

## Lecture III: Oct. 10

- cosmological news and general descript. of  $\gamma$ -cal systems
- principle of setting constraints.
- Atomic clock [ + modelisation of gp ]  
[ + seasonal variation ]
- Nuclear systems -  $\alpha/\beta$  decay  
clock.
- also



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## ① Setting constraints (slide 3)

The basic idea is that one observes a physical system and an associated observable

$$O(G_k, x)$$

↴ external parameters  
 ↴ phys. constants

usually one assumes  $G_k = G_{k_0}$  and studies the physics of the system

The idea is to allow the  $G_k \neq G_{k_0}$  and try to measure them with this physical system.

That can be achieved if  $O$  is sensitive to a change of  $G_k$  and we define

$$\frac{d \ln O}{d \ln G_k} = C_k$$

The computation of the  $C_k$  requires a description and good physical understanding of the system.

The constant  $G_k$  are not fundamental constants but can be expressed in terms of them, e.g.  $G_k$  can be b-decay energy, resonance energy etc..

$$G_k(d_i) \Rightarrow \frac{d \ln G_k}{d d_i} = d_i C_k$$

We thus have 2 steps, each of which requires a lot of theoretical physics:

① understand why the system and calculation of  $C_k$

② calculation of  $C_k$ : This may be very model-dependent  
(nuclear physics...)

### What systems have been used

The best way to visualize the different physical systems used so far is to use a spacetime diagram in which we include one past lightcone.

slide 4: describe the phys. systems

slide 5: note that for most systems we can determine the redshift.

The inference of the line variation depends on the choice of cosmology

I use the cosmological parameters of the table of slide 4, so that I have the look-back time / redshift relation depicted on this slide.

$\Delta\alpha$  refers to  $\Delta\alpha = \alpha - \alpha_0$

$\Delta\alpha_0 \Leftrightarrow$  smaller in the past

slide 6: for each system we can determine

- {  $G_k$  on which we set the primary constraint
- the observables
- ✗ external parameters requiring extra-hyptheses or data

The systems involve physics that fall into  
2 classes:

- {- QED (clock, also, CMB)
- {- nuclear physics (ucle, metrate, BBN, pop III)

in order to derive the primary constraints BUT we  
always need to use also to infer constraints  
on the fundamental constants, e.g. because we need  
to compute  
 $m_p(\alpha_i)$ ;  $g_p(\alpha_i)$  etc...

## ② Atomic clocks

Atomic spectra depend on the values of FC. In  
order to see which constants, and how they appear  
let us remind the simplest of all atoms: H.

At lowest order, neglecting spins and in NR regime,  
the spectrum of H can be obtained from the  
Schrödinger equations with Hamiltonian

$$H_0 = \frac{p^2}{2me} - \frac{e^2}{4\pi\epsilon_0 r}$$

The eigenfunctions are of the form  $\Psi_{nlm} = R_n(r) Y_{lm}(\theta, \phi)$

$n \rightarrow$  principal quantum number

The energy of the state  $(n, l, m)$  is

$$E_n = -\frac{E_I}{n^2} \left(1 + \frac{me}{mp}\right)^{-1}$$

$$\rightarrow E_I =$$

• Nonrelativistic : energy level is given by

number ( $n$ ) letter ( $s p d f g \dots f l = 0 \dots 6$ )

• There are many effects to be included to be realistic.

Theoretically they are updated by using perturbative theory  $H = H_0 + \chi$

### ① Relativistic effects

They scale as  $\alpha^4$  since in the Bohr model

$$v/c = \alpha \text{ for } n=1$$

$$\chi = -\frac{p^4}{8m_e^3 c^2} \sim \alpha^2 H_0^2$$

### ② spin-orbit interaction

neglecting  $\frac{me}{mp}$ , one has

$$\chi = \frac{\alpha}{2m_e c^2} \frac{hc}{r^3} \left( \frac{g_e \vec{L} \cdot \vec{s}}{2} \right)$$

electric gyromagnetic factor

at lowest order, the AED loop correction

$$\text{que. } \frac{g_e - 2}{2} = \frac{\alpha}{2\pi} + \dots$$

Since  $r \sim$  Bohr radius,  $\chi \sim \alpha^2 H_0$

### ③ Darwin term

It arises from the fact that in the Dirac eq.  
the interaction between  $e^-$  and  $A_p$  is local but  
the NR approx leads to a non-local eq. for the  $e^-$   
spins sensitive in a zone of  $r \sim$  Compton wavelength

$$\mathbf{W} = \frac{\pi \hbar^2 q^2}{mc^2 c^2} \delta_0(r)$$

The average in an atomic state is of order

$$\langle W_0 \rangle = \frac{\pi \hbar^2 q^2}{2 mc^2 c^2} |\psi(0)|^2 \sim mc^2 \alpha^4 \sim \alpha^2 H_0$$

All these corrections are of order  $\alpha^2$  and it gives the fine-structure level. The energy is then

$$E_{nlJ} = mc^2 - \frac{E_I}{n^2} - \frac{mc^2}{2n^4} \left( \frac{n}{J+1/2} - \frac{3}{4} \right) \alpha^4 + \dots$$

$e$ -angular ← magnetic quantum number

Note with the Dirac eq. we could have obtained

$$E_{nlJ} = mc^2 \left[ 1 + \alpha^2 (n - J - 1/2 + \sqrt{(J+1/2)^2 - \alpha^2})^{-2} \right]^{-1/2}$$

- The hyperfine structure: interaction of spins of  $e^-$   $\vec{S}$  and  $p^+$   $\vec{I}$

Their magnetic moments are

$$\Pi_s = \frac{q\hbar}{2me} \frac{\alpha e}{2} \frac{\vec{S}}{\hbar} \quad \Pi_p = -g_p \frac{q\hbar}{2mp} \frac{\vec{I}}{\hbar}$$

→ 2 new constants  $\{g_e; g_p\}$

$$\mathbf{W} = -\frac{\mu_0}{4\pi} \left\{ \frac{1}{r^3} \vec{I} \cdot \vec{\Pi}_I + \frac{8\pi}{3} \vec{\Pi}_I \cdot \vec{\Pi}_S \delta(r) \right.$$

$$\left. - \frac{1}{r^3} [3 (\vec{\Pi}_S \cdot \vec{r}) (\vec{\Pi}_I \cdot \vec{r}) - \vec{\Pi}_I \cdot \vec{\Pi}_S] \right\}$$

limit  $p \rightarrow e$

$$|W| \sim \frac{e^2 \hbar^2}{mc^2 p^2 r^3}$$

Typically 2000 times smaller than the fine structure. (sp. const apply)

It splits each  $l$  level into a series of hyperfine levels

$$F_C \{ |J-I|; J+I \}$$

Slide 8 shows the hyperfine level of  $1S_{1/2}$

For general atoms, the situation is indeed more complicated

one can get approximate analytical expressions for the energy levels for hydrogenlike / alkali but usually one relies on N-body QED simulations

we usually start

$$\{ \nu_{hfs} = R_{\infty} c A_{hfs} g_i \alpha^2 \left( \frac{m_e}{m_p} \right) F_{hfs}(\alpha)$$

$$\nu_{elec} = R_{\infty} c A_{elec} F_{elec}(Z, \alpha)$$

(Cs-133, Rb-87, K-39, ...)

- Spectra depend on  $\{ \alpha, \mu = \frac{m_e}{m_p}; g_i \}$

- dimensionless  $R_{\infty} c$  disappears in clock comparison

- we define the sensitivity

$$K_\alpha = \frac{\partial \ln F}{\partial \ln \alpha}$$

This coefficient can be calculated numerically for the relevant transitions @ an accuracy of 1% to 10%

### Experimental constraints

Many clocks have been compared, mostly to  
 $Cs - 133$

because  $F=3 \rightarrow F=4$  of  $2D_{5/2}$  ground state

at  $9.192\text{ GHz}$  is used to define the second.

- experience at  $z=0$
- can be reproduced
- high accuracy
- one limiting effect is the frequency shift due to collisions

### $Cs - Rb$ 2 hfs transition

$$\frac{\nu_1}{\nu_2} = \frac{g_{Cs}}{g_{Rb}} \frac{F_{Cs}}{F_{Rb}} \approx \frac{g_{Cs}}{g_{Rb}} \alpha^{0.49}$$

The constraints is summarized on table of slide 10

### $H - Cs$ one uses $1s-2s$ transition in H

$$\frac{\nu_{1s}}{\nu_{2s}} = g_{Cs} N \alpha^{2.83}$$

because  $\rho_H \sim \rho_{Cs}$   
 $\downarrow$   
 $Cs$  only

experiments over 44 months.

Hg - Cs

based on  $2D_{1/2} - 2d_{5/2}$  optical transition in  $^{193}\text{Hg}^+$

$$\frac{\nu_{Cs}}{\nu_{Hg}} = \frac{g_{Cs} \alpha^2 \mu F_{Cs}}{F_{Hg}} = g_{Cs} \mu \alpha^{6.05}$$

Yb - Cs

$\gamma_b$  electric quadrupole transition  $2D_{1/2} \rightarrow 2d_{3/2}$

$$\frac{\nu_{Cs}}{\nu_{\gamma_b}} = g_{Cs} \mu \alpha^{1.93}$$

Dy

electric dipole transition between 2 nearly degenerate opposite-parity states

2 transitions of same atom  $\rightarrow \frac{\nu}{\nu} \propto \alpha$

Hg - Al<sup>+</sup>

single ion optical clock (no g !)

$$\left. \begin{array}{l} {}^{27}\text{Al}^+ : 1D_0 \rightarrow 3P_0 \\ {}^{193}\text{Hg}^+ : 2D_{1/2} - 2S_{1/2} \end{array} \right\} \frac{\nu_{Hg}}{\nu_{Al}} \propto \alpha^{-3.208}$$

reaches  $10^{-17}$  over 1 year without any assumptions on other controls.

constraints on  $\{g; \alpha; \mu\}$

The constraints depend on  $\alpha, \mu, g_{Cs}, g_{Rb}$

Since Al sets a constraint on  $\alpha$  alone, we can derive independent constraints on

$$\frac{g_{Cs}}{g_b} \text{ and } g_{Cs}\mu$$

To go further we need to express the  $g$  in terms of FC.

An approximate calculation of the magnetic moment is possible in the shell model

$$g = \begin{cases} 2l g_e + g_s & \text{for } j = l + \frac{1}{2} \\ \frac{l}{J+1} (2(l+1)g_e - g_s) & = l - \frac{1}{2} \end{cases}$$

$$g_e = 1(0) \quad g_s = g_b (g_n) \quad \text{for a valence proton (neutron)}$$

$$\left\{ \begin{array}{l} \text{Rb: ground state } P_{3/2} \text{ so } l=1 \quad j=\frac{3}{2} \\ \text{Cs: } \qquad \qquad \qquad g_{7/2} \quad l=4 \quad j=\frac{7}{2} \end{array} \right.$$

Not perfect. Need to take into account polarization and non-valence nucleus

- see [slide 11]

- going further requires  $g_p \approx m_p$

[slide 12]

### • Further systems

- \* A way to get  $\mu$  is to use molecule and inter-rotation transition.

$$V = E_T (C_{\text{dip}} + C_{\text{rot}} \sqrt{\mu} + C_{\text{rot}} \mu)$$

e.g. used in  $\text{SF}_6$  vs  $\text{CO}$

It is also used in astrophysics with lines in  $\text{H}_2$ ;  $\text{HD}$ ,  $\text{RH}_3$ . (we shall discuss that)

- \* New systems
  - nuclear transition in  $\text{Th-229}$
  - hydrogen-like highly charged ion
  - diatomic molecules with same decay levels
- \* Atomic clock in space : ACES ( $^{129}\text{La}$ ,  $^{87}\text{Sr}$ ...), SAGAS
- \* Theoretical development.

### Seasonal variations

As we have seen before the var. of FC is due to some dynamical field.

Assume that we have a static sd. with syle

Plan

$$\text{KG} \quad \Delta\phi = 4\pi \frac{G\Gamma}{c^2} \delta(\vec{r}) \quad \left. \begin{array}{l} \\ \alpha = \frac{d\ln\lambda}{d\phi} \end{array} \right\} \Rightarrow \phi = \phi_0 - \frac{\alpha_0}{c^2} V$$

$$\text{Einstein} \quad \Delta V = -4\pi G \Gamma \delta^3(\vec{r})$$

if we solve iteratively

$$\boxed{\phi = \phi_0 - \alpha_0 \frac{\Phi_N(r)}{c^2}}$$

$$\text{It follows that } \frac{\Delta a_i}{a_i} = -\dot{\alpha}_i(\phi_0) \alpha_0 \frac{\Phi_N(r)}{c^2}$$

$$\frac{d\ln a_i}{d\phi}$$

The Earth moves on an elliptic orbit

$$r = \frac{a(1-e^2)}{1+e\cos\varphi} \quad \cos\varphi = \frac{\cos E - e}{1-e\cos E} \quad t = \sqrt{\frac{a^3}{GM}} (E - e\sin E)$$

so

$$\frac{\Delta a_i}{a_i}(a, \varphi) = -\dot{\alpha}_i \alpha_0 \frac{G\Gamma}{ac^2} - \dot{\alpha}_i \alpha_0 \frac{G\Gamma}{ac^2} e \cos\varphi + O(e^3)$$

Var of mean value  
 copied to cosm.  
 value

second modulus

$$\boxed{\frac{\Delta a_i}{a_i} = k_i \frac{\Delta \Phi_N}{c^2}}$$

$$\text{Dy} \rightarrow k_\alpha = (-8.7 \pm 6.6) 10^{-6}$$

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$$Al^+ \rightarrow h_a = (-5.1 \pm 5.1) 10^{-8}$$

### ③ $\alpha/\beta$ decay - fission & Meteorite dating

long lived  $\alpha/\beta$  decay isotopes can probe  
Fc on time of 4-5 Gyrs

First pointed out by Wikinson and Dym.

The sensitivity of the decay rate is defined  
as

$$\Delta_i = \frac{\partial \ln \lambda}{\partial \ln A_i}$$

$\lambda$  is usually a factor of the decay energy  $\alpha$

As we shall see when  $\alpha$  is small (due  
to almost cancellation of Nuc & EM bdy energies) the  
 $\Delta$  is increased.

We need  $\lambda \approx 2$  kev (e.g. lab & fc of the sys)

- Meteorite dating

Imagine  $X \rightarrow Y$

to meteorite containing  $X$  are forced

$$N_X(t) = N_{X_0} e^{-\lambda(t-t_0)} \quad N_Y(t) = N_{Y_0} (1 - e^{-\lambda(t-t_0)}) + N_{Y_0}$$

Indeed if  $\lambda$  varies with time

$$N_X = N_{X_0} e^{\int_{t_0}^t \lambda(t') dt'}$$

We thus define the effective decay rate

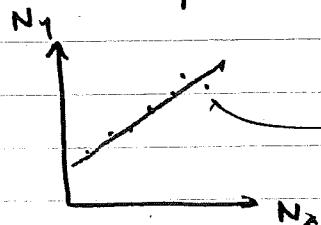
$$\bar{\lambda} = \frac{1}{t_e - t_0} \int_{t_0}^{t_e} \lambda(t') dt'$$

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Sums of meteors  $\rightarrow \{N_x(t_0), N_y(t_0)\}$  that  
are related by

$$N_y(t_0) = [e^{\bar{\lambda}(t_0 - t_0)} - 1] N_x(t_0) + N_{y_k}$$



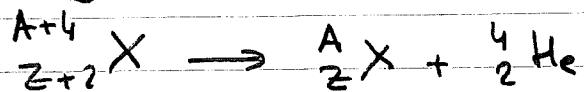
each meteor has a prim. a deflt  $N_y$

$$\text{slope} \rightarrow \bar{\lambda}(t_0 - t_0)$$

→ give a bond on the average decay rate

→ need to know  $t_0$  (other data for  
not- $\alpha$  sensitive).

### $\alpha$ decay



the decay rate is governed by the penetration  
of the coulomb barrier. (Gamow theory)

$$\lambda \approx \Lambda(\alpha, v) e^{-4\pi Z \alpha \frac{c}{v}}$$

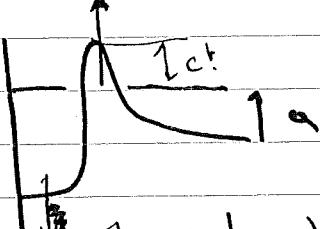
$$\frac{v}{c} = \sqrt{\frac{\alpha}{2 m_p c^2}} \quad \text{escape velocity}$$

$\Lambda$  smooth dependency on  $\alpha \ll v$  that  
we shall ignore

$$\text{So } \Delta\alpha = -4\pi^2 \frac{\alpha}{\sqrt{\alpha r_{\text{mp}}}} \left( 1 - \frac{1}{2} \frac{d \ln \alpha}{d \ln \alpha} \right)$$

the decay energy can be expressed in terms of the binding energies

$$\alpha = B(A, Z) + B_\alpha - B(A+1, Z+1)$$



$\alpha \uparrow \Rightarrow \text{height} \uparrow \Rightarrow \alpha \text{ escape at higher energy}$

As usual, we use Bethe-Veizänen

[slide 14] gives the characteristics of some  $\beta$ -decay.

[slide 15] gives examples of constraints.

### $\beta$ -decay

Decays: Rb-Sr / K-Ar /

For long lived  $\beta$ -decay, e.g.  $\frac{187}{75}\text{Re} \rightarrow \frac{187}{76}\text{Os} + \bar{\nu}_e + e^-$

$\alpha e^{-} + {}^{40}_{19}\text{K} \rightarrow {}^{40}_{18}\text{Ar} + \bar{\nu}_e + \dots$ , as long as  $\alpha$  is

small one can use then NR approximation

$$\text{get } \Delta = \Lambda \pm Q^{\frac{P}{2}} \xrightarrow[\text{smooth dependence on } \alpha]{\text{e-capture}} \begin{array}{l} P_+ = l+3 \\ P_- = 2l+2 \end{array} \quad \begin{array}{l} \text{degrees of} \\ \text{ freedom} \end{array}$$

For high- $Z$  nuclei  $p = 2 + \sqrt{1 - \alpha^2 Z^2}$  so that

$$\Delta\alpha = p \frac{d\ln\alpha}{d\ln p}$$

Then one proceeds as for  $\alpha$ -decay

Note that  $\Lambda_2 \sim G_F^2 m_S q^2$  but we see for all  $\beta$ -decay so that it disappears in the capture of 2 isotopes

Discuss the case of Re/OS.

Slides 16 e 17

## ④ Oklo

• what is oklo?

slide 19

$Z \sim 30$

discovered in 1972 by French CEA

16 natural uranium reactors have been identified

<span style="border: 1px solid black; padding: 2px;">2 most studied:</span>	$RZ_2$ : 60 bore holes 1800 kg of U-235 fissioned day $8.5 \cdot 10^5$ yr
	$RZ_{10}$ : 13 bore-holes 650 kg of U-235 fissioned day $1.6 \cdot 10^5$ yr

slide 20 predicted by Paul Kuroda

describe the conditions for the existence

$RZ_2$  &  $RZ_{10}$  operated at a depth of # thousand meters

→ water pressure  $\sim 20 \text{ MPa}$  &  $T \sim 300 \text{ K}$   
 (close to pressurised water reactors)

estimation: powered 10-50 kW

geochemistry: - U-235 depleted

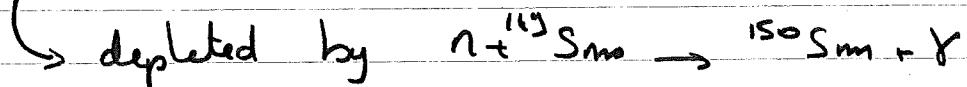
- rare-earth produced U-fission

- n-absorber as a small quantity  
 $^{149}_{62} \text{Sm}$ ,  $^{155}_{64} \text{Gd}$ .

**(slide 21)** From the measurements of isotopic abundances one can in principle reaction rates.

one key observation is  $\frac{^{149}_{\text{Sm}}}{^{147}_{\text{Sm}}} / \frac{^{147}_{\text{Sm}}}{^{149}_{\text{Sm}}}$

$$\left\{ \begin{array}{l} r = 0.9 \text{ in standard SmSm} \\ = 0.02 \text{ in ore} \end{array} \right.$$



This is dominated by a resonance at

$$E_r = 97.3 \text{ meV}$$

**(slide 22)** What needs to be done is then

- { - measure  $\Sigma_{149}$  → geochemistry
- relate  $\Sigma_{149}$  to  $E_r$  → nuclear YCs
- express  $E_r$  to capture → QCB

depends on hyp on variety of reaction  
n-species  $\propto T$ .

### • Constraining the resonance energy

For a resonant capture, the cross section takes the Lorentz-Vigier formula

$$\sigma_{(n,r)} = \frac{g_0 \pi \hbar^2}{2 m_n E} \frac{\Gamma_n \Gamma_r}{(E - E_r)^2 + \Gamma^2/4}$$

$\frac{2J+1}{(2s+1)(2I+1)}$  depends on spin  $n$   $s = 1/2$   
nuclear target I  
capture J

We have  $g_0 = \frac{9}{16}$

$\Gamma = \Gamma_n + \Gamma_\gamma$  Sum of n &  $\gamma$  partial width

$$\begin{cases} \Gamma_n = 0.533 \text{ meV @ } E_r \\ \propto \sqrt{E} \text{ in cm} \\ \Gamma_\gamma = 60.5 \text{ meV} \end{cases}$$

$\delta_{(n,\gamma)}$  is not accessible because it is averaged on the neutron flux:

$$\hat{\delta} = \frac{1}{n v_0} \int \delta_{(n,\gamma)} n(v, T) v dv \quad (\text{usual def.})$$

$$v_0 = 2200 \text{ ms}^{-1} \text{ corresponds to } E_0 = 25.3 \text{ meV} \quad v = \sqrt{\frac{2E}{m_n}}$$

while it would be clearer to define

$$\bar{\delta} = \frac{\int \delta n v dv}{\int n v dv}$$

similarly the flux is defined as

$$\hat{\phi} = v_0 \int n(v, T) dv \quad \text{instead of} \quad \phi = \int n(v, T) v dv$$

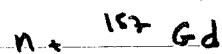
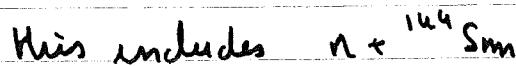
Note however that  $\bar{\delta} \phi = \hat{\delta} \hat{\phi}$

**Slide 23** summarizes the hypothesis and  $n(v, T)$ ;  $T$  can differ analyses.

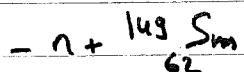
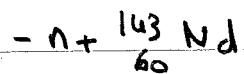
From the geochemical analysis

$$\int \hat{\phi} dt \sim 10^{21} n/cm^2 = 1 \text{ keV}^{-1}$$

→ processes with  $\delta < 1 \text{ keV}$  can be neglected



But we need to include: - fission of  ${}^{235}\text{U}_{92}$



We thus get a system for



It takes the form

$$\left\{ \begin{array}{l} \frac{dN_{147}}{\hat{\phi} dt} = -\delta_{147} N_{147} + \delta_{f235} N_{235} y_{147} \\ \frac{dN_{148}}{\hat{\phi} dt} = -\delta_{148} N_{148} \\ \frac{dN_{149}}{\hat{\phi} dt} = -\delta_{149} N_{149} \\ \frac{dN_{235}}{\hat{\phi} dt} = -\delta_5 N_{235} \end{array} \right. \quad \begin{array}{l} \text{yields fission} \\ \text{of } {}^{235}\text{U} \end{array}$$

↓ fission cross section.

take the case  ${}^{239}\text{Pu} \rightarrow {}^{235}\text{U} + \alpha$

It can be integrated and compared to obs to determine the best-fit of  $\delta$ 's

- Measure of  $E_r$

To convert from the constraint on  $\beta$  to a constraint on  $E_r$ , one needs a hypothesis on the nuclear flux

- Maxwell:  $n_m(v, T) = \left(\frac{m_n}{2\pi T}\right)^{3/2} e^{-\frac{mv^2}{2kT}}$

$\oplus$  depends only on  $T$

- above few ev  $n \sim 1/E$  tail because of the absorption of neutrons in U-resonances.

$$\rightarrow n(v) = (1-f)n_m(v, T) + f n_{ep}(v)$$

$$\text{with } n_{ep} = \begin{cases} \frac{v_{cl}^2}{v^2} & \text{for } v > v_c \\ 0 & \text{for } v < v_c \end{cases}$$

$$\text{so } \beta = g(T) \beta_0 + \gamma_0 I \quad \text{fraction of } (E_r, T)$$

Following these lines, we have the constraints of [slide 24]

- Last step: from  $E_r$  to constants

This requires a modelisation of the Samarium nucleus.

$E_r$  small results from Nuc. big,  $\sim$  EM bndy.

$$H = H_{Nuc} + H_{EM}$$

Danau-Dyon:  $\frac{\delta E_r}{\delta m_a} = -1.1 \text{ MeV}$

[slide 25]

This was imposed to take into account QM, RG...

## ⑤ Quasar absorption spectra

The idea to extend the constraints for atomic clock on astrophysical scales was prepared by T. Sandog in the 50's.

### Slide 27 2 28 - Idea of the method

- emphasize the need to look for  
achromatic effects.

- can in principle be inferred by  
many kind of spectra  
 | emission → less precise.  
 | absorption

### Slide 29 description of a quasar absorption spectrum.

### Slide 30 generalities on different techniques used

AD / RM / SIDAM

$$\omega = \omega_0 + q_1 \left[ \left( \frac{\alpha}{\alpha_0} \right)^2 - 1 \right] + q_2 \left[ \left( \frac{g}{\omega} \right)^4 - 1 \right]$$

laboratory  
precision  $\sim 0.004 \text{ cm}^{-1}$

AED calculations  
precision  $\sim 30 \text{ cm}^{-1}$

see slide 31.

## Alkali Doublet (AD) method

slide 32

It focuses on the fine structure doublet of alkali atoms

It avoids the hypothesis of homogeneity because the 2 lines of the doublet must have the same profile.

same atom  $\rightarrow$  chemical ionization homogeneity is not required

$$\Delta v \sim \frac{\alpha^2 Z^4 R_\infty}{2n^3} \Rightarrow \frac{\Delta v}{D} \propto \alpha^2$$

Inverting  $w(\lambda)$  one gets

$$\left\{ \begin{array}{l} \frac{\Delta \alpha}{\alpha} = \frac{c_r}{2} \left( \frac{\Delta \lambda / \lambda_2}{\Delta \lambda / \lambda_1} - 1 \right) \\ c_r = \frac{\delta q + \delta q_2}{\delta q + 2\delta q_2} \end{array} \right.$$

It was applied to many transitions  $C_{IV}$ ,  $N_{V}$ ,  $O_{VI}$ ,  $Mg_{II}$ ,  $Al_{III}$ ,  $Si_{II}$ ,  $Si_{IV}$

Si IV doublet

$Si IV \lambda 402$ $q = 362 \text{ cm}^{-1}$ $q_2 = -8 \text{ cm}^{-1}$  $Si IV \lambda 1393$ $q = 766 \text{ cm}^{-1}$ $q_2 = 48 \text{ cm}^{-1}$	$c_r = 0.8914$
---	----------------

2 main analysis.

• Murphy : Keck/Hires 8  $\text{km s}^{-1}$

$S/N \sim 15-40/\text{pixel}$   $R \sim 34000$

$$\Delta \frac{\alpha}{\alpha} = (-0.5 \pm 1.3) 10^{-5} \quad 2.33 < Z < 3.08$$

• chand IS Si IV ESO/UVES

$S/N \sim 60\text{-}80 / \text{pixel}$   $R \sim 45000$

$$\frac{\Delta \alpha}{\alpha} = (-0.15 \pm 0.43) \cdot 10^{-5} \quad 1.59 < z < 7.92$$

• Hanay multiplet method

Slide 33

- describes the method

- emphasizes the role of anchor.

There are 2 means analysis

• Keck → 143 abs. syst. gives the constraint on  
Slide 33

→ with 30 abs. systems based it gives  
they got

$$\left\{ \begin{array}{l} \frac{\Delta \alpha}{\alpha} = (-0.17 \pm 0.39) \cdot 10^{-5} \quad 0.6 < z < 1 \\ \end{array} \right.$$

$$\left. \begin{array}{l} \frac{\Delta \alpha}{\alpha} = (-1.86 \pm 0.53) \cdot 10^{-5} \quad 1 < z < 1.6 \end{array} \right.$$

First claim for a varying  $F_C$

Analysis based mostly on Ni II, Cr II, Zn II

and Mg I, Fe III, Al II, Al II

Note that for the  $0.2 < z < 1.2$  constraint  
it relies on different ions depending on whether  
 $z \gtrless 1.6$

The separate analysis gives

$$\frac{\Delta \alpha}{\alpha} = \begin{cases} (-0.54 \pm 0.12) \cdot 10^{-5} & 0.2 < z < 1.3 \\ (-0.76 \pm 0.17) \cdot 10^{-5} & 1.8 < z < 4.2 \end{cases}$$

The whole issue is about systematics.

I list them on slide 34

The most problematic is Hg isotopic composition.  
(see slide 34 for details)

• VLT

$$\Delta \alpha = (0.01 \pm 0.15) \cdot 10^{-5} \quad 0.4 < z < 2.3$$

• controversy / ongoing project @ VLT

• other systems slide 35

Since then many new tracers have been used.

This allows to have constraints on calibration of different constants, like with atomic clock.

$$\begin{aligned} \text{HI vs rot.} \quad hf \text{ of } n_s &\sim g_p N \alpha^2 R_\infty \\ \text{rot of deuter.} &\sim M \quad \left. \begin{aligned} \frac{v}{\nu} &\sim g_p \alpha^2 \\ &= \gamma \end{aligned} \right\} \end{aligned}$$

The constraints are summarized on slide 37  
38

- Molecular transitions

more and more obs of H<sub>2</sub>, HD, NH<sub>3</sub>

see slide 39

- Evolution

- codex

- Dipole (Australian)

## Lecture IV: Oct. 17

- Big Bang Nucleosynthesis  
[+  $B_0$ ]
- population III stars
- Cosmic microwave background
  - [+ homogeneous
  - + dipole
  - + fluctuations]
- conclusion and overview



No.

W.2

Date

## ① Cosmic Microwave Background

The CMB radiation is one of the pillar of the standard cosmological model.

As long as  $T_g$  is high and  $>$  ionisation energy, matter remains ionized so that  $\tau$  are strictly coupled to  $p^+ e^-$ .

It is today observed with a high resolution by Planck (slide 3) and has been shown to be very close to a pure black-body spectrum (slide 4).

As the temperature drops as  $T \propto 1/a$  one has recombination.

So the basis of everything is



This is a non equilibrium process that is given by the p-e interaction and that requires to solve a Boltzmann equation.

Suppose equilibrium.

$$n_e = n_p$$

$$n_b = n_p + n_H$$

$$n_p = n_e = x_e n_b$$

$$n_H = (1 - x_e) n_b$$

At equilibrium we must have

$$\frac{x_e^2}{1 - x_e} = \left( \frac{m_e T}{2\pi} \right)^{3/2} \frac{e^{-E_I/T}}{n_b}$$

$$\text{with } E_I = m_e + m_p - m_H = 13.6 \text{ eV}$$

$$T = 2.725(1+z) \times$$

$$n_b = \eta n_\gamma (1+z)^3 \text{ cm}^{-3}$$

$$\eta = \frac{n_b}{n_\gamma} \approx 6 \cdot 10^{-10}$$

It implies that

$$\log \frac{x_e^2}{1-x_e} \approx 21 - \log [n_{b_0} h^2 (1+z)^{3/2}] - \frac{25163}{1+z}$$

$$\textcircled{1} @ T = E_I \quad \text{rhs} \approx 10^{15} \Rightarrow x_e = 1$$

$\rightarrow$  recombination happens @  $T \ll E_I$

\textcircled{2} The Saha equation rewrites as

$$\ln \frac{x_e^2}{1-x_e} = \frac{3}{2} \ln \frac{m_e T}{2\pi} - \frac{E_I}{T} - \ln \left[ \eta \frac{2}{\pi^2} \zeta(3) T^3 \right]$$

so that

$$\frac{E_I}{T} = \frac{3}{2} \ln \left( \frac{m_e}{2\pi T} \right) - \underbrace{\ln \eta - \ln \left[ \frac{2}{\pi^2} \zeta(3) \frac{x_e^2}{1-x_e} \right]}_{6 \cdot 10^{-10} \text{ neglected}}$$

$$\rightarrow T \approx 3500 \text{ K}$$

The recombination mass follows as:



and  $H + \gamma$  decouple soon  
after recombination ( $P \ll H$ )

As far as we are concerned with FC, the effect will be the way it affects recombination, and the optical depth.

We expect the CRB to be dependent on  $(G, \alpha, m_e)$

I focus on  $(\alpha, m_e)$

[ $G$  later of relevance]

}  
- need to specify mass because

$(G_F, \alpha, \frac{m_e}{m_p})$  or  $(G_m, \alpha)$

① The Thomson scattering is given by

$$\tau = \overbrace{Dce N_e}^{\text{density of free electrons}} \sigma_T C$$

It enters the collision term of the Boltzmann equation -

$$L[f] = C[f]$$

$$② \sigma_T = \frac{8\pi}{3} \frac{\hbar^2}{c^2} \frac{\alpha^2}{m_e^2}$$

We neglect scattering by  $\rho$  ( $< 100$  fm/cm)

Then there is a more subtle dependence that arises for the effect on FC on  $X_e(t)$ .

In brief, recombination proceeds by

- 2 $\gamma$  emission from 2s level
- or
- Ly- $\alpha$  photons (redshifted if resonance)  
otherwise they have just the  
energy to ionize H again!

The evolution of  $x_e$  in HeII given by

$$\frac{dx_e}{dt} = C \left[ \beta (1-x_e) e^{-\frac{T_n}{T_r}} - R n_p x_e^2 \right]$$

matter density

$T_n = T_r$  at high  $z$   
but

$$\frac{dT_n}{dt} = -\frac{8\sigma_{SB}}{3mc} T_r^4 \frac{x_e}{1+x_e} (T_r - T_i) - 2\pi T_n$$

radiative const

$$\sigma_{SB} = \frac{c^2}{60\pi c^2 k^3}$$

$$B_n = -\frac{E_I}{n^2} \quad \text{energy of } n^{\text{th}} \text{ energy level}$$

$\beta$ : emissivity coefficient

$R$ : recombination coefficient

$C$ : connection constant to take into account redshifting.

$\beta$  and  $R$  are related by

$$\beta = R \left( \frac{2\pi m_e T_1}{h^2} \right)^{3/2} e^{-\frac{\beta_2}{T_1}}.$$

That is a simplified version of it. If we follow Seeger et al (1969) we have the set of eq. of

slide 5

$$x_p = \frac{n_p}{n_H}; \quad x_{He} = \frac{n_{He}}{n_H}; \quad x_e = x_{Cp} + x_{Ne}$$

↓  
prob. fraction

↓  
singly ionized  
helium.

electric fraction

→ recombination of ( $A_2$ ) = recombination through

the levels via the semi-forbidden transitions



\* spectroscopic quantities:

$\nu_{H_2S}$ : 2D-1D frequency

$\nu_{HeI, 2^3S}$  helium  $2^1P \rightarrow 1^1S$   
 $2^3P \rightarrow 1^1S$

. (c) coefficients are given by

•  $C_H = \dots$

$C_{HeI} = \dots$  (see slide 6) - they emit  $\text{1}_H$  &  $\text{1}_{HeII}$

$C_{HeI}^b = \frac{\text{Me } 2s-1s \text{ } 2 \text{ } 2^1S-1^1S \text{ for H } \& \text{ He}}{2 \text{ photon rate}}$

• recombination parameters  $(\alpha)$

$\alpha_H$  coeff B: recombination for H

$\alpha_{HeI}$ ,  $\alpha_{HeII}^L$  2 coeff for helium (sight/bright)

• photoionisation coeff  $(\beta)$

They are given by

$$\beta = \alpha \left( \frac{2\pi n_e T}{h^2} \right)^{3/2} e^{-h\nu/kT}$$

• K-coeffs  $(K)$

Scaling with  $d$  and  $m_e$

-  $\nu$  are diatomic frequencies

$$\boxed{\nu \propto d^2 m_e}$$

-  $\omega_i \propto d^3 m_e^{-3/2}$

-  $\beta_i \propto d^3$

-  $\kappa \propto d^{-6} m_e^{-3}$

-  $\Lambda \propto d^8 m_e$

-  $\delta_T \sim d^2 / m_e^{-2}$

All these scaling can be included in Recfast

The following is mostly numerical.

① we can first get the model of cascade mesh [slide 7]

②  $C_e^{TT}$ ,  $C_e^{TE}$ ,  $C_e^{EE}$  [slides 8, 9, 10]

we shall come back on the different values of  $d$  &  $m_e$

③ choose data and add Mex parameters in a standard analysis

Slide 11 summarizes the preplanck constraints  
typically of order  $10^{-2}$

### • Planck analysis

- planck TT
- WMAP polar from VTPR
- high res CMB ground exp.
- BAO (SDSS / angular, 6dF)
- HST Gaussian prior on  $H_0$

parameters  $\{w_b, w_c, H_0, \tau, n_s, A_s, d, m_e\}$

- First we let  $d$  and  $m_e$  vary alone

Slide 12 depicts the PPF and it shows

The expected gain compared to VTPR

- $d$  alone

Slide 13-15 gives the corner plots

discuss dependency with other parameters

$\{N_{eff}\}$  (slide 16)

$\{Y_p\}$  (slide 17)

The analysis of the data is summarized in slide 18 where table shows the different hypotheses

Plack imposes constraints on  $\alpha$  alone for 2% (w.r.t) to 0.6%

- This is due to the fact that it breaks the strong  $\alpha$ -He degeneracy

- The constraints on the cosm. parameters are almost affected

- Me alone similar results

- ( $\alpha$ , me) slide 23 is interesting because it shows that Plack allows to completely ~~fully~~ break the degeneracy

• can we understand why? [slide 24]

To do so we complete Ce what only part of the connections:

- hydrogen bugs energy (solid light blue)
- ly<sub>α</sub> energy (solid yellow)
- in both (solid purple)
- both + δ<sub>T</sub> (dashed dark blue)
- + --- + 2γ de ceg rate. (dashed red)
- All terms (solid green)

$$\alpha = 5\% \quad B_0 = 10.025\% \Rightarrow \text{same } B = \alpha^2 m_e \text{ so}$$

that same effect on the number of body energies

(blue / yellow comes as the sum)

and this is the domino effect.

$h\nu \nearrow \rightarrow e^{-h\nu/kT} \downarrow \Rightarrow$  easier recombination  
(K more bound)

$\Rightarrow$  • peaks move to higher l  
• early ISKs ↑

• 5th digits ↓

$\Rightarrow$  overall amplitude peaks ↑

Net flavor flux  $\begin{cases} B \sim \alpha^2 m_e \\ G_F \sim \alpha^2 / m_e^2 \end{cases}$

effect of  $m_e^2 \alpha$  on soft decays are different  
→ signature at high- $\ell$

Similar effects on EE

I refer to He Planck paper for a detailed analysis.

## ② CMB : spatial variations

As discussed last week, there has been claimed of a dipolar variation of  $F_C$ .

The goal is to characterize the effect on the CMB.

We assume

$$C_a(\vec{n}, z) = C_{a0}(z) + \sum_{l=1}^{\infty} \delta C_a^{(l)}(z) Y_{1l}(\vec{n})$$

This can be generalized to higher multipoles but the spirit is the same.

$$\delta C_a \in \Omega \quad \delta C_a^{(l+1)} = -[\delta C_a^{(l-1)}]^*$$

$$\Theta(\vec{n}) = \bar{\Theta}(\vec{n}, C_a(\vec{n}))$$

$$= \bar{\Theta}[\vec{n}, C_{a0} + \sum_l \delta C_a^{(l)} Y_{1l}(\vec{n})]$$

$$\approx \bar{\Theta}[\vec{n}] + \sum_a \sum_l \frac{\partial \bar{\Theta}}{\partial C_a} \delta C_a^{(l)} Y_{1l}(\vec{n})$$

This relates the modulation of  $C_a$  to  $\Theta$  and  $\bar{\Theta}$ .

As usual, we can decompose  $\Theta$  in spherical harmonics

$$\Theta(\vec{n}) = \sum_l a_{lm} Y_{lm}(\vec{n})$$

and we have a similar expression for  $\bar{\Theta}$

$$\text{So } a_{lm} = \int d^3n \bar{\Theta}(\vec{n}) Y_{lm}$$

Includes  $\int d^3n \bar{\Theta} Y_{lm} \rightarrow \bar{a}_{lm}$

$$\int \dots Y_{li} Y_{lm} \frac{\partial \bar{\Theta}}{\partial c_a} (\vec{n})$$

The last term behaves as  $\sum_{LM} \frac{\partial \bar{a}_{lm}}{\partial c_a} \underbrace{\{Y_{li} Y_{lm} Y_{LM}^*\}}_{3j \text{ symbols}}$

So, one can check that the algebra leads to

$$a_{lm} = \bar{a}_{lm} + \sqrt{\frac{3}{4\pi}} \sum_a \sum_i \delta_{ca}^{(ij)} (-1)^m \sum_{LM} \frac{\partial \bar{a}_{LM}}{\partial c_a}$$

$$\sqrt{(l+1)(2L+1)} \begin{pmatrix} l & L & 1 \\ -m & M & 1 \end{pmatrix} \begin{pmatrix} l & L & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

The  $3j$ -symbols are  $\neq 0$  only when  $L = l \pm 1$   
 $M = m - i$

$$\text{so that } a_{lm} = \bar{a}_{lm} - \bar{a}_{l \pm 1, m-i}$$

The effect is to induce  $l \cdot (l+1)$  correlations.

To characterize these correlations, consider

$$D_{lm}^{(i)} = \langle \alpha_{lm} \alpha_{l+m+i}^* \rangle \quad i=0, 1$$

using that  $\langle \bar{\alpha}_{lm} \bar{\alpha}_{l'm'}^* \rangle = C e^{i k l' r} \delta_{mm'},$  it is a good excuse to show

$$D_{lm}^{(i)} = f_i(l, m) \sum_a \delta C_a^{(i)} \Gamma_e^{(a)}$$

with  $f_0(l, m) = \sqrt{\frac{3}{4\pi}} \frac{\sqrt{(l+1)^2 - m^2}}{\sqrt{(2l+1)(2l+3)}}$

$$f_1(l, m) = \sqrt{\frac{3}{8\pi}} \sqrt{\frac{(l+2+ms)(l+1+m)}{(2l+1)(2l+3)}}$$

$$\Gamma_e^{(a)} = \frac{1}{2} \left( \frac{\partial \bar{C}_e}{\partial C_a} + \frac{\partial \bar{C}_{e+1}}{\partial C_a} \right)$$

$\Gamma_e^{(a)}$  is depicted on slide 27

This allows to construct an estimator of  
 $\delta_{ca}$  following Hansen and Levis

In order to interpret the measurement one needs to go through the following steps

- simulate maps with a signal
- calibrate the estimator because it is expected to be biased by the masking

500 maps Signal + noise were generated

Slides 28-29

blue  $\delta\alpha^{(1)} = 10^{-3}$

green  $= 10^{-2}$

black = planck

$$\rightarrow |\delta\alpha| < 6 \cdot 10^{-4}$$

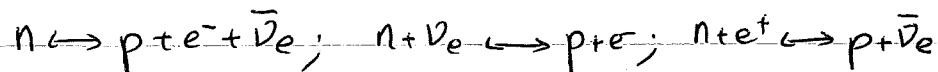
### ③ Big bang Nucleosynthesis (BBN)

- Overview: the production of light elements in the early universe is mainly determined by  $n/p$  at the freeze-out of the weak interaction.

The standard BBN scenario proceeds in 3 main steps:

$$\textcircled{1} \quad T > 1 \text{ MeV} \quad (t \leq 1 \text{ s})$$

$n, p, e^\pm, \nu$  are in statistical equilibrium



This implies that

$$\frac{n}{p} = e^{-Q/kT} \quad Q = m_n - m_p \sim 1.2 \text{ GeV}$$

The abundance of  $X_A^A$  is

$$Y_A = g_A \left( \frac{E(g)}{k\pi} \right)^{A-1} \frac{1}{2}^{(3A-5)} A^{5/2} \left( \frac{kT}{m_N c^2} \right)^{\frac{3(A-1)}{2}} \frac{1}{2} (m_p + m_n) \\ \eta^{A-1} Y_p^2 Y_n^{A-2} e^{B_A/kT}$$

$$\text{with } B_A = (2m_p + (A-2)m_n - m_A) c^2$$

is the binding energy.

$$Y_A = \frac{A n_A}{n_b} \rightarrow n_A = g_A \left( \frac{m_A T}{2\pi} \right)^{3/2} e^{-(m_A - \mu_A) T}$$

$$\text{use } \mu_A = 2\mu_p + (A-2)\mu_n$$

① freeze-out of weak interaction  $T \approx 0.8 \text{ MeV}$  ( $t \approx 2s$ )

$$\text{at } T_f : \Gamma_{\text{tot}}(T_f) \sim N(T_f) \text{ i.e.}$$

$$G_F^2 T_f^5 \sim \sqrt{GN_A} T_f^2$$

At that time

$$\left(\frac{n}{p}\right)_f \sim e^{-\alpha/T_f} \sim \frac{1}{5}$$

② free  $n$  decay into  $p$  while nuclei remain in thermal equilibrium

and  $0.5 \text{ MeV}$   $e^+ + e^- \rightarrow 2\gamma \Rightarrow$  reheat the thermal bath



nuclei are faced by 2 body reaction but this requires first  $\gamma$  to be freed



This cannot happen before

$$\eta_F e^{-B\gamma/T_N} \sim 1$$

and  $T_N \sim 0.06 \text{ MeV}$

$$\left(\frac{n}{p}\right)_N = \left(\frac{n}{p}\right)_f e^{-B\gamma/T_N} \sim \frac{1}{7}$$

All neutrons are then bound in He-4 with abundance

of order

$$\gamma_p = \frac{2 n_{dN}}{1 + n_{pN}} \approx 0.25$$

$n_{He} = n/2$  mass  $4 \times \frac{n}{2} = 2n$

Then : - Coulomb barrier ↑ for nuclei of large Z

2 P,T ↘

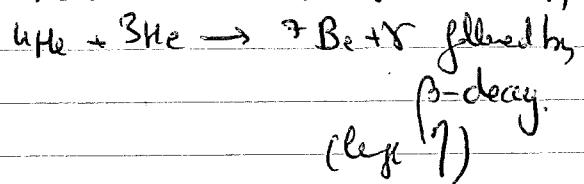
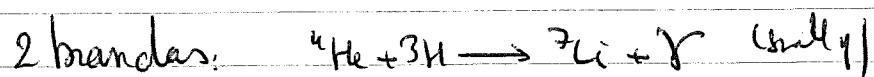
- A=5 & A=8 are not stable

⇒ very small abundance of higher ~~mass~~ nuclei

slide 31 describe He nuclear netw.

slide 32 evolution

slide 33 predictions li-7

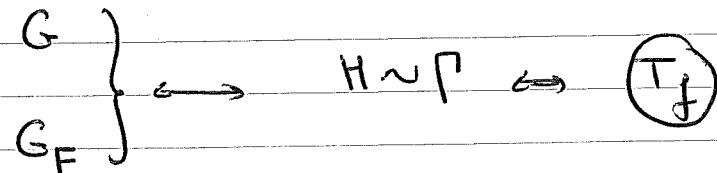


Needless to say it is quite complicated in terms of the FC involved.

We have (slide 34)

- external protons  $\propto : \begin{cases} g_x \text{ (or } N_x) \\ l = \frac{n_b}{n_r} \end{cases}$

FC



$T_n$  n-decay.



$\alpha$  & Coulomb barriers  $\rightarrow$  reaction rates.

Me:  $e^+e^-$  annihilation



Coulomb barrier

nuclei are charged so that they have to beat a coulomb barrier to interact

This is a tunnel effect and it depends on  $\alpha$  which controls the height of the barrier. (as in  $\beta$  decay)

$$\sigma(E) = \frac{S(E)}{E} e^{-2\pi\eta(E)}$$

for factor

$$\eta(E) = \alpha Z_1 Z_2 \sqrt{\frac{\pi r c^2}{2E}} \quad \pi, \text{ reduced mass}$$

$$\cdot C_F = \frac{1}{\sqrt{2\mu^2}}, \quad m_e = h\nu$$

$$\cdot Q = m_p - m_n \quad \text{depends on } h\nu \text{ and } \Lambda_{co}, \alpha$$

$-0.76 \text{ PeV}$        $\leftarrow m_d - m_u \text{ parzen?}$

$$Q = \alpha \alpha \Lambda_{co} + \underbrace{(h_d - h_u) v}_{205 \text{ PeV}}$$

binding energy

$$\frac{\delta Q}{Q} = -0.6 \left( \frac{\Delta \alpha}{\alpha} + \frac{\Delta \Lambda}{\Lambda} \right) + 1.6 \left( \frac{\Delta(h_d - h_u)}{h_d - h_u} + \frac{\Delta v}{v} \right)$$

•  $\zeta_n$  is of the form  $\zeta_n^{-1} = C_F^2 m_0^5 f(\frac{q}{m_e})$   
 actually well approximated by

$$\zeta_n^{-1} = \frac{1 + 3q_A^2}{120\pi^3} C_F^2 m_0^5 \left[ \sqrt{q^2 - 1} (2q^4 - 3q^2 - 8) \right. \\ \left. + 15 \ln \left( q + \sqrt{q^2 - 1} \right) \right]$$

for which one deduces

$$\frac{\delta \zeta_n}{\zeta_n} = -4.8 \frac{\delta v}{v} + 15 \frac{\delta h_e}{h_e} - 10.4 \frac{\delta(h_d - h_u)}{(h_d - h_u)} + 3.8 \left( \frac{\delta \alpha}{\alpha} + \frac{\delta \Lambda}{\Lambda} \right)$$

•  $B_D$  is maybe the most sensitive parameter.

Again it requires nuclear physics and it is difficult to compute

There are mainly 2 approaches

### ① Plan mass

$B_D$  can be related to  $m_\pi$  heuristically by

$$\frac{\Delta B_D}{B_D} = -r \frac{\delta m_\pi}{m_\pi} \quad r \sim 6 \rightarrow 10 \text{ fm/ft}$$

$$m_{\pi}^2 = m_q \langle \bar{u}u + \bar{d}d \rangle_{f_\pi^2} \sim \frac{1}{\lambda} \Lambda_{\text{QCD}}$$

$\frac{1}{2}(m_u + m_d)$

↓ π decay const

$$\therefore \frac{\delta B_p}{B_p} \sim \# \frac{\delta \Lambda}{\Lambda} + \# \frac{\delta m_q}{m_q}$$

## ② Sigma-model

$$V_{\text{Nuc}} = -\frac{g_s^2}{4\pi r} e^{-m_s r} + \frac{g_r^2}{4\pi r} e^{-m_r r}$$

$g_s, g_r$  scalar/vector coupling const

Solving for a Schrödinger eq. one can get  
 bound states and then energy and then  
 deduce it

$$\frac{\delta B_p}{B_p} = -48 \frac{\delta m_s}{m_s} + 50 \frac{\delta m_r}{m_r} + 6 \frac{\delta m_N}{m_N}$$

Then the mass of the Δ depends on the slope quick

so that one ends with a formula of  $m_{\Delta}$

$$\frac{\delta B_p}{B_p} = 18 \frac{\delta A}{\Lambda} - 17 \left( \frac{\delta v}{v} + \frac{\delta g_s}{g_s} \right)$$

- focusing on the effective parameters  $\{B_0, \gamma_n, Q\}$   
we can determine their value to be compatible  
with observational data.

(slide 37)

→ typically  $\sim 10^{-2}$

(slide 38) note however that with one of  $B_0$  one  
can also the L-7 pb.

- The next step is to use all the previous  
relations to set constraints on  $\{h_i, v, d, \Lambda\}$

As an example, I consider what we get in the framework  
of Unification theory -

Many developments       $A=5 / A=8 / \text{CNO etc.}$

No.

IV.2u

Date

### ③ Stellar Dynamics

modif. of  $\alpha$  and nuclear interactive affect.  
reactors have instan.

A crucial part is production of  $e^{12}$ . via 3 $\alpha$  reaction.

BDN steps at  $A=7$  (because  $A=5$  &  $A=8$  are unstable)

whatever will need.

① phenomenological description of  $^8Be$  and  $^{12}C$  nuclei

② integration of reaction rates over  $E$  to get  
flux as function of  $T$

③ include in a stellar model

We shall concentrate on red II stars.

- feed  $\sim 10^8$  years after big bang.  $Z \sim 10^{-3.5}$

- zero metallicity

-  $10 - 100 M_\odot$

- no initial conv. They must contract until 3 $\alpha$   
is triggered.

Production of  $^{12}\text{C}$  via  $^{3}\text{He}$  requires of triple beta decay:

① The decay lifetime of  $\text{Be}-8$  is  $10^{-16}$  s

$\sim 4$  orders of mag. larger than  $\text{He}$  beta decay to scatter  
 $\rightarrow$  allows second  $\alpha$ -capture

② An excited state of  $^{12}\text{C}$  lies just above the energy of  $^{6}\text{Be} + \alpha$

③ The energy level of  $0^-$  at  $7.1197$  MeV is non-resolved and below the energy of  $^{12}\text{C} + \alpha$  ( $7.16$  MeV)  
 $\rightarrow$  that most  $^{12}\text{C}$  is not destroyed

$\rightarrow$  predicted by Keppe in 54

observed by Dubau 53 and decay Cook 57.

Slide 62 shows the energy level of  $^{40}\text{K}(\alpha, \gamma) ^{12}\text{C}$

as of  $^{3}\text{He}$ ,  $^{8}\text{Be}$  &  $^{12}\text{C}$

excited state of  $^{12}\text{C}$  is an  $l=0$  resonance

it decays to first  $l=2$  excited level ( $4.44$  MeV)

Defining:

$E_n(^8Be)$  energy of Be ground state with respect to  $\alpha + \alpha$

$E_n(^{12}C)$  energy of the He Kyle level with respect to  $^8Be + \alpha$

$$E_n(^{12}C) = ^{12}C(O_2^+) + Q_{\alpha\alpha}(^{12}C)$$

$Q_{\alpha\alpha} = E_n(^8Be) + E_n(^{12}C)$  energy of Kyle with 3d shell.

$\Gamma_d(^8Be)$  partial width of Be decay.

$\Gamma_d(^{12}C)$  partial width of  $^8Be + \alpha \rightarrow ^{12}C + \gamma$

Assume Nernst equilibrium between He-4 & Be-8 so that

their abundances are related by the Saha eq.

+ sheep resources.

The rate of  ${}^4He(\alpha, \gamma){}^{12}C$  rate can be expressed

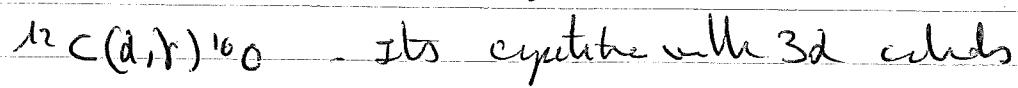
by

$$N_A? \langle \sigma v \rangle^{3d} = 3^{3/2} 6M^2 \left( \frac{2\pi}{m k_B T} \right)^{3/2} \omega r e^{-\frac{Q_{\alpha\alpha}}{k_B T}}$$

$\omega = 1$  spin factor

$$\gamma = \frac{\Gamma_d(^{12}C)}{\Gamma_d(^8Be)}$$

During helium burning. The only expected reaction is



The  $^{12}\text{C}/^{16}\text{O}$

### Microsopic model [slide 63]

In order to analyse the sensitivity of 3d rate to van of strength of electrostatic and NN interaction we use a microscopic model.

$$\alpha \sim N_F \langle \delta v \rangle$$

$$S = \frac{d \ln \alpha}{d \ln \alpha_0} = - \frac{\alpha_{3d}}{\tau} \sim - \frac{6.4}{T_g}.$$

We derive a Schrödinger equation with

$$H = \sum_i^A T(r_i) + \sum_{i>j=1}^A V(r_{ij})$$

kinetic of nuclei

Nuclei-nuclei interact.

$$V = V_C(r_{ij}) + V_N(r_{ij})$$

↓            ↓  
electro -      Nuclei interaction.

$$V_N = \left( V_R(r) + \frac{1}{2} (1 + p^2) V_+ - \frac{1}{2} (1 - p^2) V_S \right) \\ \times \frac{1}{2} [u + (2-u)p^r]$$

$p^2$ : spin exchange operator

$$H \psi^{J\pi} = E^{J\pi} \psi^{J\pi}$$

↓

wave function is a fraction of the  $A-1$  coordinates  $r_{ij}$

when  $A > 6$  no exact solutions can be found.

cluster approx: Brüdy energy if  $\alpha$  is large

$$\psi^{J\pi}_{BZ} = A \phi_\alpha \phi_\alpha g_1^{J\pi}$$

↓

wave function of  $\alpha$

fraction.

antym. cluster

$\alpha \rightarrow \alpha$

$$\psi^{J\pi}_{nc} = \alpha \phi_\alpha \phi_\alpha \phi_\alpha g_3^{J\pi} (\rho, R)$$

$\alpha \rightarrow \alpha$

To take into account varying  $F_C$  we defn.  $V_R$  as

$$V = (1 + \delta_\alpha) V_C + (1 + \delta_{MN}) V_N$$

First, we deduce that

$$\frac{\delta \beta_B}{\beta_B} = 5.716 \delta_{MN} \quad \text{This relation allows} \\ \delta_{MN} \rightarrow (h, v, \lambda).$$

Then

$$\left. \begin{aligned} E_R(^B\!B_2) &= (0.09 - 12 \delta_{MN}) \text{ MeV} \\ E_n(^n\!C) &= (0.28 - 20 \delta_{MN}) \text{ MeV} \end{aligned} \right\} \\ \Rightarrow Q_{3\alpha} = (0.37 - 32.6 \delta_{MN}) \text{ MeV}$$

Then the reaction rate can be obtained by integrating over energy

$$N_A^2 \langle \sigma v \rangle^{3d} \propto \int \frac{\sigma_{3\alpha}(E)}{P_\alpha(E)} e^{-E/kT} N_A \langle \sigma v \rangle^{2s} e dE \\ \propto \frac{P_\alpha^2(E)}{(E-E_h)^2 + P_\alpha^2(E)}$$

We obtain slide 44.

$$y = \log \frac{d\sigma_{3\alpha}(\delta_{MN})}{d\ln \omega} = \frac{1}{\ln 10} \underbrace{\frac{S \delta_{MN}}{d \ln \frac{d\sigma_{3\alpha}}{d \ln \delta_{MN}}}}$$

$y = 1.644 \left( \frac{\delta_{MN}}{10^{-3}} \right) \left( \frac{T}{10^8 K} \right)^{-1}$

What is written here is that

$$\frac{\delta Q_{3d}}{\delta \delta m} \sim 10^2 Q_{3d}$$

concentrations of  $10 \delta_m$  MeV on an energy of 0.1 MeV

$\Rightarrow$  exploring constraint

Then this is introduced in a Sheller codes

- slide 45 : HR degree for 15  $\pi\circ$

- slide 46 60  $\pi\circ$

- slide 47 gives more constraints.