi = jAuto-CorrelationCross-Correlation



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Density

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RSD

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Lensing

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 $\ell = 71$

How good can a future survey determine the cosmological parameters?

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Cosmological parameters λ_{lpha}

Covariance matrix

$$\operatorname{Cov}_{[\ell,\ell'][(ij),(pq)]} = \delta_{\ell,\ell'} \frac{C_{\ell}^{\operatorname{obs},ip} C_{\ell}^{\operatorname{obs},jq} + C_{\ell}^{\operatorname{obs},iq} C_{\ell}^{\operatorname{obs},jp}}{f_{\operatorname{sky}} \left(2\ell+1\right)}$$

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Observed spectrum

$$C_{\ell}^{\text{obs},ij} = C_{\ell}^{ij} + \frac{\delta_{ij}}{n(i)}$$

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number of galaxies n(i) per steradian in the i-th redshift bin

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Observed spectrum

$$C_{\ell}^{\text{obs},ij} = C_{\ell}^{ij} - \frac{\delta_{ij}}{n(i)}$$

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Figure of Merit

FoM =
$$\left[\det(F^{-1})\right]^{-1/2}$$

$$\operatorname{FoM}_{\text{fixed}} = \left[\det(\hat{F}^{-1})\right]^{-1/2}$$
$$\operatorname{FoM}_{\text{marginalized}} = \left[\det(\widehat{F}^{-1})\right]^{-1/2}$$

Binning Strategy

To recover the 3D information we need to split the redshift range in many bins and to consider the cross-correlations between different redshift bins.

Spectroscopic Survey

Photometric Survey





The redshift bins are chosen such that there is the same number of galaxy for each bin.

We can not correlate two galaxies separated by a distance shorter than the non-linear scale λ_{\min}

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Transverse distance

$$d(z)\theta(\ell) \sim d(z)\frac{2\pi}{\ell}$$

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$$\frac{d(z)\theta(\ell)}{z_i - z_j} \sim d(z) \frac{2\pi}{\ell}$$
$$\frac{z_i - z_j}{H(\bar{z})}$$

$$\ell_{\max}^{ij}$$

$$\frac{2\pi d(z)/\lambda_{\min}}{\sqrt{\lambda_{\min}^2 - \left(\frac{\delta z_{ij}}{H(\bar{z})}\right)^2}}$$

 $\begin{array}{l} \text{if } \delta z_{ij} \leq 0 \,, \\ \text{if } 0 < \frac{\delta z_{ij}}{H(\bar{z})} < \lambda_{\min} \,, \end{array} \end{array}$

otherwise.

Cosmological Parameter Forecast Increasing the number of bins, we get a better redshift resolution. But shot-noise increases too.



Since also the signal increases, shot-noise starts dominating for smaller bin size than naively expected.

Cosmological Parameter Forecast spectroscopic DES-like









only redshift bins auto-correlations with redshift bins cross-correlations

Cosmological Parameter Forecast 2D vs 3D



Cosmological Parameter Forecast Euclid

Spectroscopic Survey

Photometric Survey



MOH = 100 MOH

$\lambda_{\rm min} = 68 \ {\rm Mpc}/h$

only redshift bins auto-correlations with redshift bins cross-correlations

Cosmological Parameter Forecast

spectroscopic DES-like



4 bins, 8 bins, 16 bins, 32 bins

Cosmological Parameter Forecast Euclid

Spectroscopic Survey

Photometric Survey



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Different terms encode different information

Density Redshift-space distortion Lensing Potential

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Large scale matter distribution

Redshift-space distortion Lensing Potential

Density

Different terms encode different information

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Lensing Potential

Cosmic velocity field

Different terms encode different information

Density Redshift-space distortion Lensing

Geometry of the universe

Potential

S

Different terms encode different information

Density Redshift-space distortion Lensing Potential

Signal to Noise

$$\left(N \right)_{\ell} \qquad \sigma_{\ell}$$

$$\sigma_{\ell} = \sqrt{\frac{2}{(2\ell+1) f_{\text{sky}}}} \left(C_{\ell} + \frac{1}{n} \right)$$

 $C_\ell - \tilde{C}_\ell$



 \overline{z} =1, Δz =0.01







 \overline{z} =1, Δz =0.1







 \overline{z} =1, Δz =0.5



Euclid



0.5 < z < 2.5



ł

Cumulative Signal to Noise

$$\left(\frac{S}{N}\right)^2 = \sum_{\ell=2}^{\ell_{\max}} \left(\frac{C_{\ell} - \tilde{C}_{\ell}}{\sigma_{\ell}}\right)^2$$







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*l*_{max}



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CLASSgal Next..

Main parts of CLASSgal will be merged with CLASS (v2.1): - cross-correlation with CMB - implementation for non-flat universe

http://cosmology.unige.ch/tools

For CLASS http://class-code.net

Conclusion

So far galaxy surveys have mainly determined P(k). Easy to measure it, but requires a fiducial model.

Future surveys (like Euclid) will be able to measure the spectra $C_{\ell}(z, z')$ directly from the data.

These spectra contain information about the matter distribution (density), the velocity (redshift space distortion) and the spacetime geometry (lensing).

The spectra depend sensitively and in several different ways on dark energy, on the matter and baryon densities, bias, etc. Their measurements provide a new estimation of cosmological parameters.

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