Lecture 1

Jean-Philippe UZAN

What is a constant?

Constant : PHYS., Numerical value of ***some*** quantity that allows to characterize a body. Quantity whose value is fixed (***e.g.*** mass and charge of the electron, speed of light) and that plays a ***central*** role in physical theories.

This definition asks more questions than it gives answers!

- How many constants?
- Are they all on the same footing?
- What role do they play in laws of physics?
- Can they vary? *(according to the dictionary, NO!)*

Making a list of constants

Let us start to look in a book of physics (probably the best place to find constants)
depends on *when* and by *who* the book was written

Any parameter not determined by the theories at hand

It has to be assume constant (no equation/ nothing more fundamental)

Reproductibility of experiments.

It does not show our *knowledge* but our *ignorance*

Studying the constant of a theory =
To study the limits of this theory

How many fundamental constants should we consider today?

Reference theoretical framework

The number of physical constants depends on the level of description of the laws of nature.

In our present understanding [*General Relativity + SU(3)xSU(2)xU(1)*]:

- G : Newton constant (**1**)

- **6** Yukawa coupling for quarks
- **3** Yukawa coupling for leptons

- mass and VEV of the Higgs boson: **2**

- CKM matrix: **4** parameters
- Non-gravitational coupling constants: **3**
- Λ_{uv}: **1**

- c, ħ : **2**

- cosmological constant

22 constants
19 parameters

Constant	Symbol	Value
Speed of light	c	299 792 458 m s^{-1}
Planck constant (reduced)	$\hbar$	1.054 571 628(53) × 10^{-34} J s
Newton constant	G	6.674 28(67) × 10^{-11} m^2 kg^{-1} s^{-2}
Weak coupling constant (at m_Z)	$g_2(m_Z)$	0.6520 ± 0.0001
Strong coupling constant (at m_Z)	$g_3(m_Z)$	1.221 ± 0.022
Weinberg angle	$\sin^2\theta_W(91.2\ \text{GeV})_{\overline{\text{MS}}}$	0.23120 ± 0.00015
Electron Yukawa coupling	h_e	2.94 × 10^{-6}
Muon Yukawa coupling	h_μ	0.000607
Tauon Yukawa coupling	h_τ	0.0102156
Up Yukawa coupling	h_u	0.000016 ± 0.000007
Down Yukawa coupling	h_d	0.00003 ± 0.00002
Charm Yukawa coupling	h_c	0.0072 ± 0.0006
Strange Yukawa coupling	h_s	0.0006 ± 0.0002
Top Yukawa coupling	h_t	1.002 ± 0.029
Bottom Yukawa coupling	h_b	0.026 ± 0.003
Quark CKM matrix angle	$\sin\theta_{12}$	0.2243 ± 0.0016
	$\sin\theta_{23}$	0.0413 ± 0.0015
	$\sin\theta_{13}$	0.0037 ± 0.0005
Quark CKM matrix phase	δ_{CKM}	1.05 ± 0.24
Higgs potential quadratic coefficient	$\hat{\mu}^2$	?
Higgs potential quartic coefficient	λ	?
QCD vacuum phase	θ_{QCD}	< 10^{-9}

Number of constants may change

This number is « time-dependent ».

Neutrino masses

Add **3** Yukawa couplings + **4** MNS parameters = **7** more

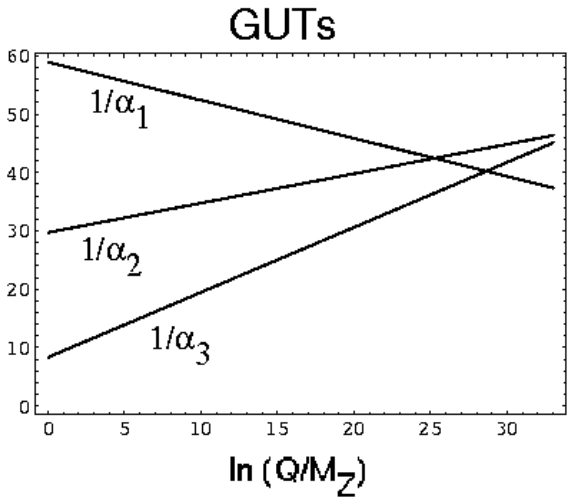
Unification

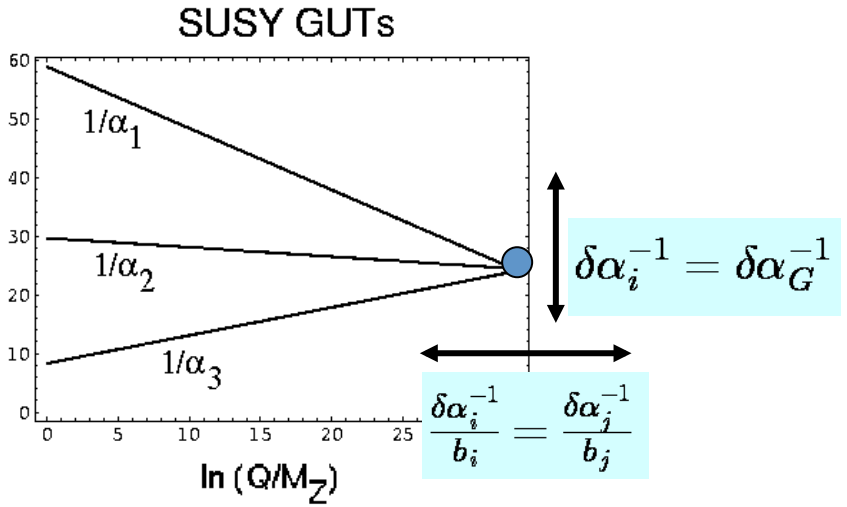
$$\alpha_i^{-1}(E) = \alpha_{GUT}^{-1} + \frac{b_i}{2\pi}\ln\frac{M_{GUT}}{E}$$

$$\text{SM}: \quad b_i = (41/10, -19/6, -7)$$

$$\text{MSSM}: \quad b_i = (33/5, 1, -3)$$

$$\alpha^{-1} = \tfrac{5}{3}\alpha_1^{-1} + \alpha_2^{-1}$$





Three classes of constants

Does this mean that all constants are to be put on the same footing?

- **Class A** : characterizes a *given* physical system,

  e.g. : mass of the electron

- **Class B** : characterizes a *class* of phenomena,

  e.g.: charge of the electron

- **Class C** : universal constant,

  e.g.: speed of light, Planck constant, gravitation constant

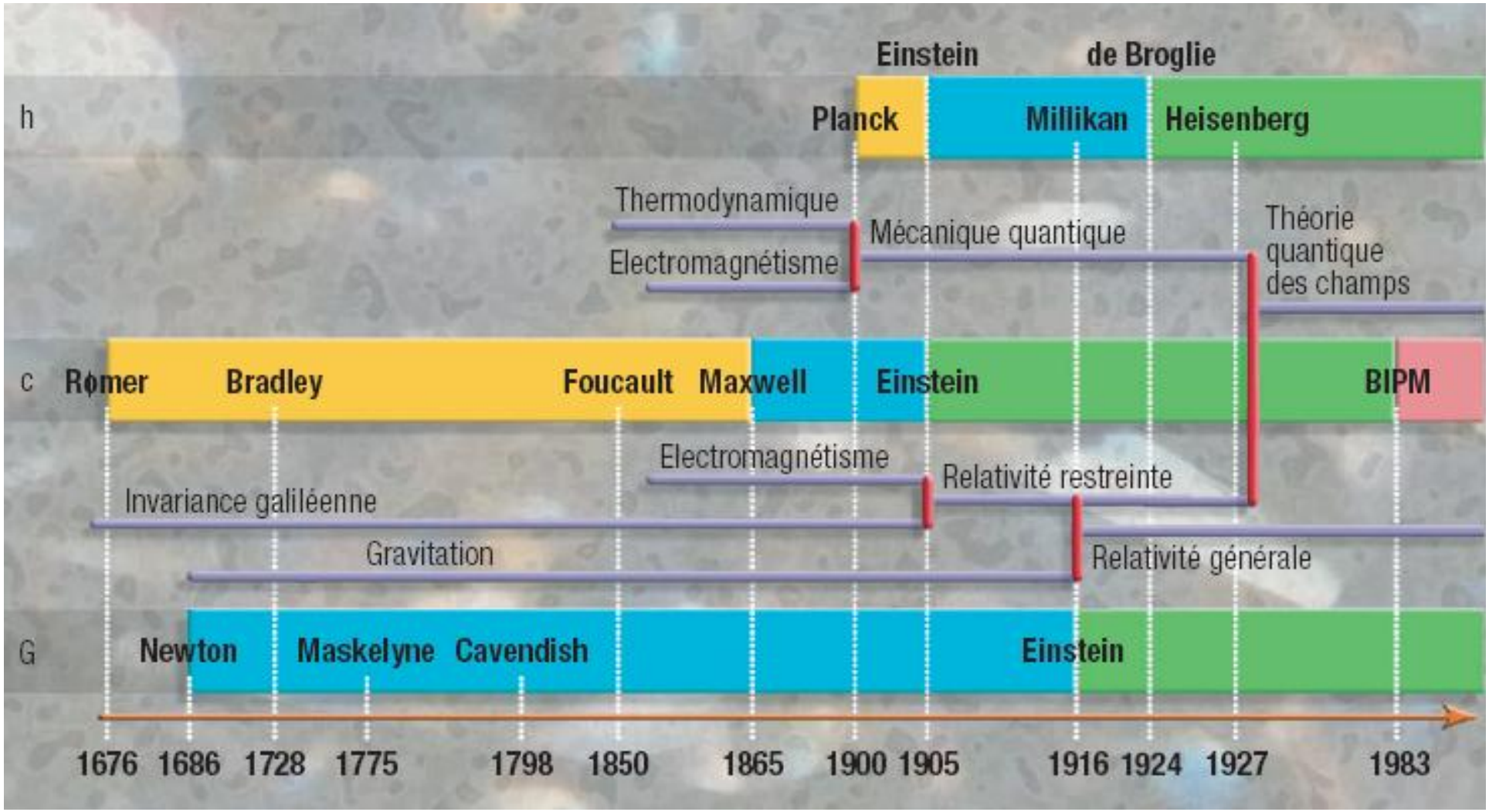
The classification depends on time!

The 3 fundamental constants played a role of ***concept synthesizers***:
they created bridges between concept that were incompatible before

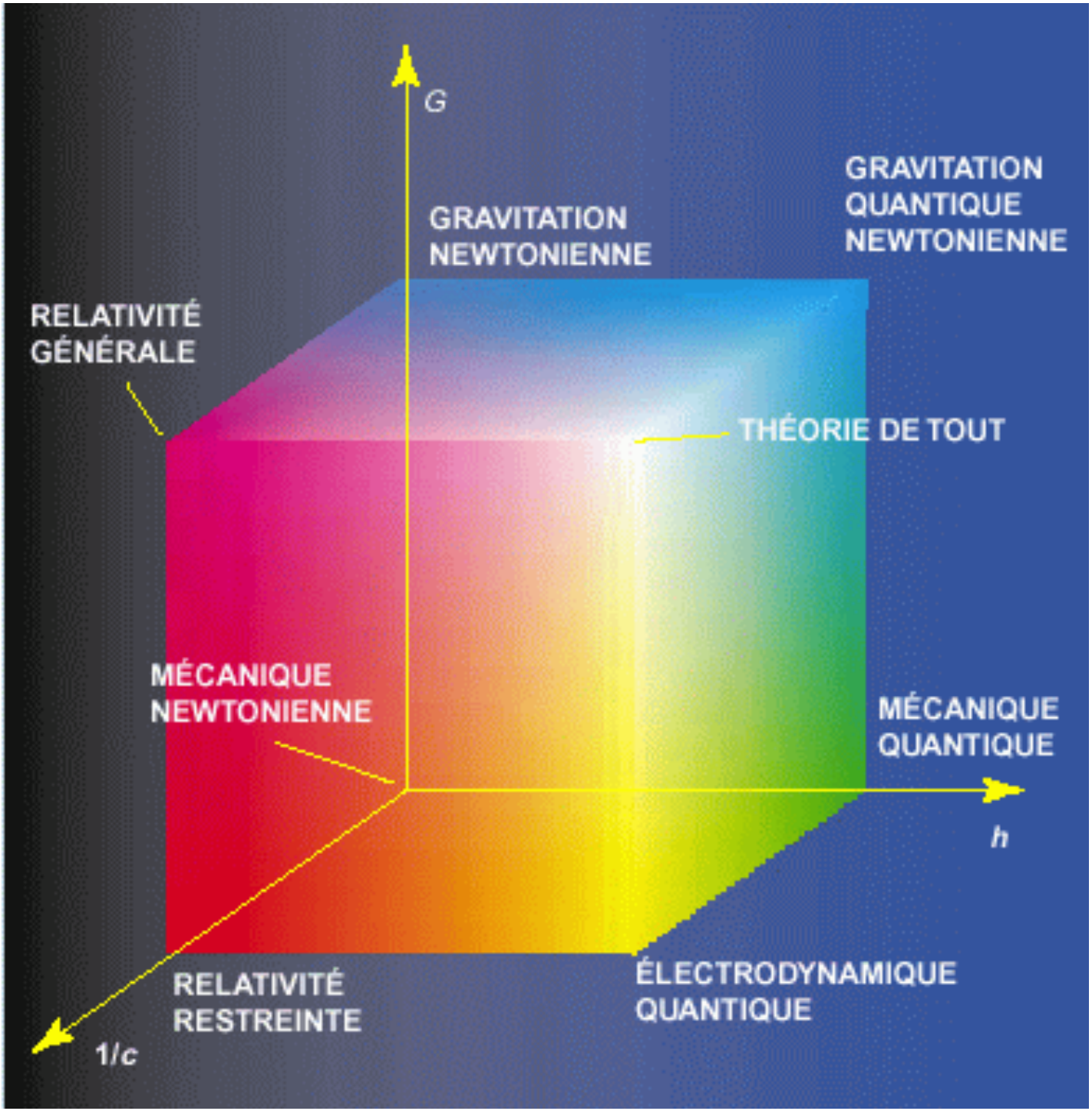
space & time ⟶ spacetime

particle & waves ⟶ wave function

Change of classes and history of physics

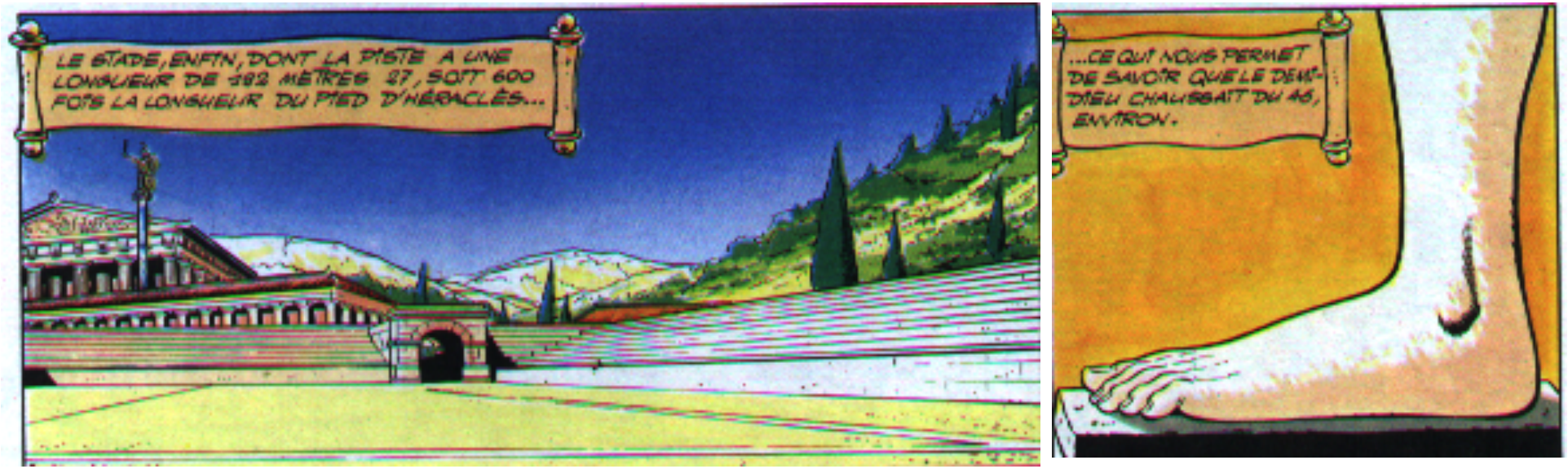


JPU, B. Leclercq, De *l'importance d'être une constante* (Dunod, 2005)
translated as "*The natural laws of the universe*" (Praxis, 2008)


G
GRAVITATION
NEWTONIENNE
GRAVITATION
QUANTIQUE
NEWTONIENNE
RELATIVITÉ
GÉNÉRALE
THÉORIE DE TOUT
MÉCANIQUE
NEWTONIENNE
MÉCANIQUE
QUANTIQUE
h
RELATIVITÉ
RESTREINTE
ÉLECTRODYNAMIQUE
QUANTIQUE
1/c

From units to constants

Units systems were initially very *anthropomorphic*



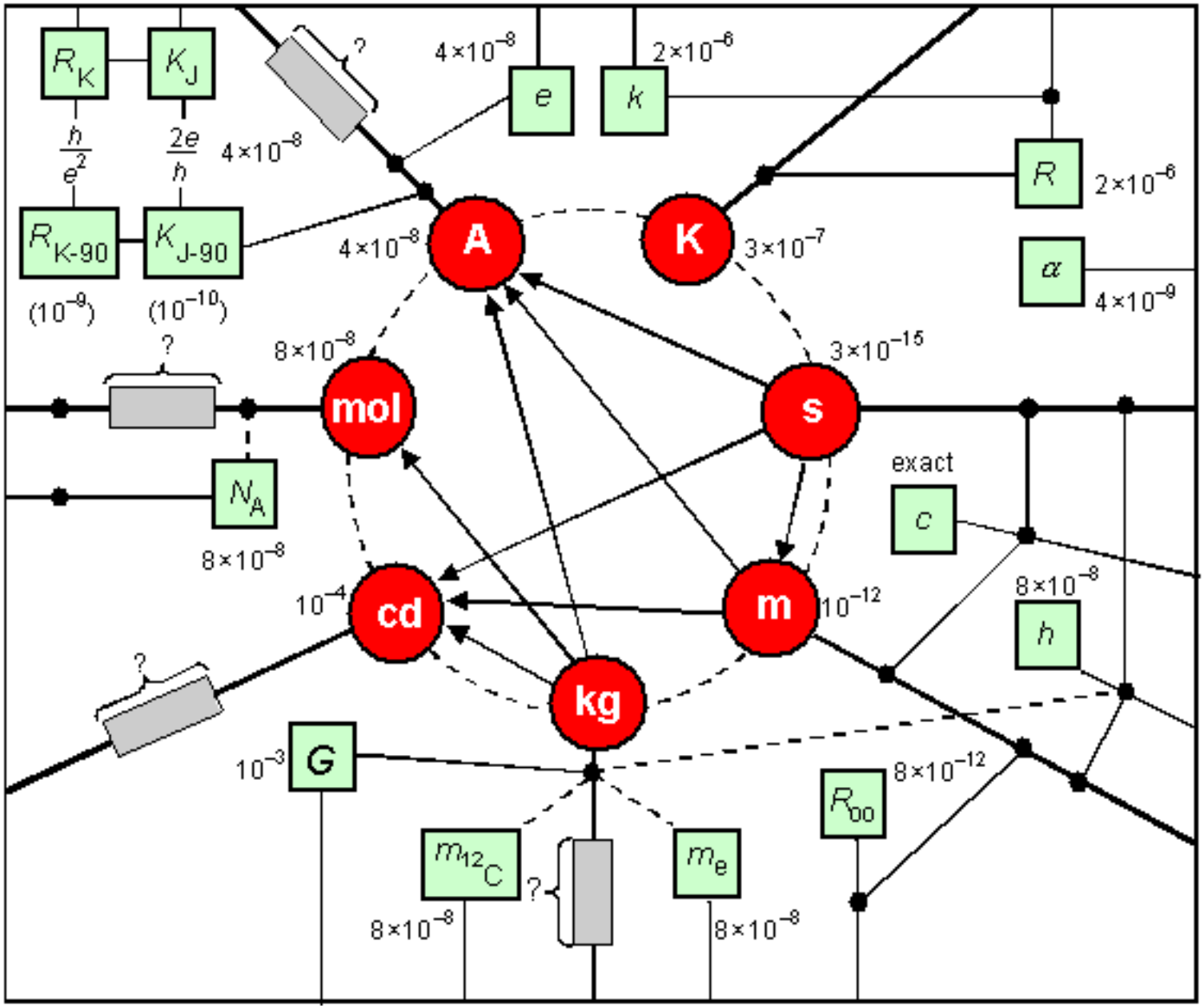
They depend on some reference person
Vary from a region to another, confusion of names etc…

French revolution

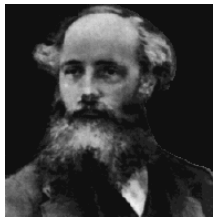
26 March 1791, pushed by Charles Maurice Talleyrand, the **meter** is defined as 1/40,000,000 of the length of a meridian

The metre

International system of units



From units to constants

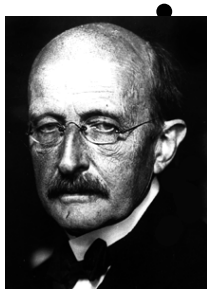


J.C. Maxwell (1870)

« If we wish to obtain standards of length, time and mass which shall be absolutely permanent, we must seek them not in the dimensions, or motion or the mass of our planet, but in the wavelength, the period of vibration, and absolute mass of these imperishable and unalterable and perfectly similar molecules. »

G. Johnstone-Stoney (1881)

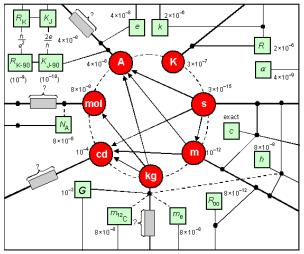
« Nature presents us with 3 such units »



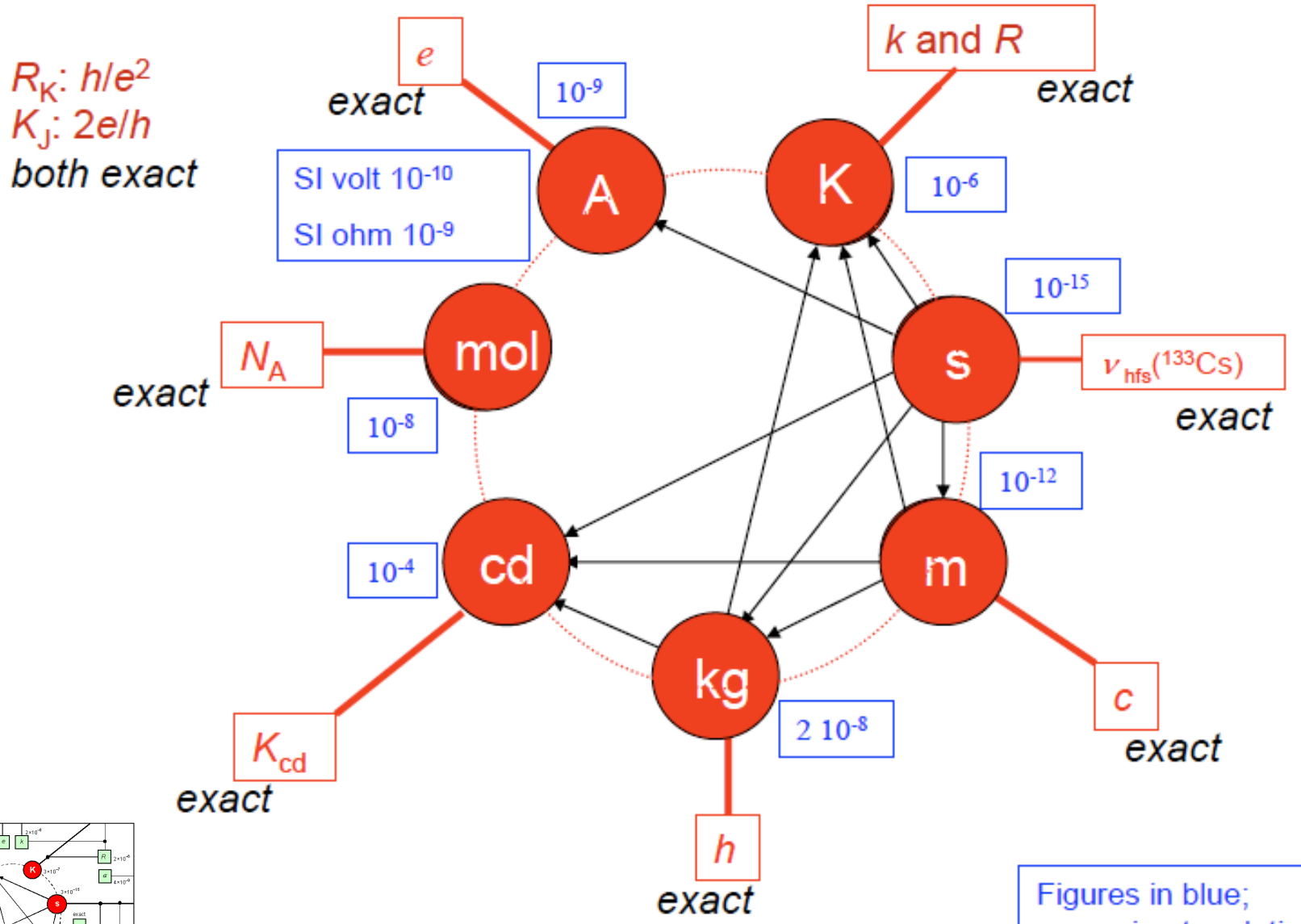
Planck (1900)

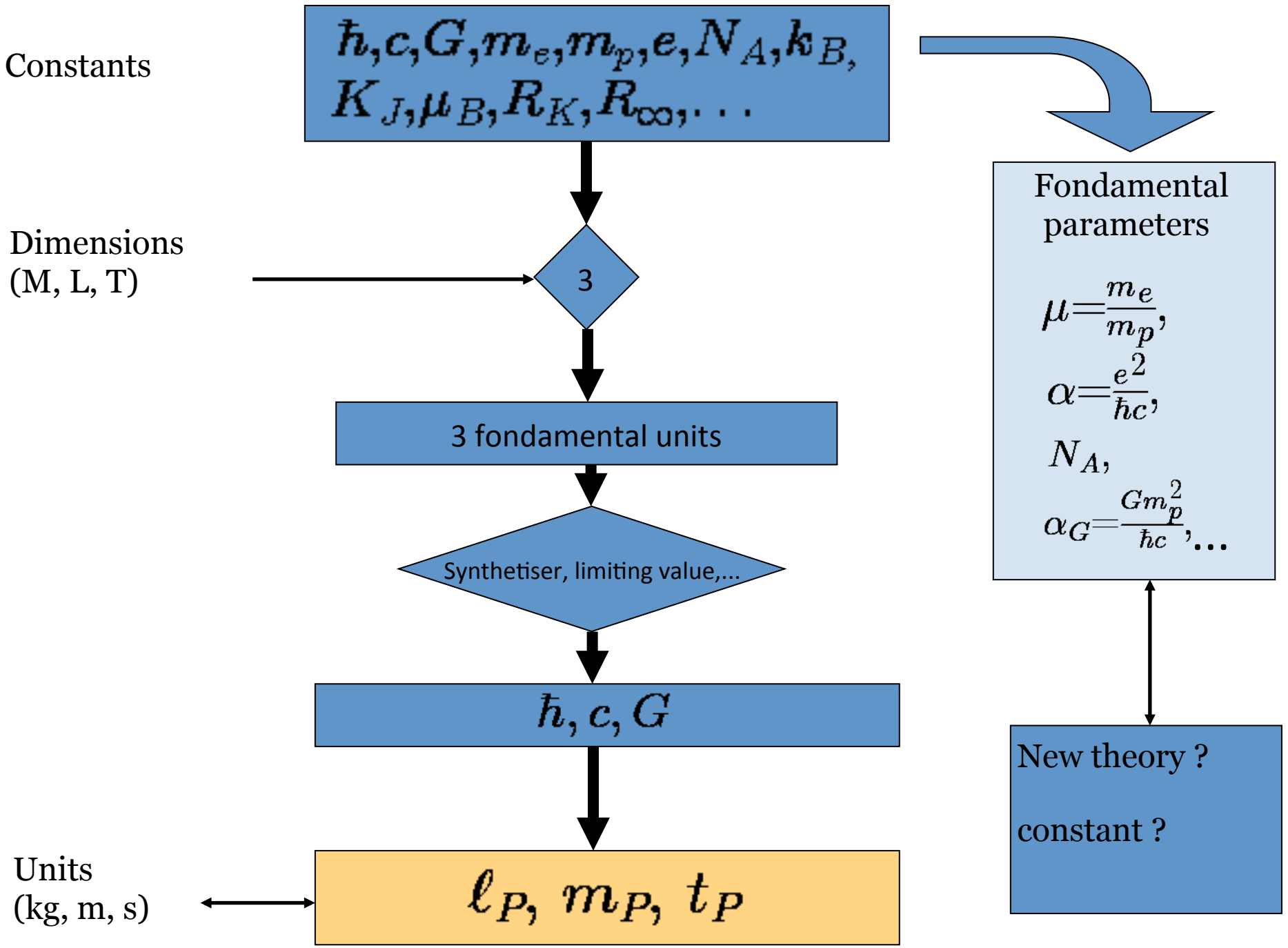
« It offers the possibility of establishing units for length, mass, time and temperature which are independent of specific bodies or materials and which necessarily maintain their meaning for all time and for all civilizations, even those which are extraterrestrial and nonhuman, constants which therefore can be called fundamental physical units of measurement »

Proposal for the new SI



http://www.bipm.org/en/si/new_si/




Constants
$\hbar, c, G, m_e, m_p, e, N_A, k_B, K_J, \mu_B, R_K, R_\infty, \ldots$
Dimensions
(M, L, T)
3
3 fondamental units
Synthetiser, limiting value,...
$\hbar, c, G$
Units
(kg, m, s)
$\ell_P,\ m_P,\ t_P$
Fondamental parameters
$\mu = \frac{m_e}{m_p}$,
$\alpha = \frac{e^2}{\hbar c}$,
N_A,
$\alpha_G = \frac{G m_p^2}{\hbar c}$,...
New theory ?
constant ?

Summary

Any parameter not determined by the theories at hand

- *Hence, it depends on our knowledge of physics*

All the constants do not have the same status

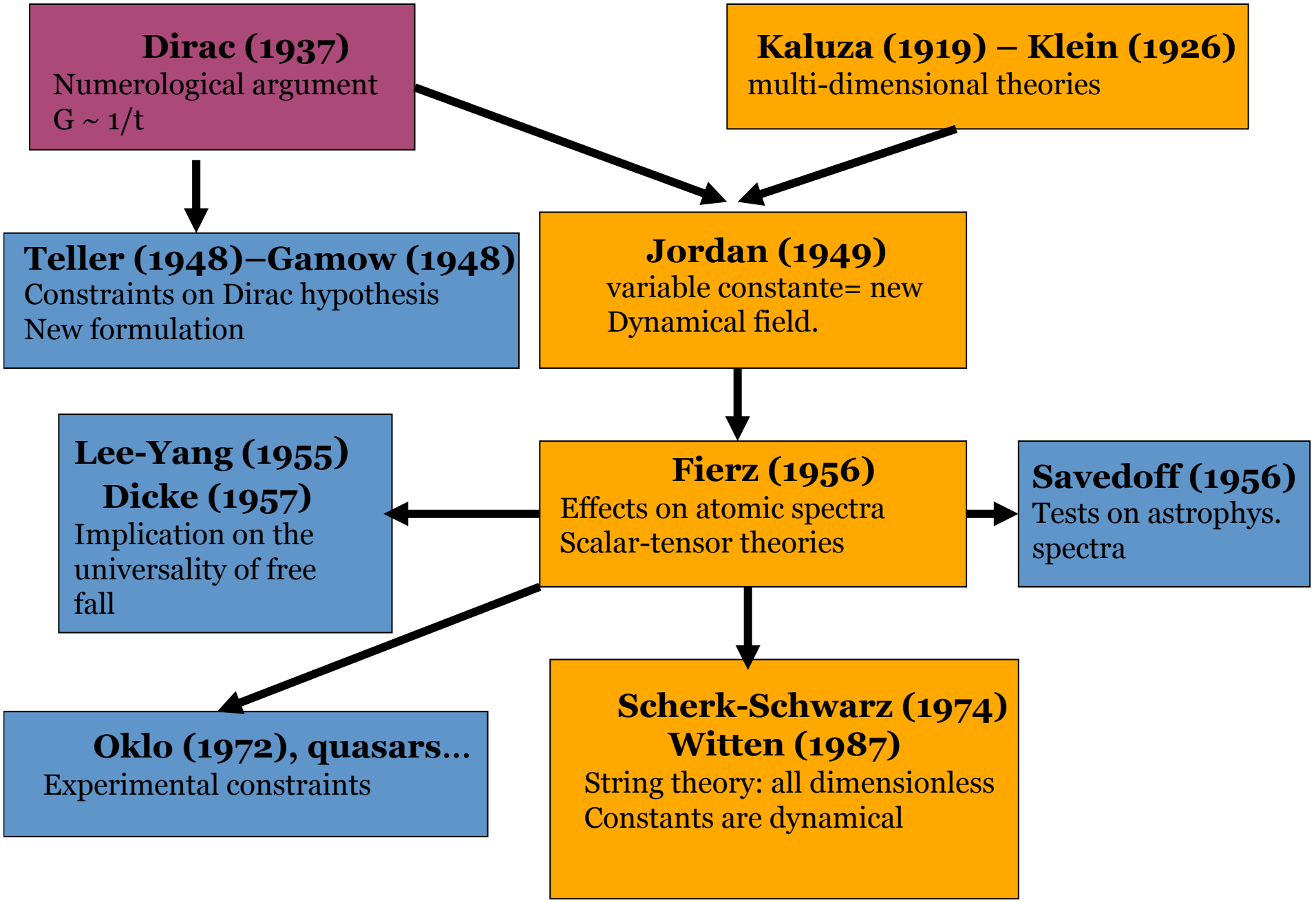
- *The change of status told us about the evolution of physics*

We can define units from constants

- *Recent evolution in metrology and SI*

We are left with pure numbers

- *Why are they constant?*
- *Can we explain their value?*


Dirac (1937)
Numerological argument
G ~ 1/t
Kaluza (1919) – Klein (1926)
multi-dimensional theories
Teller (1948)–Gamow (1948)
Constraints on Dirac hypothesis
New formulation
Jordan (1949)
variable constante= new
Dynamical field.
Lee-Yang (1955)
Dicke (1957)
Implication on the
universality of free
fall
Fierz (1956)
Effects on atomic spectra
Scalar-tensor theories
Savedoff (1956)
Tests on astrophys.
spectra
Oklo (1972), quasars…
Experimental constraints
Scherk-Schwarz (1974)
Witten (1987)
String theory: all dimensionless
Constants are dynamical

Fundamental constants & gravity

Equivalence principle

« C'est alors, considérant ces faits, qu'il me vint à l'esprit que si l'on supprimait totalement la résistance du milieu, tous les corps descendraient avec la même vitesse. »

Galilée, *in Discours concernant deux sciences nouvelles*, 1638
Traduction de Maurice Clavelin, PUF, 1995.

« Il y a une puissance de la gravité, qui concerne tous les corps, proportionnelle aux différentes quantités de matière qu'ils contiennent. »

« Cette force est toujours proportionnelle à la quantité de matière des corps, & elle ne diffère de ce qu'on appelle l'inertie de la matière que par la manière de la concevoir. »

« La force de la pesanteur entre les différentes particules de tout corps est inversement proportionnelle au carré des distances des positions des particules. »

Isaac Newton, *in Principia*, Londres, 1687
Traduction d'Émilie du Châtelet, Paris, 1759.

The equivalence principle in Newtonian physics

Inertial mass is the mass that appears in Netwon's law of motion.

$$F = m_I\, a\,,$$

Passive gravitational mass is the mass that characterizes the response to a gravitational field (notion of weight)

$$F = m_G\, g$$

Active gravitational mass characterizes the strength of the gravitational field created by an object

$$F_{AB} \propto m_{G,A}^{act} m_{G,B}^{pass}$$

Action-reaction law implies that $m_{G,A}^{act} m_{G,B}^{pass} = m_{G,B}^{act} m_{G,A}^{pass}$

And thus, m_G^{act}/m_G^{pass} is a constant, that can be chosen to be 1.

The equivalence principle in Newtonian physics

The deviation from the universality of free fall is characterized by

$$\eta \equiv 2\frac{|a_1 - a_2|}{|a_1 + a_2|}$$

Second law: $F = m_I a$

Definition of weight $F = m_G g$

$$\Big\} \quad a = (m_G/m_I)g,$$

So that $$\eta = 2\frac{|m_G^1/m_I^1 - m_G^2/m_I^2|}{m_G^1/m_I^1 + m_G^2/m_I^2}$$

Consider a pendumum of length L in a gravitational field g,

$$\ddot{\theta} + \omega^2\theta = 0 \quad \text{où} \quad \omega \equiv \omega_0\sqrt{\frac{m_G}{m_I}} \quad \text{et} \quad \omega_0 \equiv \sqrt{\frac{g}{L}}.$$

Then $$\eta \approx 2\frac{|\omega_B - \omega_A|}{\omega_0}$$

On the basis of general relativity

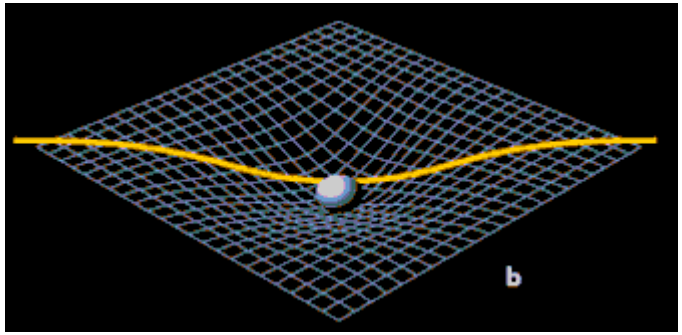
The equivalence principle takes much more importance in general relativity

It is based on **Einstein equivalence principle**

universality of free fall
local Lorentz invariance
local position invariance

Not a basic principle of physics but mostly an empirical fact.

If this principle holds then gravity is a consequence of the geometry of spacetime



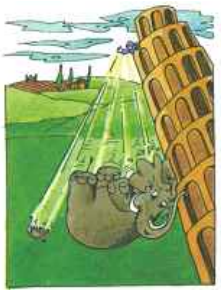
This principle has been a driving idea in theories of gravity from Galileo to Einstein

GR in a nutshell

Underlying hypothesis

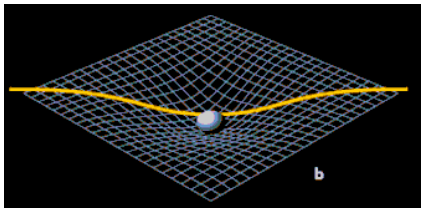
Equivalence principle

- Universality of free fall
- Local lorentz invariance
- Local position invariance



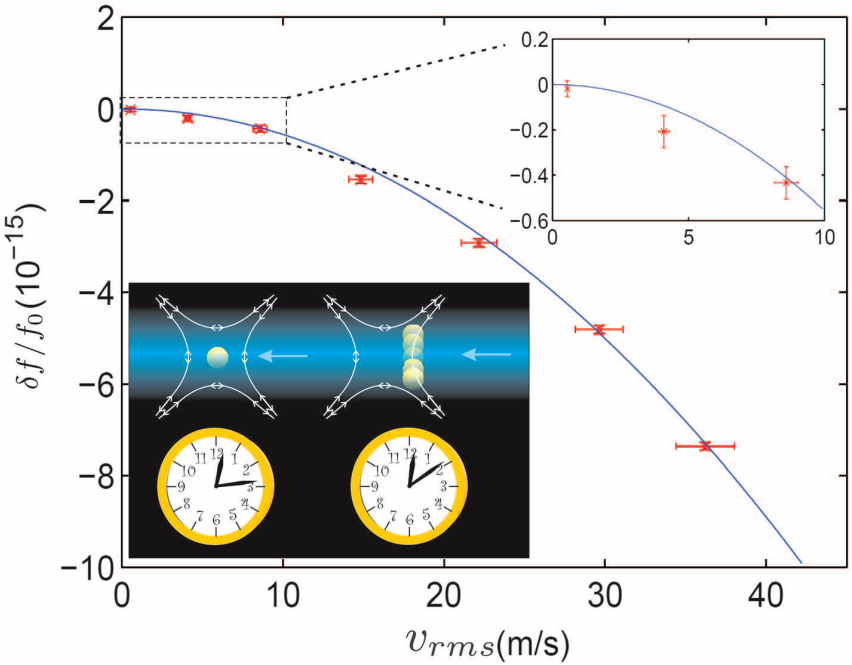
Physical metric

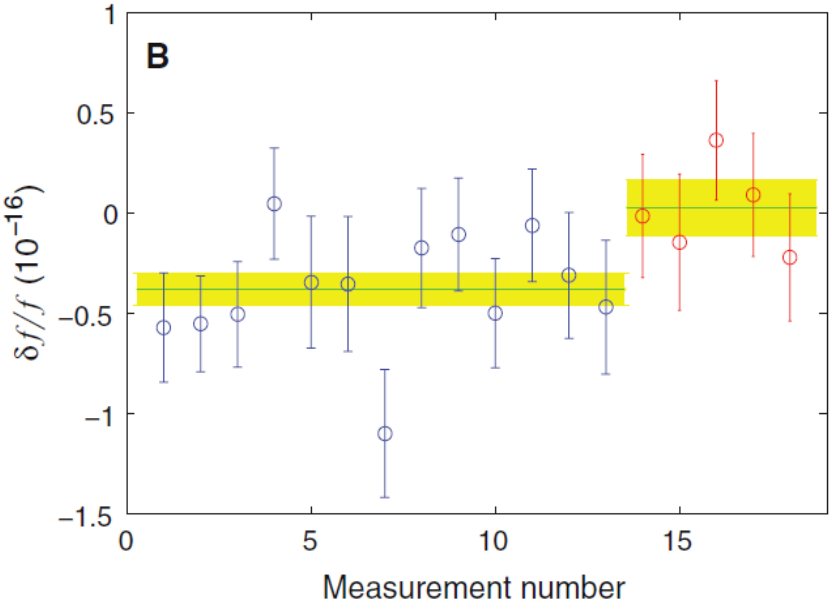
$$S_{matter}(\psi, g_{\mu\nu})$$



Gravirational redshift at 30 cm level

C. W. Chou,* D. B. Hume, T. Rosenband, D. J. Wineland, Science 329, 1630, (2010)





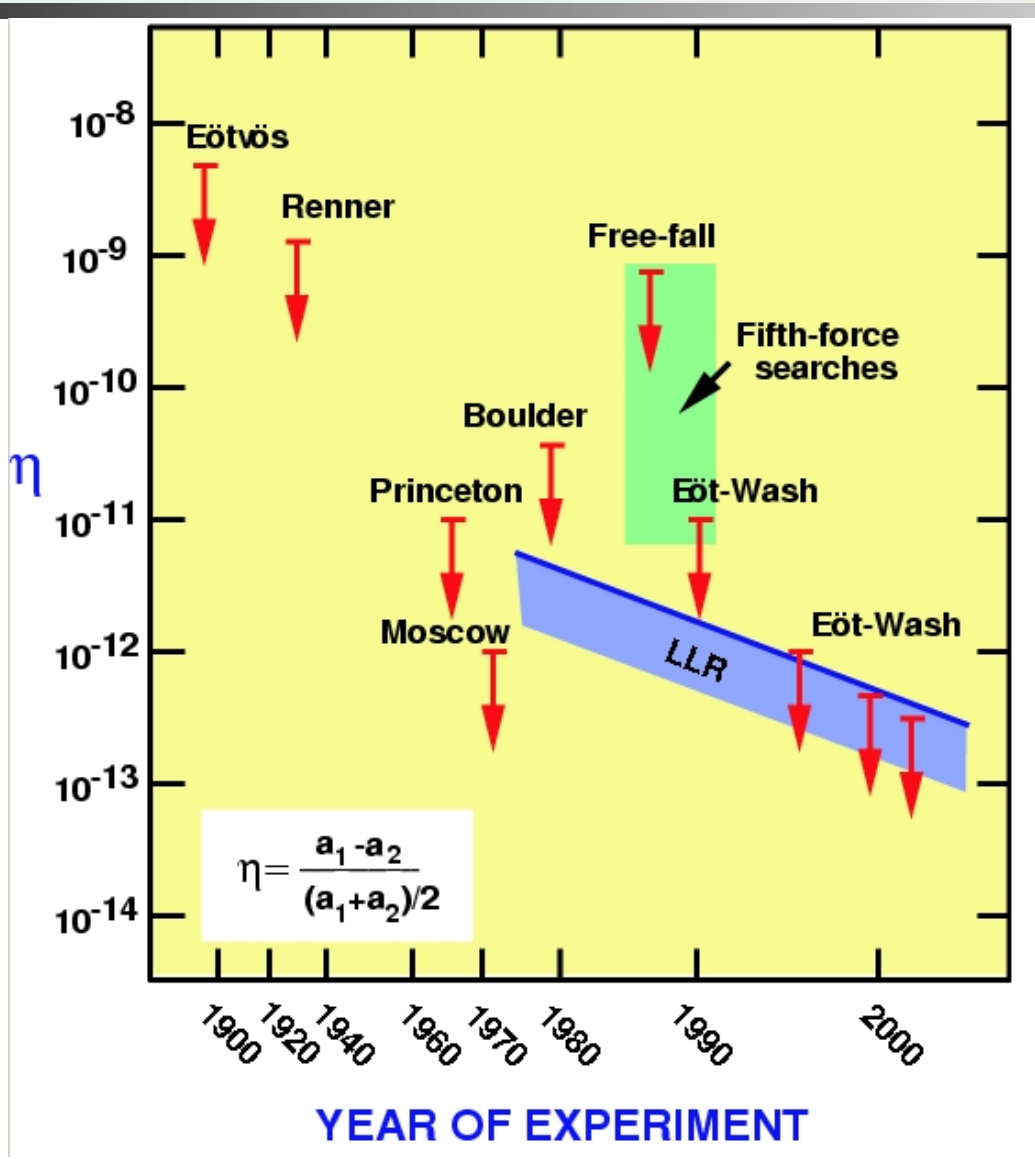
Time dilation

Gravitational shift

Clock B is lifted up by 33 cm

its rate is increased by 3. 4 10^{-17}

Current contraints



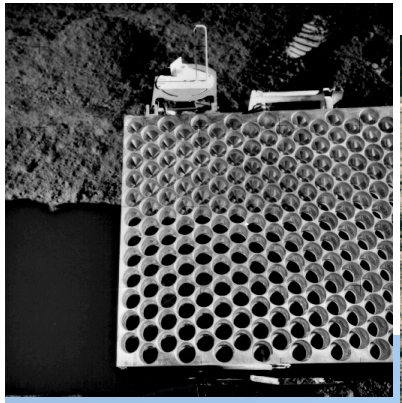
$$\eta_{\mathrm{Te,Bi}} = (0.3 \pm 1.8) \times 10^{-13}. \quad \text{[Schlamminger, 2008]}$$

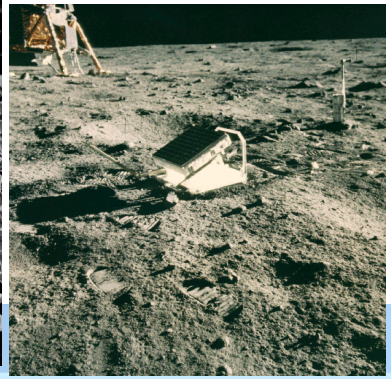
Current contraints

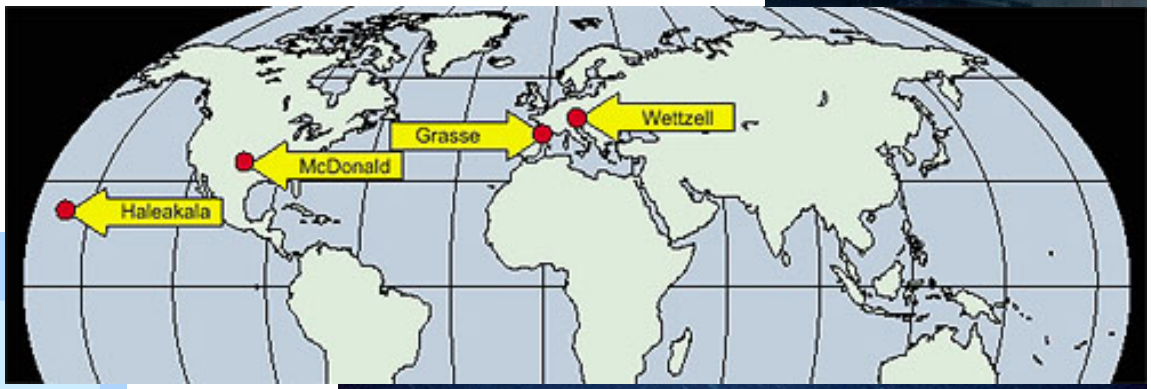
Table 3: Summary of the constraints on the violation of the universality of free fall.

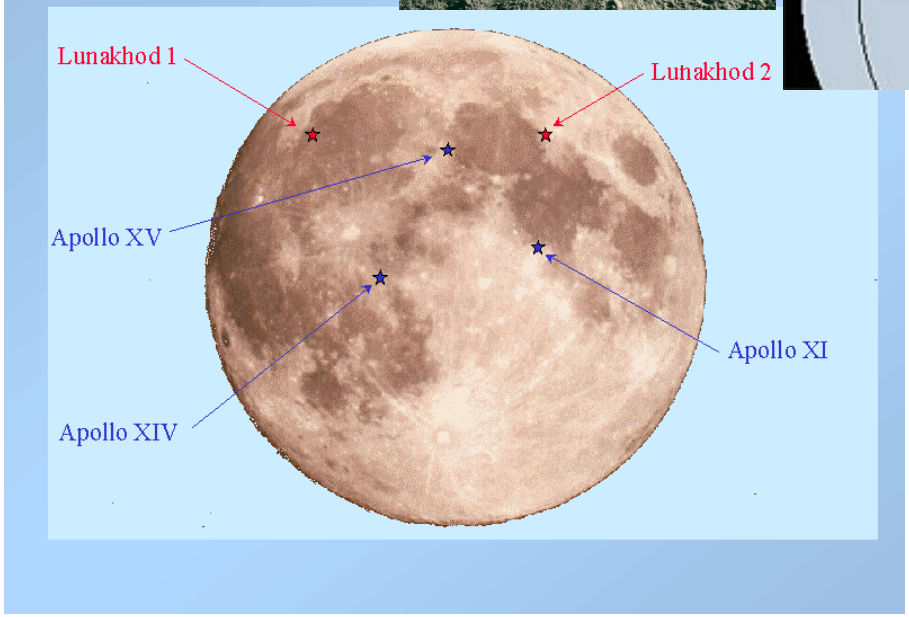
Constraint	Body 1	Body 2	Ref.
$(-1.9 \pm 2.5) \times 10^{-12}$	Be	Cu	[4]
$(0.1 \pm 2.7 \pm 1.7) \times 10^{-13}$	Earth-like rock	Moon-like rock	[23]
$(-1.0 \pm 1.4) \times 10^{-13}$	Earth	Moon	[543]
$(0.3 \pm 1.8) \times 10^{-13}$	Te	Bi	[450]
$(-0.2 \pm 2.8) \times 10^{-12}$	Be	Al	[481]
$(-1.9 \pm 2.5) \times 10^{-12}$	Be	Cu	[481]
$(5.1 \pm 6.7) \times 10^{-12}$	Si/Al	Cu	[481]

Lunar laser ranging



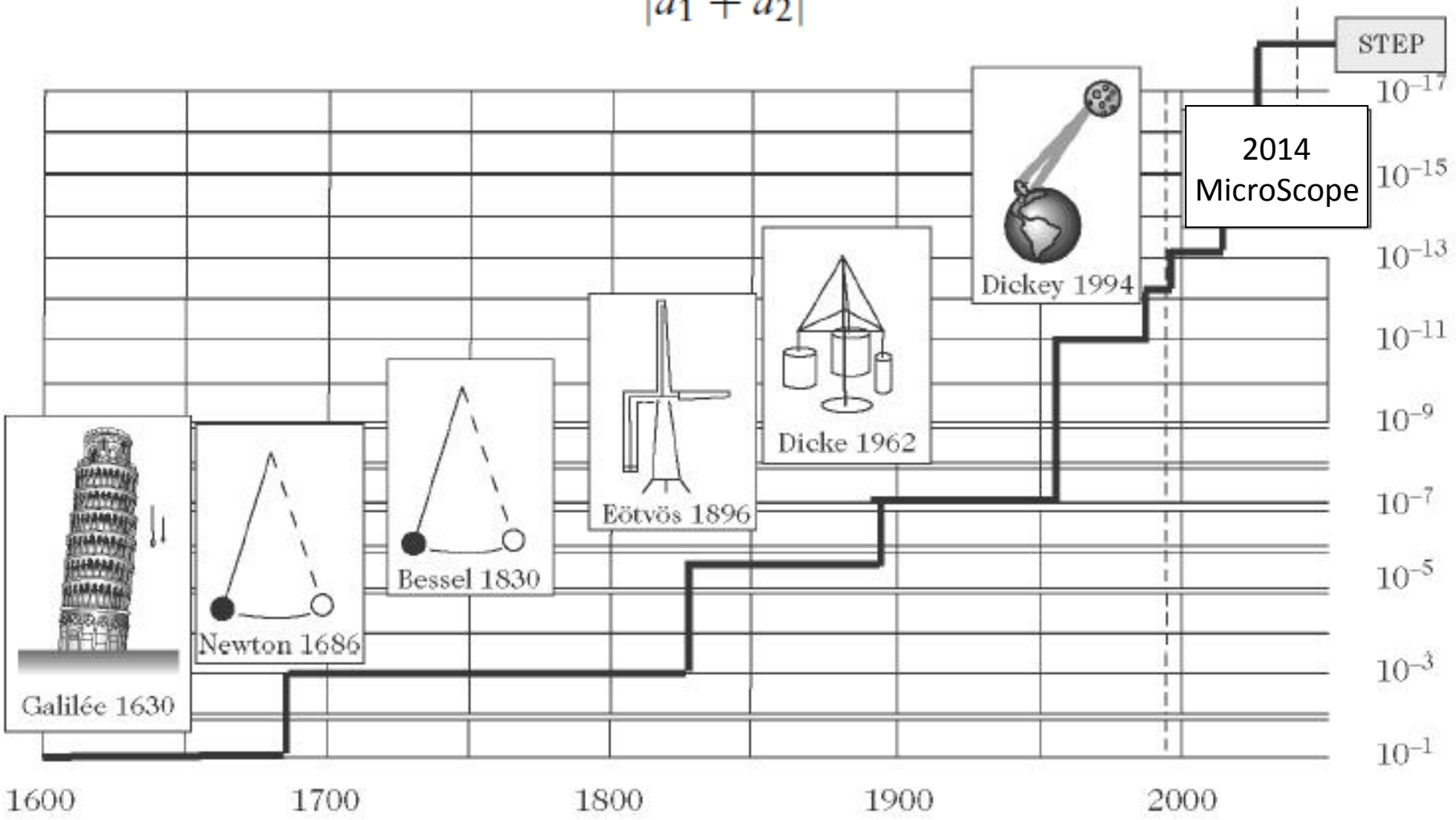






Tests on the universality of free fall

$$\eta \equiv 2\frac{|a_1 - a_2|}{|a_1 + a_2|}$$

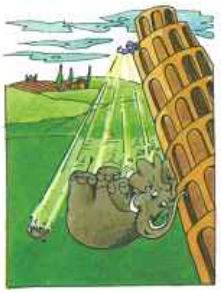


GR in a nutshell

Underlying hypothesis

Equivalence principle

- Universality of free fall
- Local lorentz invariance
- Local position invariance



Physical metric

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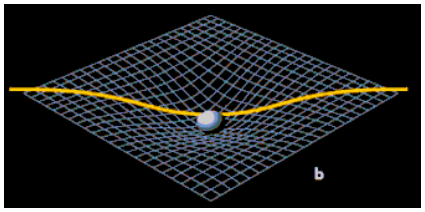


Dynamics

gravitational metric

$$S_{grav} = \frac{c^3}{16\pi G}\int \sqrt{-g_*}\, R_*\, d^4x$$

Relativity

$$g_{\mu\nu} = g^*_{\mu\nu}$$

Solar system tests

Metric theories are usually tested in the PPN formalism

$$\mathrm{d}s^2 = (-1 + 2U + 2(\beta - \gamma)U^2)\mathrm{d}t^2 + (1 + 2\gamma U)\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2$$

Light deflection

$$\Delta\theta = 2(1+\gamma)\frac{GM}{bc^2}$$

Perihelion shift of Mercury

$$\Delta\varphi = \frac{2\pi GM}{a(1-e^2)}(2 + 2\gamma - \beta)$$

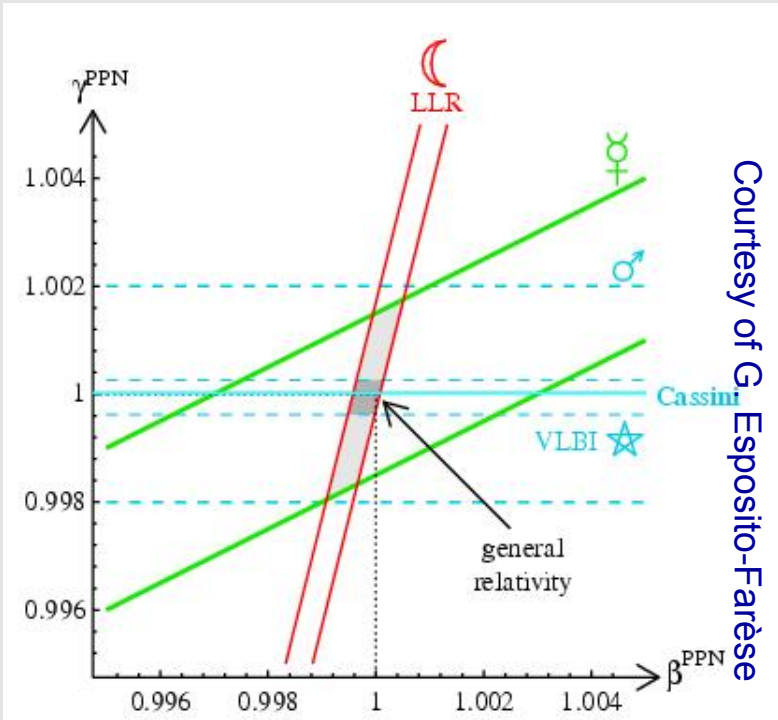
Nordtvedt effect

$$\delta r \sim 13.1(4\beta - \gamma - 3)\cos(\omega_0 - \omega_s)t \quad (\mathrm{m})$$

Shapiro time delay

$$\delta t \propto (1+\gamma)$$

[Will, Liv. Rev. Relat. 2006-3]



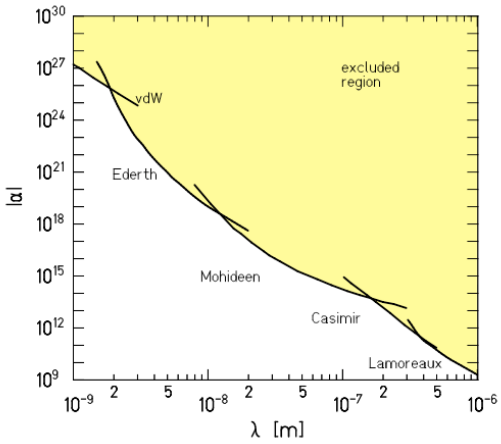
$$\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$$

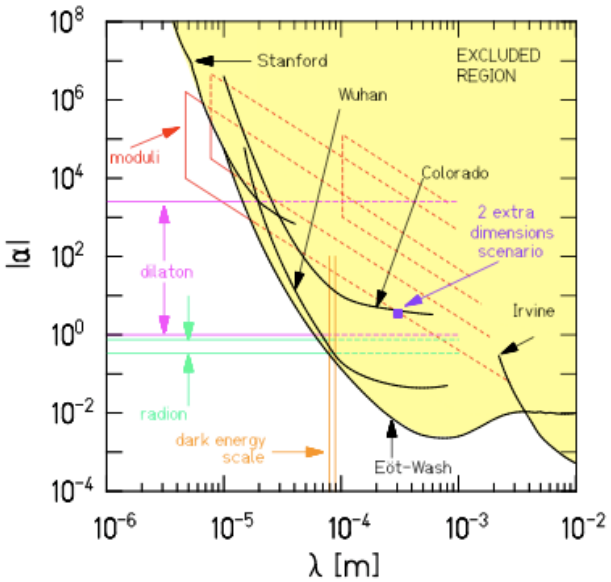
$$|2\gamma - \beta - 1| < 3 \times 10^{-3}$$

Fifth force

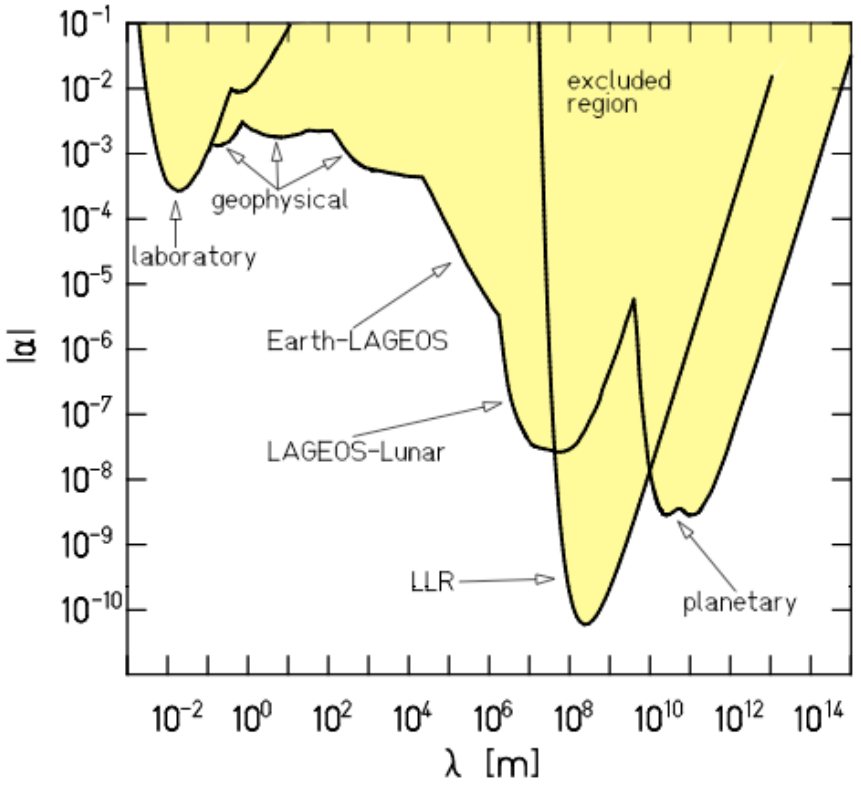
The PPN formalism cannot be applied if the modification of General relativity has a range smaller than the Solar system scale.

Fifth force experiments





Adelberger et al., Ann. Rev. Nucl. Part. Sci., 53 77 (2003)
Adelberger et al., Prog. Part. Nucl. Phys 62, 102 (2009)



Equivalence principle and constants

In general relativity, any test particle follow a geodesic, which does not depend on the mass or on the chemical composition

Imagine some constants are space-time dependent

1- Local position invariance is violated.

2- Universality of free fall has also to be violated

Mass of test body = mass of its constituants + binding energy

In Newtonian terms, a free motion implies $\frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} = \vec{0}$

But, now

$$\frac{d\vec{p}}{dt} = \vec{0} = m\vec{a} + \underbrace{\frac{dm}{d\alpha}\dot{\alpha}\vec{v}}_{m\vec{a}_{\text{anomalous}}}$$

Needs to test GR

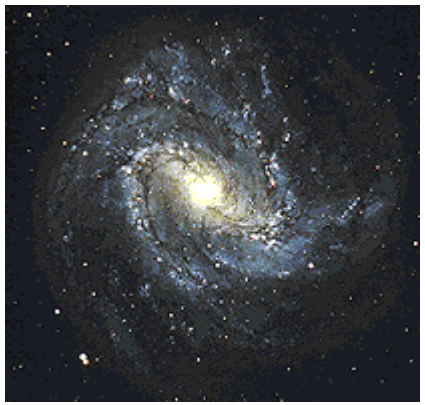
- The variation of the constants,
- the deviations from Newton law (or general relativity),
- the violation of the universality of free fall

are tied together.

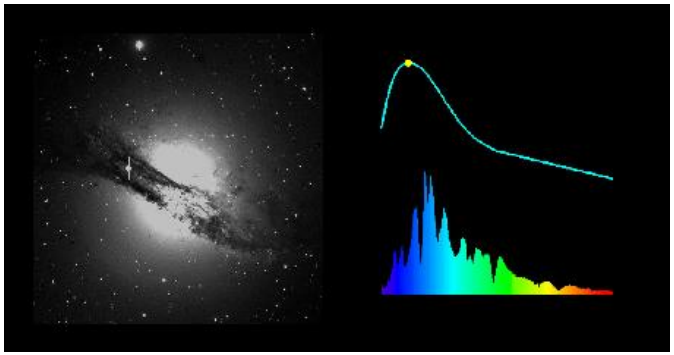
Testing the constants, is testing gravity

There is a growing need to test general relativity on astrophysical scales

*dynamics of galaxies and **dark matter***



*acceleration of the universe and **dark energy***



but also theoretical motivations…

Parameter space

Tests of general relativity on astrophysical scales are needed

- *galaxy rotation curves: low acceleration*
- *acceleration: low curvature*

Solar system:

$$\frac{R}{\phi^3} = \frac{c^4}{G^2 M_\odot^2}$$

Cosmology:

$$R = 3H_0^2\{\Omega_m(1+z)^3 + 4\Omega_\Lambda\}$$

Dark energy:

$$R < R_\Lambda = 12H_0^2\Omega_\Lambda$$

Dark matter:

$$a < a_0 \sim 10^{-8}\mathrm{cm.s}^{-2}$$

$$a^2 = \phi R < a_0^2$$

[Psaltis, 0806.1531]