

The Standard Model (SM) in the Sky

DaNi G. FiGuERoA

Theoretical Physics Dep., UniGE

DGF, [arXiv:1402.1345](#) (Today!)

Swiss Cosmology Days, Zürich, Feb 6-7, 2014.

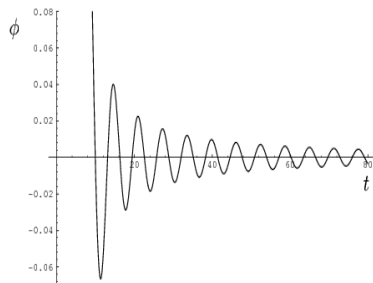
I shall be talking about ...

1. **Parametrically Excited Fermions** \rightarrow **GWs**
2. **SM Higgs** \rightarrow **SM Fermions** \rightarrow **GWs**

1.1 Fermion Parametric Excitation

Scalar Field (condensate) after Inflation:

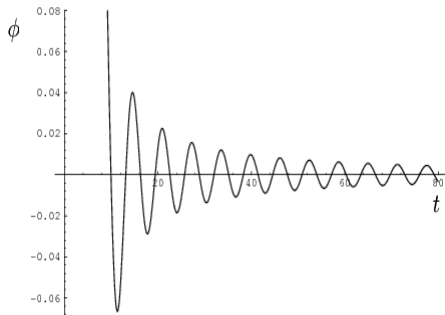
Coherent Oscillations: $\phi(t) \approx \Phi(t)f(t)$, $f(t+T) = f(t)$



1.1 Fermion Parametric Excitation

Fermions: $y\phi\bar{\psi}\psi$: Oscillations \rightarrow ψ – Particle Creation
(Non-Pert., Out-of-Eq.)

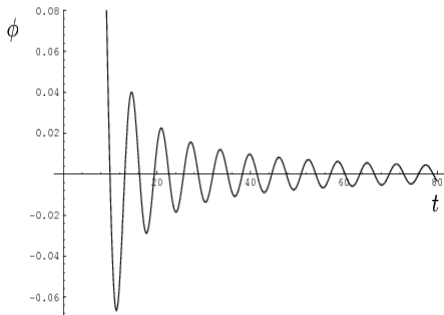
$$\psi(\mathbf{x}, t) = \int \frac{d\mathbf{k}}{(2\pi)^3} e^{-i\mathbf{k}\cdot\mathbf{x}} \left[\hat{a}_{\mathbf{k},r} \mathbf{u}_{\mathbf{k},r}(t) + \hat{b}_{-\mathbf{k},r}^\dagger \mathbf{v}_{\mathbf{k},r}(t) \right],$$



1.1 Fermion Parametric Excitation

Fermions: $y\phi\bar{\psi}\psi$: Oscillations \rightarrow ψ – Particle Creation
(Non-Pert., Out-of-Eq.)

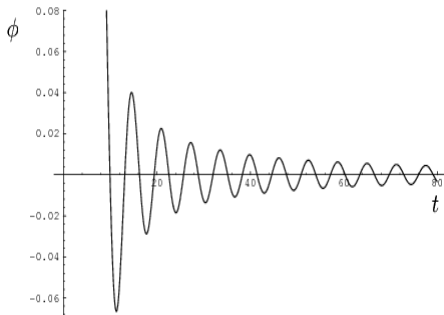
$$\psi(\mathbf{x}, t) = \int \frac{d\mathbf{k}}{(2\pi)^3} e^{-i\mathbf{k}\cdot\mathbf{x}} \left[\hat{a}_{\mathbf{k},r} \begin{pmatrix} u_{\mathbf{k},+}(t) S_r \\ u_{\mathbf{k},-}(t) S_r \end{pmatrix} + \hat{b}_{-\mathbf{k},r}^\dagger \begin{pmatrix} v_{\mathbf{k},+}(t) S_r \\ v_{\mathbf{k},-}(t) S_r \end{pmatrix} \right],$$



1.1 Fermion Parametric Excitation

Fermions: $y\phi\bar{\psi}\psi$: Oscillations \rightarrow ψ – Particle Creation
(Non-Pert., Out-of-Eq.)

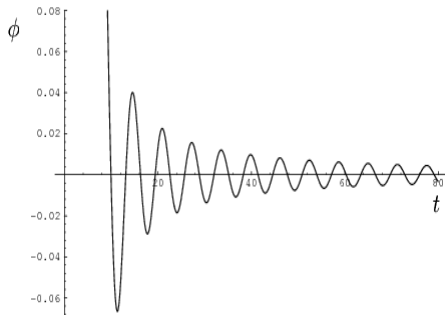
$$\frac{d^2}{dt^2}u_{\mathbf{k},\pm} + \left(\omega_{\mathbf{k}}^2(t) \pm i\frac{d(am_{\psi})}{dt}\right)u_{\mathbf{k},\pm}(t) = 0, \quad \omega_{\mathbf{k}}^2(t) = k^2 + a^2(t)m_{\psi}^2(t)$$



1.1 Fermion Parametric Excitation

Fermions: $y\phi\bar{\psi}\psi$: Oscillations \rightarrow ψ - Particle Creation
(Non-Pert., Out-of-Eq.)

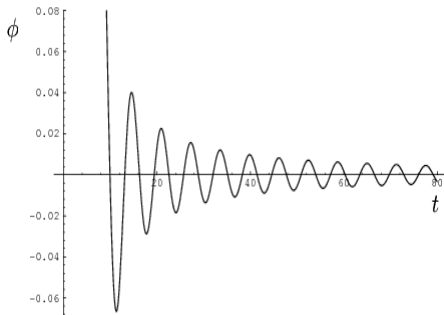
$$\frac{d^2}{dt^2}u_{\mathbf{k},\pm} + \left(\omega_{\mathbf{k}}^2(t) \pm i\frac{d(am_{\psi})}{dt}\right)u_{\mathbf{k},\pm}(t) = 0, \quad \omega_{\mathbf{k}}^2(t) = k^2 + a^2(t)m_{\psi}^2(t)$$



1.1 Fermion Parametric Excitation

Fermions: $y\phi\bar{\psi}\psi$: Oscillations \rightarrow ψ - Particle Creation
(Non-Pert., Out-of-Eq.)

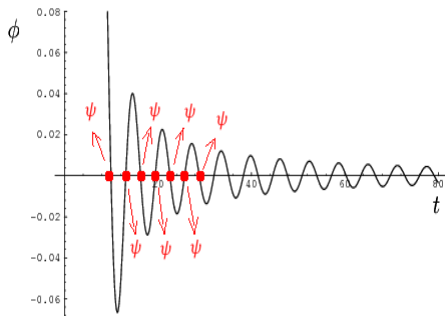
$$\frac{d^2}{dt^2}u_{\mathbf{k},\pm} + \left(\omega_{\mathbf{k}}^2(t) \pm i\frac{d(am_\psi)}{dt}\right)u_{\mathbf{k},\pm}(t) = 0, \quad \omega_{\mathbf{k}}^2(t) = k^2 + a^2(t)y^2\phi^2(t)$$



1.1 Fermion Parametric Excitation

Fermions: $y\phi\bar{\psi}\psi$: Oscillations \rightarrow ψ – Particle Creation
(Non-Pert., Out-of-Eq.)

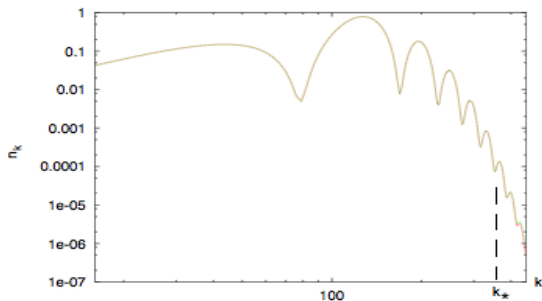
$$\frac{d^2}{dt^2} u_{\mathbf{k},\pm} + \left(\omega_{\mathbf{k}}^2(t) \pm i \frac{d(am_{\psi})}{dt} \right) u_{\mathbf{k},\pm}(t) = 0, \quad \frac{d}{dt} \omega_{\mathbf{k}} \gg \omega_{\mathbf{k}}^2(t)$$



1.1 Fermion Parametric Excitation

Fermions: $y\phi\bar{\psi}\psi$: Oscillations \rightarrow ψ - Particle Creation
(Non-Pert., Out-of-Eq.)

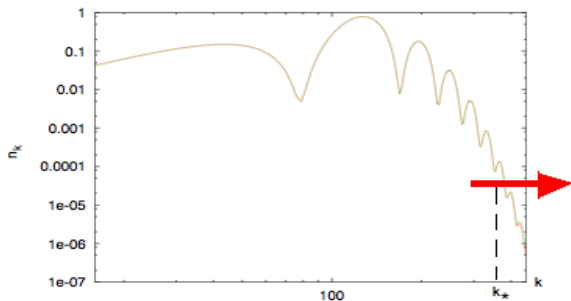
$$\frac{d^2}{dt^2} u_{\mathbf{k},\pm} + \left(\omega_{\mathbf{k}}^2(t) \pm i \frac{d(am_{\psi})}{dt} \right) u_{\mathbf{k},\pm}(t) = 0, \quad \frac{d}{dt} \omega_{\mathbf{k}} \gg \omega_{\mathbf{k}}^2(t)$$



1.1 Fermion Parametric Excitation

Fermions: $y\phi\bar{\psi}\psi$: Oscillations \rightarrow ψ - Particle Creation
(Non-Pert., Out-of-Eq.)

$$\frac{d^2}{dt^2} u_{\mathbf{k},\pm} + \left(\omega_{\mathbf{k}}^2(t) \pm i \frac{d(am_{\psi})}{dt} \right) u_{\mathbf{k},\pm}(t) = 0, \quad \frac{d}{dt} \omega_{\mathbf{k}} \gg \omega_{\mathbf{k}}^2(t)$$



1.2 GWs from Fermions

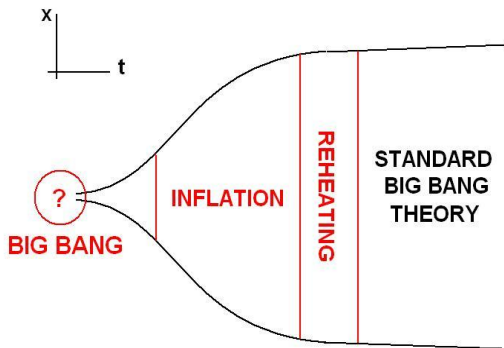
$$\text{Fermi-Sphere: } \left\{ \begin{array}{l} n_k(k \lesssim k_*) \lesssim 1, \\ n_k(k \gg k_*) \rightarrow 0 \end{array} \right\} \Rightarrow T_{\mu\nu}^{(\psi)} \sim (\bar{\psi} \gamma_{(\mu} D_{\nu)} \psi) [u_{k,\pm}(t)]$$
$$\left[\left(T_{\mu\nu}^{(\psi)} \right)^{\text{TT}} \rightarrow \text{GW Source !} \right]$$

$$\text{GWs: } \frac{d\rho_{\text{GW}}}{d \log k}(k, t) = \# \frac{Gk^3}{a^4(t)} \int_0^t \int_0^t dt_1 dt_2 \mathcal{G}(k, t_2 - t_1) \Pi^2(k, t_1, t_2)$$

$$\text{UTC: } \langle T_{ij}^{\text{TT}}(\mathbf{k}, t_1) T_{ij}^{\text{TT}}(\mathbf{k}', t_2) \rangle \equiv (2\pi)^3 \Pi^2(k, t_1, t_2) \delta^{(3)}(\mathbf{k} - \mathbf{k}')$$

2.1 SM Higgs during/after Inflation

Inflation \rightarrow Reheating \rightarrow hot Big Bang



2.1 SM Higgs during/after Inflation

Inflation: $dS(H_*)$, $(H_* \gg v \equiv 246 \text{ GeV})$

SM Higgs: $\Phi = \frac{\varphi}{\sqrt{2}} \rightarrow V(\varphi) = \frac{\lambda(\mu)}{4}\varphi^4$, $\mu = \varphi \gg v$

Prob. Dist: φ light ($|V''| < H_*^2$) \Rightarrow $\begin{cases} \text{Random Walk } (k < aH_*) \\ P_{\text{eq}}(\varphi) \propto \text{Exp}\{-c\frac{(\varphi/H_*)^4}{\lambda_*}\} \end{cases}$

End of Inflation: $\varphi_* = \alpha H_*/\lambda_*^{1/4}$ $\alpha \in [0.01, 1]$ (98 %)

2.1 SM Higgs during/after Inflation

Inflation: $dS(H_*)$, $(H_* \gg v \equiv 246 \text{ GeV})$

SM Higgs: $\Phi = \frac{\varphi}{\sqrt{2}} \rightarrow V(\varphi) = \frac{\lambda(\mu)}{4}\varphi^4$, $\mu = \varphi \gg v$

Prob. Dist: φ light ($|V''| < H_*^2$) \Rightarrow $\begin{cases} \text{Random Walk } (k < aH_*) \\ P_{\text{eq}}(\varphi) \propto \text{Exp}\{-c\frac{(\varphi/H_*)^4}{\lambda_*}\} \end{cases}$

End of Inflation: $\varphi_* = \alpha H_*/\lambda_*^{1/4}$ $\alpha \in [0.01, 1]$ (98 %)

2.1 SM Higgs during/after Inflation

Inflation: $dS(H_*)$, $(H_* \gg v \equiv 246 \text{ GeV})$

SM Higgs: $\Phi = \frac{\varphi}{\sqrt{2}} \rightarrow V(\varphi) = \frac{\lambda(\mu)}{4}\varphi^4$, $\mu = \varphi \gg v$

Prob. Dist: φ light ($|V''| < H_*^2$) \Rightarrow $\begin{cases} \text{Random Walk } (k < aH_*) \\ P_{\text{eq}}(\varphi) \propto \text{Exp}\{-c\frac{(\varphi/H_*)^4}{\lambda_*}\} \end{cases}$

End of Inflation: $\varphi_* = \alpha H_*/\lambda_*^{1/4}$ $\alpha \in [0.01, 1]$ (98 %)

2.1 SM Higgs during/after Inflation

Inflation: $dS(H_*)$, $(H_* \gg v \equiv 246 \text{ GeV})$

SM Higgs: $\Phi = \frac{\varphi}{\sqrt{2}} \rightarrow V(\varphi) = \frac{\lambda(\mu)}{4}\varphi^4$, $\mu = \varphi \gg v$

Prob. Dist: φ light ($|V''| < H_*^2$) \Rightarrow $\begin{cases} \text{Random Walk } (k < aH_*) \\ P_{\text{eq}}(\varphi) \propto \text{Exp}\{-c\frac{(\varphi/H_*)^4}{\lambda_*}\} \end{cases}$

End of Inflation: $\varphi_* = \alpha H_*/\lambda_*^{1/4}$ $\alpha \in [0.01, 1]$ (98 %)

2.1 SM Higgs during/after Inflation

Inflation: $dS(H_*)$, $(H_* \gg v \equiv 246 \text{ GeV})$

SM Higgs: $\Phi = \frac{\varphi}{\sqrt{2}} \rightarrow V(\varphi) = \frac{\lambda(\mu)}{4}\varphi^4$, $\mu = \varphi \gg v$

Prob. Dist: φ light ($|V''| < H_*^2$) \Rightarrow $\begin{cases} \text{Random Walk } (k < aH_*) \\ P_{\text{eq}}(\varphi) \propto \text{Exp}\{-c\frac{(\varphi/H_*)^4}{\lambda_*}\} \end{cases}$

End of Inflation: $\varphi_* \neq 0$ ($V \propto \varphi^4$) \rightarrow **Higgs Oscillations (!)**

Higgs Osc. \rightarrow SM Ψ 's Param. Exc. \rightarrow GWs

$$y_j \varphi \bar{\psi}_j \psi_j : \frac{d^2}{d\tau^2} u_{k,\pm}^{(j)} + (\kappa^2 + q_j (a\varphi)^2 \pm i\sqrt{q_j} \frac{d}{d\tau}(a\varphi)) u_{k,\pm}^{(j)} = 0, \quad q_j \equiv \frac{y_j^2}{\lambda_I}$$

Higgs Osc. \rightarrow SM Ψ 's Param. Exc. \rightarrow GWs

$$j = \left\{ \begin{array}{l} \{t, b, c, s, u, d\} \\ \{e, \mu, \tau\} \end{array} \right\} \Rightarrow \Omega_{\text{GW}}^{(j)}(k) \equiv \frac{1}{\rho_c} \frac{d\rho_{\text{GW}}}{d\log k}(k; q_j), \quad q_j \equiv \frac{y_j^2}{\lambda_I}$$

Higgs Osc. \rightarrow SM Ψ 's Param. Exc. \rightarrow GWs

$$n_k^{(j)}(k \lesssim k_*^{(j)}) \rightarrow \Omega_{\text{GW}}^{(j)}(k) : \left\{ \begin{array}{l} k_p \sim k_*^{(j)} \text{ (Max.)} \\ \propto k^3, k \ll k_p \\ \propto k^{-1.5}, k \gg k_p \end{array} \right\}$$

Higgs Osc. \rightarrow SM Ψ 's Param. Exc. \rightarrow GWs

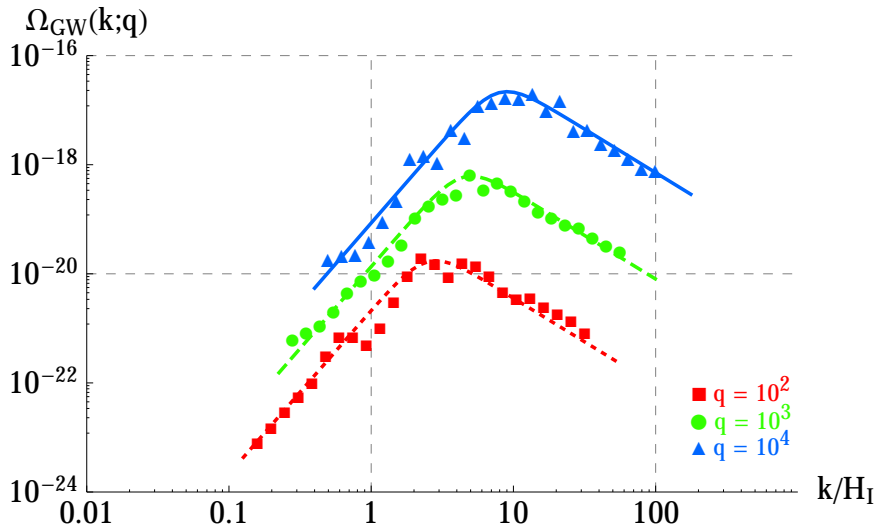
$$n_k^{(j)}(k \lesssim k_*^{(j)}) \rightarrow \Omega_{\text{GW}}^{(j)}(k) : \left\{ \begin{array}{l} k_p \sim k_*^{(j)} \text{ (Max.)} \\ \propto k^3, k \ll k_p \\ \propto k^{-1.5}, k \gg k_p \end{array} \right\} \propto q_j^{\frac{3}{2} + \delta}$$

$(\delta \ll 1)$

Higgs Osc. \rightarrow SM Ψ 's Param. Exc. \rightarrow GWs

$$n_k^{(j)}(k \lesssim k_*^{(j)}) \rightarrow \Omega_{\text{GW}}^{(j)}(k) : \left\{ \begin{array}{l} k_p \sim k_*^{(j)} \text{ (Max.)} \\ \propto k^3, k \ll k_p \\ \propto k^{-1.5}, k \gg k_p \end{array} \right\} \propto q_j^{\frac{3}{2} + \delta}$$
$$k_*^{(j)} \simeq q_j^{\frac{1}{4}} \sqrt{\lambda_I} \varphi_I \quad (\delta \ll 1)$$

Higgs Osc. \rightarrow SM Ψ 's Param. Exc. \rightarrow GWs

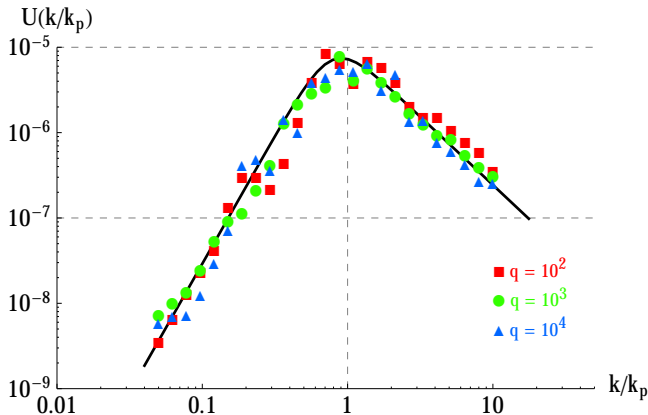


SCALING :

$$\Omega_{\text{GW}}(k; q_j) = (H_{\text{I}}/M_{\text{p}})^4 (a_{\text{I}}/a_{\text{F}})^{1-3w} \times q_j^{1.55} \mathcal{U}(k/k_p)$$

Higgs Osc. \rightarrow SM Ψ 's Param. Exc. \rightarrow GWs

$$\mathcal{U}(x) \equiv \mathcal{U}_1 \frac{x^3}{(\alpha + \beta x^{4.5})}, \quad \begin{cases} \mathcal{U}_1 \equiv \mathcal{U}(1) [\sim 10^{-5}(\text{RD}), \sim 10^{-6}(\text{MD})] \\ \alpha + \beta = 1 [\alpha = 0.25, \beta = 0.75 (\text{RD, MD})] \end{cases}$$



Total GWs :

$$h^2 \Omega_{\text{GW}}^{(0)}(f) \simeq \epsilon_{\text{I}} 10^{-6} (H_{\text{I}}/M_p)^4 \sum_j q_j^{1.55} \mathcal{U}(q_j^{-1/4}(k/H_{\text{I}}))$$

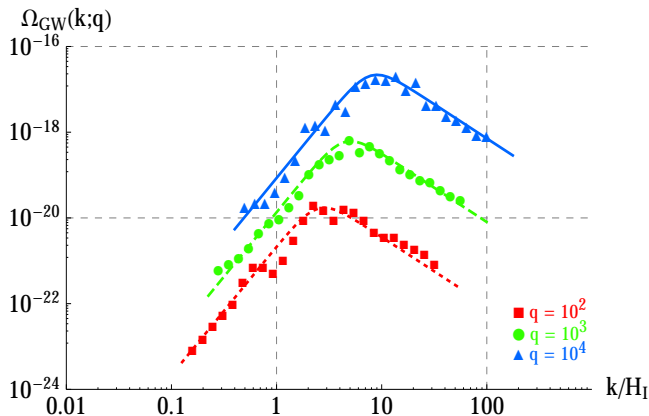
Higgs Osc. \rightarrow SM Ψ 's Param. Exc. \rightarrow GWs

SM Yukawa Couplings : $y_t > y_b > y_\tau > y_c > y_\mu \gtrsim y_s > y_d > y_u > y_e$

$$h^2 \Omega_{\text{GW}}^{(0)}(f) \propto q^{3/2} \propto y^3 \Rightarrow \text{Top Quark dominates (!)}$$

Higgs Osc. \rightarrow SM Ψ 's Param. Exc. \rightarrow GWs

Top Quark dominates (!)



Top Quark GW Peak:

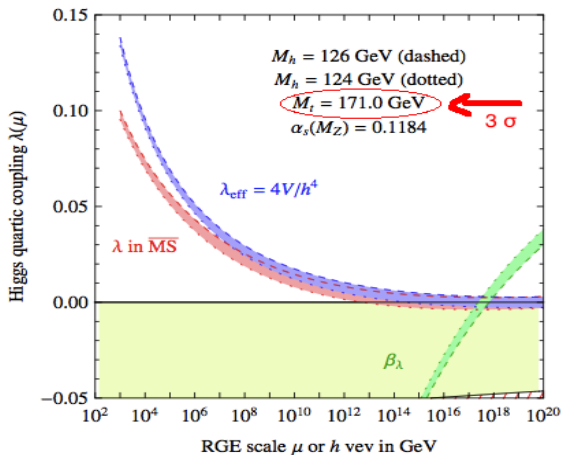
$$f_p^{(t)} \sim (H_*/H_*^<)^{1/2} \times 10^7 \text{ Hz}$$

$$h^2 \Omega_{\text{GW}}^{(p)}|_t \sim (H_*/H_*^<)^4 \lambda_I^{-1.55} 10^{-30}$$

$$[\text{CMB: } H_* \leq H_*^< \equiv 9 \cdot 10^{13} \text{ GeV}]$$

Higgs Osc. \rightarrow SM Ψ 's Param. Exc. \rightarrow GWs

$$\lambda_I \lesssim 10^{-7}, 10^{-10}, 10^{-13} \Rightarrow h^2 \Omega_{\text{GW}}^{(p)}|_t \gtrsim 10^{-20}, 10^{-15}, 10^{-10}$$



(Degrassi et Al, 2012)

Conclusions

- 1 Inflation, $dS(H_*) \Rightarrow$ Higgs Osc. $\Rightarrow \Psi$'s (Param. Excitation).
- 2 $\{\Psi_a\} \rightarrow h^2 \Omega_{\text{GW}}^{(0)}(f) \propto q_a^{3/2} \propto y_a^3 \Rightarrow$ Top Quark dominates (!)
- 3 $0 < \lambda_I \lll 1$, $H_* = 9 \cdot 10^{13}$ GeV, Peak's Frequency: $f_* \sim 10^7$ Hz
 $h^2 \Omega_{\text{GW}}^{(p)}|_t \gtrsim 10^{-20}, 10^{-15}, 10^{-10}$ ($\lambda_I \lesssim 10^{-7}, 10^{-10}, 10^{-13}$).
- 4 Similar Conclusions for Higgs Inflation ! Also expected in BSM !
- 5 Universal Effect: Inflation + SM Higgs \Rightarrow GWs: Spectroscopy of Particle Physics: Probing the Most Strongly Interacting Particle (!)

Conclusions

- 1 Inflation, $dS(H_*) \Rightarrow$ Higgs Osc. $\Rightarrow \Psi'$ s (Param. Excitation).
- 2 $\{\Psi_a\} \rightarrow h^2 \Omega_{\text{GW}}^{(0)}(f) \propto q_a^{3/2} \propto y_a^3 \Rightarrow$ Top Quark dominates (!)
- 3 $0 < \lambda_I \lll 1$, $H_* = 9 \cdot 10^{13}$ GeV, Peak's Frequency: $f_* \sim 10^7$ Hz
 $h^2 \Omega_{\text{GW}}^{(p)}|_t \gtrsim 10^{-20}, 10^{-15}, 10^{-10}$ ($\lambda_I \lesssim 10^{-7}, 10^{-10}, 10^{-13}$).
- 4 Similar Conclusions for Higgs Inflation ! Also expected in BSM !
- 5 Universal Effect: Inflation + SM Higgs \Rightarrow GWs: Spectroscopy of Particle Physics: Probing the Most Strongly Interacting Particle (!)

Conclusions

- 1 Inflation, $dS(H_*) \Rightarrow$ Higgs Osc. $\Rightarrow \Psi'$ s (Param. Excitation).
- 2 $\{\Psi_a\} \rightarrow h^2 \Omega_{\text{GW}}^{(0)}(f) \propto q_a^{3/2} \propto y_a^3 \Rightarrow$ Top Quark dominates (!)
- 3 $0 < \lambda_I \lll 1$, $H_* = 9 \cdot 10^{13}$ GeV, Peak's Frequency: $f_* \sim 10^7$ Hz
 $h^2 \Omega_{\text{GW}}^{(p)}|_t \gtrsim 10^{-20}, 10^{-15}, 10^{-10}$ ($\lambda_I \lesssim 10^{-7}, 10^{-10}, 10^{-13}$).
- 4 Similar Conclusions for Higgs Inflation ! Also expected in BSM !
- 5 Universal Effect: Inflation + SM Higgs \Rightarrow GWs: Spectroscopy of Particle Physics: Probing the Most Strongly Interacting Particle (!)

Conclusions

- 1 Inflation, $dS(H_*) \Rightarrow$ Higgs Osc. $\Rightarrow \Psi'$ s (Param. Excitation).
- 2 $\{\Psi_a\} \rightarrow h^2 \Omega_{\text{GW}}^{(0)}(f) \propto q_a^{3/2} \propto y_a^3 \Rightarrow$ Top Quark dominates (!)
- 3 $0 < \lambda_I \lll 1$, $H_* = 9 \cdot 10^{13}$ GeV, Peak's Frequency: $f_* \sim 10^7$ Hz
 $h^2 \Omega_{\text{GW}}^{(p)}|_t \gtrsim 10^{-20}, 10^{-15}, 10^{-10}$ ($\lambda_I \lesssim 10^{-7}, 10^{-10}, 10^{-13}$).
- 4 **Similar Conclusions for Higgs Inflation ! Also expected in BSM !**
- 5 Universal Effect: Inflation + SM Higgs \Rightarrow GWs: Spectroscopy of Particle Physics: Probing the Most Strongly Interacting Particle (!)

Conclusions

- 1 Inflation, $dS(H_*) \Rightarrow$ Higgs Osc. $\Rightarrow \Psi's$ (Param. Excitation).
- 2 $\{\Psi_a\} \rightarrow h^2 \Omega_{\text{GW}}^{(0)}(f) \propto q_a^{3/2} \propto y_a^3 \Rightarrow$ Top Quark dominates (!)
- 3 $0 < \lambda_I \lll 1$, $H_* = 9 \cdot 10^{13}$ GeV, Peak's Frequency: $f_* \sim 10^7$ Hz
 $h^2 \Omega_{\text{GW}}^{(p)}|_t \gtrsim 10^{-20}, 10^{-15}, 10^{-10}$ ($\lambda_I \lesssim 10^{-7}, 10^{-10}, 10^{-13}$).
- 4 Similar Conclusions for Higgs Inflation ! Also expected in BSM !
- 5 **Universal Effect: Inflation + SM Higgs \Rightarrow GWs: Spectroscopy of Particle Physics: Probing the Most Strongly Interacting Particle (!)**