Lecture II: Sept. 26

Today, the goal is to discuss several theoretical framework of the litterature in which FC are dynamical, from standard field theory to string theory.

This will give a second motivation for this test: they allow to constraint many theoretical framework of gravity beyond GR.

• Field Theory

As already explained, if a constant is varying this means it has to be replaced by a dynamical field.

This has to be done at the level of the action

AND

it is a common <u>mistake</u> to let it vary in equations that have been derived under the assumption that this constant was actually constant

The famous example is VSL (at least in their early version):

- They assume

$$H^2 = \frac{8\pi G}{c^4}\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$$

- assume e.g. $G(t)$; $c(t)$ or $\Lambda(t)$

- Then use $\nabla_\mu \Big(\underbrace{T^{\mu\nu}\frac{8\pi G}{c^4} + \Lambda g_{\mu\nu}}_{T^{\mu\nu}_{tot}} \Big) = 0$

<u>can you tell me what is wrong here?</u>

one will get a modified equation of conservation including $\dot{\rho}$, $\dot{c}$ etc..

But there will miss one equation

Usually one then <u>postulates</u> $c(t)$ or $G(t)$.

In field theory, this is different

$$\mathcal{L}(\underbrace{\phi,\dots}_{x^\mu};\alpha) \longrightarrow \mathcal{L}(\phi\dots\underset{\uparrow\, x^\mu}{\alpha})$$

Then

① $\dfrac{\delta\mathcal{L}}{\delta\phi} = \partial_\mu \dfrac{\delta\mathcal{L}}{\delta\partial_\mu\phi}$ will include terms in $\partial_\mu\alpha$

so the "δd" equations will be modified

② $\dfrac{\delta\mathcal{L}}{\delta\alpha} = \partial_\mu \dfrac{\delta\mathcal{L}}{\delta\partial_\mu\alpha}$ There is a <u>new</u> equation

so that there is no freedom to choose $\alpha(t)$ at hand!

This is important since many wrong papers are based on this mistake.

- <u>Scalar-tensor theories:</u>

Historically, the first "varying G" theory was formulated by Jordan (1937) who considered

$$S = \int \sqrt{-g}\, d^4x\, \phi^1 \left[R - \xi \left(\frac{\nabla\phi}{\phi}\right)^2 - \frac{\phi}{2} F^2 \right]$$

What would you say about this action?

[to be discussed with students / compare with

$$S = \int \sqrt{-g}\, d^4x \left[\frac{R}{2\kappa} - \frac{1}{4\alpha} F^2 \right]$$

$$\begin{cases} \bullet\ \frac{Gm^2}{\hbar c};\ \alpha \longrightarrow (\eta, \xi) \\ \bullet\ 1 \text{ new d.o.f } \phi \end{cases}$$

To understand better the structure of such theories, I concentrate on scalar-tensor theories.

In the Jordan frame, they take the form

$$S = \int \frac{d^4x}{16\pi G_*} \sqrt{-g} \left[F(\varphi) R - g^{\mu\nu} Z(\varphi) \partial_\mu \varphi \partial_\nu \varphi - 2U(\varphi) \right] + S_m [\psi; g_{\mu\nu}]$$

G_*: constant of the theory
Not to be confused with G_N

(Z, F, U): 3 arbitrary functions
only 2 are physical
$F > 0$ for gravity to carry pos. energy.

matter fields are all coupled to $g_{\mu\nu}$ minimally
$\Rightarrow$ satisfies the EEP in its weak form.

The equation of motions are obtained by
$\delta g_{\mu\nu}$ [Einstein] $\delta\varphi$ [conservation]
$\delta\varphi$ [New eq.]
and are

$$\begin{cases} F(\varphi)\, G_{\mu\nu} = 8\pi G_* T_{\mu\nu} + Z(\varphi)\left[\partial_\mu\varphi\,\partial_\nu\varphi - \frac{1}{2} g_{\mu\nu}(\partial_\alpha\varphi)^2\right] \\ \qquad\qquad + \nabla_\mu\partial_\nu F - g_{\mu\nu}\Box F - g_{\mu\nu}\, U(\varphi) \\ 2Z\,\Box\varphi = -F' R - Z'(\partial_\alpha\varphi)^2 + 2U' \\ \nabla_\mu T^\mu{}_\nu = 0 \end{cases}$$

with $T^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S_m}{\delta g_{\mu\nu}}$

- if $\varphi = c^t$. Then we are back to RG (Einstein + conservation equations)
- $\frac{G_*}{F(\varphi)} = G_{eff}(\varphi)$ it looks like a varying G theory
- we see explicitely the difference with letting G vary in Einstein equation because there are less in $\partial\varphi$ and the new (KG) equation.

- $\nabla_\mu T^{\mu\nu} = 0$ because of the minimal coupling to $g_{\mu\nu}$

This action is usually rewritten in Einstein frame after the change of variables

$$\begin{cases} A(\varphi_*) = F^{-1/2}(\varphi) \\ 2V(\varphi_*) = U(\varphi)\, F^{-2}(\varphi) \\ g^*_{\mu\nu} = F(\varphi)\, g_{\mu\nu} \\ \left(\frac{d\varphi_*}{d\varphi}\right)^2 = \frac{3}{4}\left(\frac{d\ln F(\varphi)}{d\varphi}\right)^2 + \frac{1}{2F(\varphi)} \end{cases}$$

so that the action reads

$$S = \frac{1}{16\pi G_*}\int d^4x \sqrt{-g_*}\left[R_* - 2g^{\mu\nu}_* \partial_\mu\varphi_* \partial_\nu\varphi_* - 4V\right] + S_m\left(A^2 g^*_{\mu\nu}; \psi\right)$$

when we compare to the Jordan frame action, we see:

- φ_* is not directly coupled to R_* explicitly
- RG + φ + coupling to matter to $A^2 g^*_{\mu\nu}$
- consider a massive scalar field

J.F: $-\frac{1}{2}\int d^4x \sqrt{-g}\left(g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi + m^2\psi^2\right)$

EF: $-\frac{1}{2}\int d^4x \sqrt{-g^*}\left[A^2(\varphi)\, g^{*\mu\nu}\partial_\mu\psi\partial_\nu\psi + m^2 A^4(\varphi)\psi^2\right]$

consider the action of part particle

JF $\quad -\int mc\sqrt{-g_{\mu\nu}dx^\mu dx^\nu} \qquad m = c^t$

EF $\quad -\int \underbrace{mA(\varphi_\alpha)}_{m(\varphi_\alpha)}\sqrt{-g_{\alpha\mu\nu}dx^\mu dx^\nu}$

But
$$Gm^2 = \begin{cases} \frac{G_*}{F}m^2 = G_* A^2 m^2 & \text{in JF} \\ G_\alpha m^2(\varphi_\alpha) = G_\alpha A^2 m^2 & \text{in EF} \end{cases}$$

The theory can be thought as

- varying G theory
- varying m theory (with $\frac{m_i}{m_j} = c^t \ \forall ij$)

But this is the same theory because the dimensionless quantity

$$\boxed{\alpha_G = \frac{Gm^2}{\hbar c} \text{ is the same in both frames.}}$$

- part particle follows geodesics of JF metric but not of the EF
- observables in 2 frames cannot be distinguished if dimensionless.

Note also that for electromagnetism

$$S_{Maxwell} = \frac{1}{4}\int \sqrt{-g^*}\, g^{*\mu\nu} g^{*\rho\sigma} F_{\mu\rho} F_{\nu\sigma}\, d^dx$$

$$= \frac{1}{4}\int \sqrt{-g}\, g^{\mu\nu} g^{\rho\sigma} A^{d-4}(\phi) F_{\mu\rho} F_{\nu\sigma}\, d^dx$$

In 4 dimension the action is invariant (conformal invariance)

$\Rightarrow$ Null geodesic of $g_{\mu\nu}$ & $g^*_{\mu\nu}$ are the same.

Deflection angle is $\delta\theta = \frac{4 G_* M}{b c^2}$ ← careful here!!

- The field equations are

$$\begin{cases} G^*_{\mu\nu} = 8\pi G_* T^*_{\mu\nu} + 2\partial_\mu\varphi_* \partial_\nu\varphi_* - g^*_{\mu\nu}\left(g_*^{\alpha\beta}\partial_\alpha\varphi_* \partial_\beta\varphi_*\right) - 2V g^*_{\mu\nu} \\ \Box_* \varphi_* = -4\pi G_* \alpha(\varphi_*) T_* + V'_* \\ \nabla^*_\mu T_*{}^\mu{}_\nu = \alpha(\varphi_*) T_* \partial_\nu \varphi_* \end{cases}$$

- Einstein equation are the same as in GR

- Eq. of conservation 2 KG are coupled so that we explicitely see the fifth force in this frame.

$\alpha \equiv \frac{d\ln A}{d\varphi_*}$ strength of coupling

$\beta = \frac{d\alpha}{d\varphi_*}$

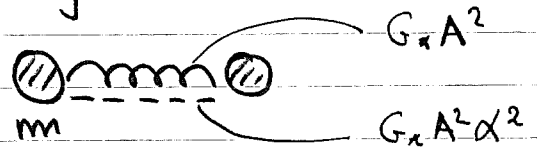
- <u>What is the gravitational constant?</u>

  - Not G_* !

  - It is dynamical and one needs to define it.

  - $G_{eff} = G_* A^2(\varphi_*)$ is a guess for the field equation

  - the analysis in weak field leads to definition of G_{cav}



[see Damour-Esp 92]

$$\begin{cases} G_N^{(0)} = G_* A_0^2 (1+\alpha_0^2) & \text{today} \\ G_N = G_N^{(0)} \dfrac{A^2}{A_0} \dfrac{1+\alpha^2}{1+\alpha_0^2} & \text{is a funct of } \varphi(x^\mu) \end{cases}$$

It can be expressed in JF as

$$G_N = \frac{G_*}{F}\left(1 + \frac{F_\phi^2}{2F+3F_\phi^2}\right)$$

In weak field, one can apply the PPN formalism and derives the deviation from GR by equating the PPN parameter

$$\gamma^{PPN} - 1 = -\frac{2\alpha_0^2}{1+\alpha_0^2} \qquad \beta^{PPN} - 1 = \frac{1}{2}\frac{\beta_0 \alpha_0^2}{(1+\alpha_0^2)^2}$$

and the time var. of G

$$\frac{\dot{G}_N}{G_N} = 2\alpha_0\left(1 + \frac{\beta_0}{1+\alpha_0^2}\right)\dot{\varphi}_{*0}.$$

This shows on a particular exaple that

- deviation from GR
- var. of FC

} are related and 2 aspects of the same modification.

Today in the Solar system $\alpha_0^2 < 10^{-5}$

. we need to be close to GR today but in principle can be far in the early universe.

- <u>What about other constants</u>

It seems a priori easy to make a constant dynamical:

- add a scalar field with kinetic term $(\partial\phi)^2$
- do $\alpha_i \cancel{\to} \alpha_i(\phi)$

let us try with the fine structure constant

$$S = \int \left\{ \frac{R}{16\pi G} - 2(\partial_\mu\phi)^2 - \frac{1}{4}B(\phi)F_{\mu\nu}^2 - V(\phi) \right\} \sqrt{-g}\, d^4x$$

So that

$$\alpha(\phi) = B^{-1}(\phi)$$

we want to emphasize that it has dramatic effects on the UFF.

Indeed, p, n, nuclei have an EM binding energy so their masses will be time dependent.

As we explained last week, the mass of any nuclei has a contribution

$$m_A(\varphi) \supset 98.25\, \alpha \frac{Z(Z-1)}{A^{1/3}} \text{ MeV}$$

consider sources of point masses

$$T^{\mu\nu} = \frac{1}{\sqrt{g}} \sum_i \int m_i(\varphi(y_i))\, u_i^\mu u_i^\nu\, ds_i\, \delta^{(4)}(x-y_i)$$

The field equation reads

$$R_{\mu\nu} = 2\partial_\mu\varphi\partial_\nu\varphi + 8\pi G\left(T_{\mu\nu} - \frac{1}{2}T g_{\mu\nu}\right)$$

$$\Box\varphi = -4\pi G\sigma$$

$$\sigma(x) = -\frac{1}{\sqrt{g}}\sum_i \int ds_i \frac{\partial m_i}{\partial\varphi}\delta^4(x-y_i)$$

$$\equiv \sum \alpha_i(\varphi)\, T_i(x)$$

$$\hookrightarrow g_{\mu\nu}T_i^{\mu\nu}$$

$$a_i^\mu = -\alpha_i[\varphi]\nabla^\alpha\varphi_i\,(g_\alpha{}^\mu + u_\alpha u^\mu) \quad (\text{cf. I.29})$$

It can be shown that the Newtonian interaction potential is

$$V_{int} = -G\frac{m_i m_j}{r_{ij}}(1 + \alpha_i\alpha_j)$$

So

$$\gamma_{ij} = \frac{(\alpha_i - \alpha_j)\alpha_E}{1 + \frac{1}{2}(\alpha_i + \alpha_j)\alpha_E} \sim (\alpha_i - \alpha_j)\alpha_E$$

$$\alpha = \frac{\partial \ln m_{A,Z}/m_p}{\partial \varphi}$$

$$= \frac{1}{A m_p} \; 98.25 \, \frac{Z(Z-1)}{A^{1/3}} \, \frac{d\alpha}{d\varphi} \, \mathrm{MeV}$$

$$m_{A,Z} \simeq (A-Z) m_n + Z m_p + 98.25 \, \alpha \, \frac{Z(Z-1)}{A^{1/3}} \, \mathrm{MeV} + \dots$$

[Note: we will have to be more carefull because $m_p = \dots m_u \dots m_d \dots E_s \dots E_\alpha$]

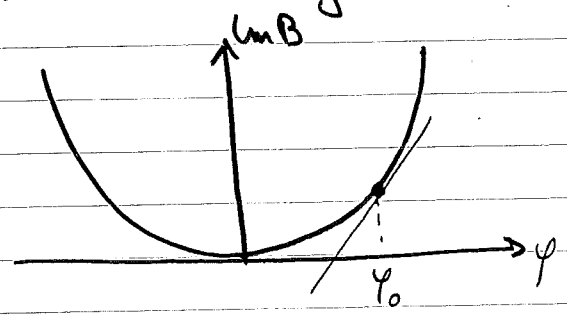
$$\alpha = 98.25 \left(\frac{\mathrm{MeV}}{m_p}\right) \frac{Z(Z-1)}{A^{4/3}} \, \alpha \, (\ln B)'_0$$

$$\simeq c_\alpha \frac{Z(Z-1)}{A^{4/3}} (\ln B)'_0 \qquad \text{with } c_\alpha \sim 98.25 \, \frac{1}{938} \, \frac{1}{137}$$

$$\sim 10^{-3}$$

$$\eta_{ij} \sim 10^{-6} \underbrace{f[Z_i, A_i, Z_j, A_j] \, f[Z_E, A_E]}_{\mathcal{O}(10^{-1};\, 10)} \left[(\ln B)'_0\right]^2$$

It means that in order to satisfy the bound in the solar system we need $(\ln B)'_0 \ll 1$



i.e. we need to be close to the minimum of the coupling function.

- This is a generic problem of VFC theories

- Note that we have been considering <u>only</u> the contribution of the EM binding energy of the nuclei.

In fact we have to include similar contributions for n & p so that in full generality

$$m_A^Z = \Lambda_{QCD}\, f_m\Big(\underbrace{\frac{m_u}{\Lambda};\ \frac{m_d}{\Lambda};\ \frac{m_e}{\Lambda}}_{k_i};\ \alpha\Big)$$

so that

$$\alpha = \frac{\partial \ln \Lambda_{QCD}/m_p}{\partial \varphi} + \sum \frac{\partial \ln k_i}{\partial \varphi} \cdot \frac{\partial \ln f_m}{\partial k_i}$$

we shall discuss these contributions later but they are dependent of knowledge of QCD for the $\frac{\partial \ln f_m}{\partial k_i}$ and of the particular coupling of φ for the $\frac{\partial \ln k_i}{\partial \varphi}$

$Q_i^{(A,Z)} \equiv \frac{\partial f_m^{A,Z}}{\partial k_i}$ are effective charges measuring the sensitivity of the mass to var. of a constant

This shows that in general we expect a large violation of the universality of free fall.

There are 2 classes of mechanism that allows to pass around this:

- least coupling mechanism (Damour-Nordtvedt)
- chameleon-like

I will describe them at the end of the lecture.

- <u>Motivation from high-energy physics</u>

Historically one motivation arises from extra-dimension theories.

consider a 5D Einstein-Hilbert action

$$S_5 = \frac{1}{12\pi^2 G_5} \int \bar{R} \sqrt{-\bar{g}}\, d^5x$$

and decompose the 5-dimensional metric as

$$\bar{g}_{AB} = \begin{pmatrix} g_{\mu\nu} + \frac{A_\mu A_\nu}{M^2}\phi^2 & \frac{A_\mu}{M}\phi^2 \\ \frac{A_\nu}{M}\phi^2 & \phi^2 \end{pmatrix} \begin{matrix} [A_\mu] = M \\ \text{dimensionless} \end{matrix}$$

So the 15 dof of $\bar{g}_{AB}$ are decomposed in

$10 + 4 + 1$ dof

$x^A = (x^\mu, y)$

<u>Kaluza cylender condition</u> : the theory is capacitified on a circle

It assumes that nothing depend on y (ie $\partial_y X = 0$)

$$X(x^\mu, y) = \sum \chi_n(x^\mu)\, e^{2\pi i \frac{n}{L} y}$$

This means that we consider scales $\gg L$, the scale of capacitification

i.e. At low energy we can neglect $n \neq 0$ modes

The action S_5 can be rewritten in terms of $(\underset{\substack{\downarrow \\ R}}{g_{\mu\nu}}, \phi, A_\mu)$ as

$$\boxed{S = \frac{1}{16\pi G} \int d^4x \sqrt{-g}\, \phi \left(R - \frac{\phi^2}{4\pi^2} * F_{\alpha\beta} F^{\alpha\beta} \right)}$$

with $\quad G = \dfrac{3\pi \bar{G}_5}{4 V_{(5)}} \qquad V_{(5)} = \int dy$

As we discussed earlier this is a theory with varying (G, α) if A_μ is the $U(1)$ of the standard model.

Note that there is no term with ϕ alone since

$R \supset \phi^{-1} \Box \phi$ but $\sqrt{\bar{g}} = \phi \sqrt{g}$ so that we have $\Box \phi$, i.e. a total derivative

If needed, one can use the form of the Ricci tensor

$$\bar{R}_{44} = \frac{1}{4\pi^2}\phi^4 F_{\alpha\beta}F^{\alpha\beta} - \phi\left(\partial_\alpha\partial^\alpha + \Gamma^\alpha_{\beta\alpha}\partial^\beta\right)\phi$$

$$\bar{R}_{4\mu} = \frac{1}{\pi}\left[\bar{R}_{44}A_\mu + \frac{1}{2}\phi^2\left(\nabla^\alpha F_{\mu\alpha} + 3\frac{\partial^\alpha\phi}{\phi}F_{\mu\alpha}\right)\right]$$

$$\bar{R}_{\mu\nu} = R_{\mu\nu} - \frac{1}{\phi}\nabla_\mu\nabla_\nu\phi - \frac{\phi^2}{2\pi^2}F^\alpha_\mu F_{\nu\alpha} + \frac{1}{\pi}\left(A_\mu\bar{R}_{4\nu} + A_\nu\bar{R}_{4\mu}\right)$$
$$- \frac{1}{\pi^2}\bar{R}_{44}A_\mu A_\nu$$

- Generically, any theory with extra-dimension will reduce to an effective 4D theory with varying coupling

- In this sytlest form, one expects an unpotential variation of α.

In particular $\phi = ct$ is not a solution of the field equations

$$\begin{cases} \Box\phi = \dfrac{\phi^3}{4\pi^2}F_{\alpha\beta}F^{\alpha\beta} \\ \nabla^\alpha F_{\mu\alpha} = -3\dfrac{\nabla^\alpha\phi}{\phi}F_{\mu\alpha} \\ G_{\mu\nu} = \dfrac{\phi^2}{2\pi^2}T^{(EM)}_{\mu\nu} + \dfrac{1}{\phi}\left(\nabla_\mu\nabla_\nu\phi - g_{\mu\nu}\Box\phi\right) \end{cases}$$

unless $A_\mu = 0$.

So one cannot recover Einstein + Maxwell

If $A_\mu = 0$, which a solution of the field equation, then we recover a scalar-tensor theory with no potential

Note that usually, the scalar field is redefined as

$\phi = e^{4\sqrt{\frac{\pi}{3}}\frac{\varphi}{M_P}}$ and φ is the dilaton with $[\varphi] = M$

$$\left(\begin{array}{ccc} \text{Scalar-tensor theories} & & \text{string theory} \\ \phi & \longleftrightarrow & \varphi \\ \text{Jordan frame} & & \text{string frame} \end{array}\right)$$

<u>Note</u>: • Even if ϕ has no explicit kinetic term in the action, it is a dynamical field, as can be seen from equation $D\phi = \cdots$

• one can, as in ST theories, diagonalize the d.o.f. by switching to Einstein frame. one performs

$$g^*_{AB} = \phi^{-1} g_{AB}$$

$$S = \frac{1}{12\pi^2 G_s} \int d^4x \sqrt{-g_*}\, R_*$$

$$= \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} \left(R - \frac{\phi^2}{4\pi^2} F_{\alpha\beta} F^{\alpha\beta} - \frac{3}{2} \frac{\nabla_\mu \phi \nabla_\nu \phi}{\phi^2} \right)$$

$$= \int d^4x \sqrt{-g} \left\{ \frac{1}{16\pi G_N} \left(R - \frac{\phi^2(\varphi)}{4\pi^2} F^2 \right) - \frac{1}{2} \nabla_\mu \varphi \nabla^\mu \varphi \right\}$$

so that φ behaves as a canonical field.

It follows that (if A_μ is the electromagnetic A_μ)

$$\boxed{\alpha^* = \frac{1}{4\pi} e^{-\frac{4\pi}{\sqrt{3}M_p}\varphi}}$$

Note that $\alpha \sim \frac{1}{137}$ if $\varphi \sim 10^{-2} M_p$

but φ dynamical so is α and one needs to explain this coincidence.

In the case of D extra-dim $\boxed{G \sim \phi^{-D} \qquad \alpha_i \sim K_i(D)\, \phi^{-2}}$

During the compactification, we have a typical mass scale

$$M \sim R^{-1}$$

If M is large enough, low energy observables are not sensitive to the fifth dimension.

At higher energy, there appear a series of excited modes.

consider a scalar field

$$S_\Sigma = -\frac{1}{2}\int d^5x \sqrt{g_*}\, g^{AB}\partial_A \Sigma \partial_B \Sigma$$

$$\Sigma(x_\mu, y) = \sum_{n=-\infty}^{+\infty} \sigma_n(x_\mu)\, e^{inMy} \qquad \sigma_{-n} = \sigma_n^*$$

and expand the action (assuming $\partial_y \phi = 0 = \partial_y A_\mu$, i.e. considering only the zero modes of ϕ & A_μ) Then, we find

$$S_\Sigma = -\frac{2\pi}{M}\int d^4x \sqrt{-g}\left\{ \frac{1}{2}\partial_\mu \sigma_0 \partial^\mu \sigma_0 + \sum_1^\infty \left[|\partial_\mu - inA_\mu \sigma_n|^2 + \frac{n^2 M^2}{\phi^2}|\sigma_n|^2 \right]\right\}$$

By renormalising σ_n as $\sigma_n = \sqrt{\frac{M}{4\pi}}\,\tilde{\sigma}_n$ we obtain the action for

- massless field $\tilde{\sigma}_0$
- series of massive fields $\tilde{\sigma}_n$; $m_n = \frac{nM^{3/2}}{\phi\sqrt{2\pi}}$

So the masses depend explicitely on the dilaton and are thus <u>dynamical</u>

No.

Date

Some motivations from string theory

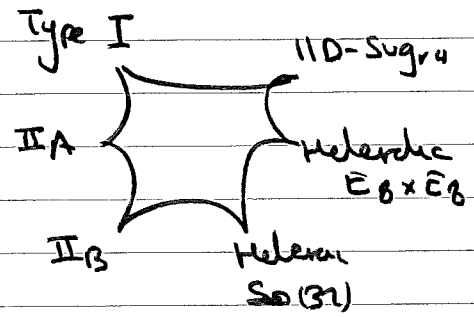
String theory is based on the idea that fundamental objects are no more point particles but 1-dimensional objects.

Superstring theory including bosons and fermions is found to be consistent in $D = 10$.

$\Rightarrow$ need of 6 spatial extra-dimensions

So Kaluza-Klein effects, including dynamical constants, are generic of string theory.

There exist different theories, only 5 are consistent to which one needs to add 11-d supergravity



strings can be of 2 types: open and closed

There exist dualities between these theories.

My goal is indeed not to describe these theories but to have a look to their low energy effective limit.

<u>For the 2 heterotic string theories</u>

they contain closed string and the <u>tree-level</u> action is

$$S_H = \int d^{10}x \sqrt{-g_{10}}\, e^{-2\Phi} \left\{ M_H^8 \left(R_{10} + 4\Box\phi - 4(\nabla\phi)^2 \right\} - \frac{M_H^6}{4} F^{AB} F_{AB} \right.$$
$$\left. + \dots \right\}$$

This action needs to be compactified. Assuming a 6D Kalabi-Yau space, it leads to

$$S_H = \int d^4x \sqrt{-g_4}\, \phi \left[M_H^8 \left\{ R_4 + \left(\frac{\nabla\phi}{\phi}\right)^2 - \frac{1}{6}\left(\frac{\nabla V_6}{V_6}\right)^2 \right\} - \frac{M_H^6}{4} F^2 + \dots \right.$$

with $\phi = V_6\, e^{-2\Phi}$

so we have $\boxed{M_4^2 = M_H^8\, \phi \qquad g_{YM}^{-2} = M_H^6\, \phi}$

if the matter fields (in the dots) are minimally coupled to g_4

<u>For type I</u> it contains open strings and its low energy action is

$$S_I = \int d^{10}x \sqrt{-g_{10}}\, M_I^6\, e^{-\Phi} \left[e^{-\Phi} M_I^2 R_{10} - \frac{F^2}{4} + \dots \right]$$

terms describing the dynamics of the dilaton + fermions.

contrary to S_H, the dilaton couples differently to R and F^2

type I requires Dg-Branes to be consistent

After compactification, one gets at tree level

$$S_I = \int d^{10}x \sqrt{-g_{10}}\, e^{-2\phi} M_I^8 R_{10} - \int d^4x \sqrt{-g}\, e^{-\phi} \frac{1}{4} F^2 + \dots$$

"gravity sector" — YM fields localized on the branes

This 2 examples shows that at tree-level, the existence of a dilaton implies that effective 4D constant are dynamical.

- effect of loop corrections: include KK excitations and one gets corrections

$$g_{YM}^{-2} = M_H^6 \phi - \frac{b_a}{2} (M_H R)^2 + \dots$$

and the coupling is no more universal.

- At tree level, there is no stabilisation mechanism

$\Rightarrow$ $\boxed{\text{why are the constants so constant}}$

- Witten (1987), in string theory <u>all</u> dimensionless constants are dynamical.

- A large set of models

These theoretical motivations have motivated the formulation of a series of phenomenological models.

I just summarize some of them.

Bekenstein model (1982)

it introduces of modification of the electromagnetic sector by introducing a scalar field ε such that

$$e_i = e_{0i}\, \varepsilon(x^\alpha)$$

Its dynamics is described by

$$S_\varepsilon = -\frac{\hbar c}{2\ell^2} \int g^{\mu\nu} \frac{\partial_\mu \varepsilon\, \partial_\nu \varepsilon}{\varepsilon^2} \sqrt{-g}\, d^4x$$

constant length scale

invariant under $\varepsilon \to \lambda\varepsilon$

The electromagnetic sector is modified by

$$e_i A_\mu \to e_{0i}\, \varepsilon A_\mu$$

em potential

The gauge invariance is preserved if $\varepsilon A_\mu \to \varepsilon A_\mu + \lambda_{,\mu}$ for all λ.

It follows that the action for A_μ is

$$S = -\frac{1}{16\pi} \int F^{\mu\nu} F_{\mu\nu} \sqrt{-g}\, d^4x$$

$$F_{\mu\nu} \equiv \frac{1}{\varepsilon}\left[(\varepsilon A_\nu)_{,\mu} - (\varepsilon A_\mu)_{,\nu}\right]$$

The gravitational sector is described by the standard Einstein-Hilbert action.

It can be rewritten in terms of $a_\mu \equiv \epsilon A_\mu$ $\psi = \ln \epsilon$ as

$$S = \frac{c^3}{16\pi G}\int R\sqrt{-g}\, d^4x - \frac{1}{16\pi}\int e^{-2\psi} f^{\mu\nu} f_{\mu\nu} \sqrt{-g}\, d^4x$$

$$- \frac{1}{8\pi K^2}\int (\partial_\mu \psi)^2 \sqrt{-g}\, d^4x$$

$$K = \frac{1}{(4\pi \hbar c)} \qquad f_{\mu\nu} = \partial_{[\mu} a_{\nu]}$$

under this form the structure of the theory is clear and our previous discussion shows that it will be in trouble with UFF

The model can then easily generalized to allow the coupling of ψ to other fields.

$$S = \frac{1}{2} M_4^2 \overbrace{\int R\sqrt{-g}\, d^4x}^{\text{H.E.}} - \int\Big[\frac{1}{2} M_*^2 \partial_\mu \phi \partial^\mu \phi + \frac{1}{4} B_F(\phi) F^2\Big]\sqrt{-g}\, d^4x$$

$$- \int \Big\{ \sum \bar{N}_i (i \not{D} - m_i B_N(\phi)) N_i + \frac{1}{2}\bar{\chi}\partial\lambda$$

$\not{D} = \gamma^\mu(\partial_\mu - i e_0 A_\mu)$ photon hotic

~~neutron~~ Dark matter

$$+ M_4^2 B_\Lambda(\phi)\Lambda + \frac{1}{2} M_\lambda B_\lambda(\phi)\chi^T\lambda\Big\}\sqrt{-g}\, d^4x$$

$\Lambda \to$ effective potential

~~gaugino~~.

The models of free functions B_F, B_{N_i}, B_Λ, B_χ usually they can be extended as

$$B_\chi = 1 + \xi_\chi \phi + \frac{1}{3} \xi_\chi \phi^2$$

for instance

$$\alpha = \frac{e_0^2}{4\pi B_F(\phi)} \Rightarrow \frac{\Delta\alpha}{\alpha} = \xi_F \phi + \frac{1}{2}\left(\xi_F - 2\xi_F^2\right)\phi^2$$

It is a very general framework that includes Bekenstein, Brans-Dicke...

The dynamics is modified <u>cosmologically</u> by the coupling to Λ & DM

If B_i' have a minimum then it can dynamically lower the variation of UFF.

$\boxed{\text{String Inspired models:}}$

The other phenomenological framework was proposed by Damour & Polyakov.

Starting from the tree-level actions of string theory, they argued that the full string loop expansion should lead to an action of the form

$$S = \int d^4x \sqrt{-\hat{g}} \Big[M_s^2 \{ B_g(\phi) \hat{R} + 4 B_\phi(\phi) [\Box\phi - (\nabla\Phi)^2] \}$$

stringy scale

$$- B_F(\phi) \frac{k}{4} \hat{F}^2 - B_\psi(\phi) \bar{\psi} \not{D} \psi + \dots$$

$B_g, B_\phi, B_F, B_\psi \dots$ are not known functions of ϕ

Assuming that they are obtained from a genus expansion, in the limit $\Phi \to -\infty$ they shall be of the form

tree level

$$B_i(\phi) = e^{-2\Phi} + c_0^{(i)} + c_1^{(i)} e^{2\Phi} + \dots$$

Again, one can shift to the Einstein frame by

$$g_{\mu\nu} = C B_g \hat{g}_{\mu\nu} \; ; \; \psi = \sqrt{C B_g}^{-3/4} B_\psi^{1/2} \hat{\psi}$$

$$\phi = \int \left[\frac{3}{4} \left(\frac{B_g'}{B_g} \right)^2 + 2 \frac{B_\Phi'}{B_\Phi} + \frac{2 B_\Phi'}{B_g} \right] d\Phi$$

So that

$$S = \int \frac{d^4x}{16\pi G} \sqrt{-g} \Big[R - 2(\nabla\phi)^2 - \frac{k}{4} B_F(\phi) F^2 - \bar{\psi} \not{D} \psi + \dots$$

Thus $\boxed{g_{YM}^{-2} = k B_F(\phi)}$

This is a very general framework that allows to discuss the conditions on the functions B_i so that one avoids problems with the UFF.

I shall come back on this later after a

discussion on the coupled variation of constants

- <u>Least coupling mechanism</u>

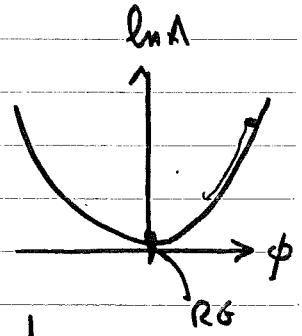
Let me come back on pure Scalar-Tensor theories.

The equation of evolution in Einstein frame for ϕ is

$$\Box \varphi = -4\pi G\, \alpha(\varphi)\, T_* + V'$$

Assume

$$\begin{cases} \cdot\ V = 0 & \text{(massless dilaton)} \\ \cdot\ A = e^{a(\varphi)} & a = \frac{1}{2}\beta\varphi^2 \end{cases}$$



Consider the dynamics in a FL spacetime

$$ds^2 = -dt^2 + a^2(t)\gamma_{ij}\, dx^i dx^j$$

then

$$\begin{cases} \ddot{\varphi} + 3H\dot{\varphi} = -4\pi G_* \alpha(\varphi)(\rho - 3P) \\ \dot{\rho} + 3H(\rho + P) = \alpha(\varphi)(\rho - 3P)\dfrac{d\varphi}{dt} \end{cases}$$

$\beta\varphi$

assume $P = w\rho$

use $p = -\ln(1+z)$ as time variable

The KG equations rewrites as

$$\underbrace{\frac{2}{3-\varphi'^2}}_{m(\varphi)}\varphi'' + \underset{\text{damping}}{(1-w)\varphi'} = \underset{\text{potential}}{-\alpha(\varphi)\,(1-3w)}$$

if $\beta > 0$ and $w \neq \frac{1}{3}$ $\quad \varphi \to 0$

- During RDU, $1-3w=0$ $\quad \varphi \to$ ct.

  is it always so? $\rightsquigarrow$ Mass threshold: $\boxed{\text{Slide 2}}$

  No

- matter era : the source term is active

  There exists an exact solution but we can discuss what happens on a plot

  - dependence of β, Ω_{cdm}. $\boxed{\text{Slides 3-5}}$

[we shall later discuss how it generalizes to the general models

• Chameleon

Another interesting idea is to play with the potential of the field.

$$\Box \phi = -\frac{dV_{eff}}{d\phi} \qquad V_{eff} = V(\phi) + A^2(\phi)\rho$$

Assume $V(\phi) \searrow$ while $\ln A \nearrow$

runaway potential $\qquad A^2 \sim e^{\beta\phi/M_p}$

then V_{eff} has a minimum that depends on the local matter density!

$\Rightarrow$ field light in space

massive on Earth

if it is massive enough then one may avoid laboratory constraints.

Coupled variations of the constants

- gauge coupling unification

In quantum field theory, the computation of scattering processes include higher order corrections of the coupling constants related to loop corrections, that introduce integral over momenta.

Depending on the theory, these integrals may be finite, log or power law divergent in terms of a UV cut-off

renormalizable: physical quantities do not depend on the choice of the cut-off.

But may depend on the energy E of the process.

The renormalisation group allows to compute the dependence of a coupling constant as a function of E

$$\frac{d \ln g_i(E)}{d \ln E} = \beta_i(E)$$

depend on gauge group particle content

In the standard model

$$\beta_1(g_1) = \frac{g_1^3}{4\pi^2} \frac{5 n_g}{9}$$

number of generations of fermions

$$\beta_2(g_2) = -\frac{g_2^3}{4\pi^3}\left(\frac{11}{6} - \frac{n_g}{3}\right)$$

$$\beta_3(g_3) = -\frac{g_3^3}{4\pi^2}\left(\frac{11}{4} - \frac{n_f}{6}\right)$$

number of quark flavor with mass < E

• at large momentum, coupling & 2 loop correction become less and less important: QCD is said asymptotically free.

• integrating $\Rightarrow$ $\alpha_3(E) = \dfrac{6\pi}{(33 - n_f)\ln E/\Lambda}$

Λ_{QCD} is the scale at which α_3 diverges.

$\boxed{\text{slide 6}}$

→ idea that the 3 non-grav. couplings may have a single common coupling strength at high energy.

→ driving idea of Grand Unified theories

$SO(10)$; $SU(5)$ → ... → $SU(3)\times SU(2)\times U(1)$

It has 2 consequences for us:

- there exist relations between the Yukawa couplings.

- ∃ scale M_U such that

$$\alpha_1(M_U) = \alpha_2(M_U) = \alpha_3(M_U) = \alpha_U(M_U).$$

The variation of α has to be accompagned by a variation of the couplings.

From the renormalisation group equation, one gets

$$\alpha_i^{-1}(E) = \alpha_i^{-1}(M_U) - \frac{b_i}{2\pi} \ln \frac{E}{M_U}$$

$\left(\frac{41}{10}, \frac{-19}{6}, 7\right)$ standard model

$\left(\frac{33}{5}, 1, -3\right)$ N=1 SUSY

b_i depends on the particle content and jump when there is a mass threshold.

e.g. $E_- \xrightarrow{m_{th}} E_+$

$b^{(+)} - b^{(-)}$

$$\alpha_i^{-1}(E_-) = \alpha_i^{-1}(E_+) - \frac{b_i^{(-)}}{2\pi} \ln \frac{E_-}{E_+} - \frac{b_i^{(th)}}{2\pi} \ln \frac{m_{th}}{E_+}$$

This implies that the running depends on the mass spectrum

For α_3 ; for $E > m_t$ on has

$$\Lambda_{QCD} = E \left(\frac{m_c\, m_b\, m_t}{E}\right)^{2/27} e^{-2\pi / 9\alpha_3(E)}$$

Thus Λ_{QCD} shall also vary.

$m_i = h_i v$ so that variation of Yukawa & Λ_{QCD} & α_i is correlated.

we can deduce that

$$\frac{\Delta \Lambda_{QCD}}{\Lambda_{QCD}} = R \frac{\Delta \alpha}{\alpha} + \frac{2}{27}\left(3\frac{\Delta v}{v} + \frac{\Delta h_c}{h_c} + \frac{\Delta h_b}{h_b} + \frac{\Delta h_t}{h_t}\right)$$

$$R = \frac{\delta \ln \Lambda_{QCD}}{\delta \ln \alpha_{em}}$$

This number depends
- on GUT model
- on particle content

typically $\boxed{R = 30 - 50}$

In some models, one assumes a relation between v & h_t (dimensional transmutation models) as

$$v = M_p e^{-\frac{8\pi^2 c}{h_t}} \quad c \sim O(1)$$

so that weak scale is related to M_p.

one can also define

$$S = \frac{\delta \ln v}{\delta \ln h}$$

. Typically $\boxed{S \sim 160}$

The situation is thus as follow

$$\{\alpha_i, h, v, \Lambda_{QCD}\} \xrightarrow[(R,S)]{} \{\alpha ; v\}$$

model-dependent.

There is a large litterature on the computation of these numbers (see Refs in the review).

<u>What we shall recall:</u>

- in most GUT and realistic models, one cannot just set 1 constant to vary.
- the correlations are model-dependent.

We will come back on that later but it has important consequences:

- in the chiral limit $m_p = \Lambda_{QCD}\left(1 + \mathcal{O}\left(\frac{m_q}{\Lambda_{QCD}}\right)\right)$
- $m_e = h_e v$

$$\Rightarrow \quad \frac{\delta\mu}{\mu} = -0.8R\frac{\delta\alpha}{\alpha} + 0.6(S+1)\frac{\delta h}{h}$$

both $\frac{\delta\alpha}{\alpha}$ & $\frac{\delta\mu}{\mu}$ are in principle observable

$\Rightarrow$ ① cross-check if one is not varying

② allow to constrain R, i.e. window on Unif models.

Masses

The local tests of UFF involves macroscopic object

$\Rightarrow$ we need to compute $m(\alpha_A^Z)$ or m_p in function of the fundamental constants.

we start from

$$m(A,Z) = Z\, m_p + (A-Z)\, m_p + Z\, m_e + E_S + E_{EM}$$

2 difficulties : • $\frac{\delta m_p}{m_p}$; $\frac{\delta m_n}{m_n}$

• E_S

$\longrightarrow$ link between QCD and nuclear physics

there are a lot of phenomenological models

- chiral perturbation theory
- non-relativistic constituent quark model
- combinations.

For E_{EM}, we still use Bethe-Weizäcker formula

$$E_{EM} = 98.25 \frac{Z(Z-1)}{A^{1/3}} \alpha \text{ MeV}$$

Then

$$m_{(p,n)} = \underset{\text{pure cusler}}{\mu_3} + b_{(u,d)} m_u + b_{(d,u)} m_d + \underset{\text{binding energy.}}{B_{(p,n)}} \alpha$$

so

$$m_{A,Z} = (A\mu_3 + E_S) + (Z b_u + N b_d) m_u \quad (m_u = h_u v)$$
$$+ (Z b_d + N b_u) m_d \quad (m_d = h_d v) \quad (N = A - Z)$$
$$+ \left(Z B_p + N B_n + 98.25 \frac{Z(Z-1)}{A^{1/3}} \text{ MeV}\right) \alpha$$
$$+ Z m_e \quad (m_e = h_e v)$$

B_n and B_p can be computed (There is a full Phys Rep on that!)

$$\boxed{B_p \alpha = 0.63 \text{ MeV} \qquad B_n \alpha = -0.3 \text{ MeV}}$$

<u>Note</u> Even if one assumes only $B_F(\phi) F^2$ then it induces a coupling to p, n in the action and cosmologically to ρ_b. This cannot be avoided. [Many mistakes here!]

ex $$S = \int \Big[\frac{1}{2} M_p^2 R - \frac{1}{2}(\partial\phi)^2 + V(\phi) + \frac{1}{4} B_F(\phi) F^2 + \sum i \bar{\psi}_j \not{D} \psi_j - m_j \bar{\psi}_j \psi_j \Big] \sqrt{-g}\, d^4x$$

$$B_i(\phi) = e^{\xi_i \frac{\phi}{M_p}}$$

Because of the radiative coupling, it induces a coupling to nucleons:

$$\xi_N = m_N^{-1} \langle N | (\xi_F/4) F^2_{\mu\nu} | N \rangle \quad \boxed{\begin{array}{l} \xi_p = -0.0007\, \xi_F \\ \xi_n = 0.00015\, \xi_F \end{array}}$$

$\Rightarrow$ ① masses are ϕ dependent (Strong extract from VFF)

② dynamics is modified $\quad \Box\phi + m_\phi^2 \phi = B'_N(\phi)\, \rho_N$

We now need to determine E_S

- I will describe deuterium in BBN (easier because simpler nuclei).
- For larger nuclei, there is no consensus in the literature.

- liquide drop model

$$\frac{E_S}{A} = a_V - \frac{a_S}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A^2} + a_P \frac{(-1)^A + (-1)^Z}{A^{3/2}}$$

$$(a_V, a_S, a_A, a_P) = (15.7,\ 17.8,\ 23.7,\ 11.2)$$

- Damour-Donoghue (~ chiral approach)

$$\frac{E_S}{A} = a_3 + \frac{b_3}{A^{1/3}} \quad \text{with } a_3 = a_3^{\text{chiral lim}} + m_\pi^2 \frac{\partial a_3}{\partial m_\pi}$$

- fitting formula derived for effective field theories (semi-empirical)

$$\frac{E_S}{A} = -\left(120 - \frac{97}{A^{1/3}}\right)\eta_S + \left(67 - \frac{57}{A^{1/3}}\right)\eta_V$$

scalar (attractive) — vector (repulsive)

nuclear coload interaction

The order of magnitude are similar but there is a dispersion of the numbers due to the model-dependence and to the difficulty to relate QCD to nuclear physics.

<u>Two applications</u>

① for the universality of free fall, we define

$$\frac{\partial \ln h_i}{\partial \varphi} \times \underbrace{\frac{\partial f_m}{\partial h_i}}$$

↳ these terms can be estimated by the following developments.

assuming liq. drop, we have the values in the slide 7 for the standard expects.

② in the string dilaton model, the field ϕ can couple to the charges

$$\alpha, \quad B = N + Z, \quad D = N - Z \quad \& \quad E = \frac{Z(Z-1)}{A^{1/3}}$$

if all B have the same minimum then

$$\eta_{AB} \propto (\phi_0 - \phi_m)^2 \left[c_B\, \delta\left(\frac{B}{M}\right) + c_D\, \delta\left(\frac{D}{M}\right) + c_E\, \delta\left(\frac{E}{M}\right) \right]$$

But if they do not have the same minimum one expect large variation of η $\Rightarrow$ strong constraint on the stabilisation mechanism.

To finish, I want to highlight the links with dark energy and discuss and alternative model.

① Dynamical DE models involve a ϕ slow-rolling.
if it also sources var. of Fc then

$\dot{\alpha}$ is related to $\dot{\phi}$ and thus to $\phi(t)$

from $w_h = \frac{P}{\rho} \Rightarrow \dot{\phi} = H\sqrt{3\Omega_\phi(1+w_h)}$

. This implies that constraints on $\dot{\alpha}$ set constraints on (w_h, Ω_ϕ) even when $\rho_\phi < \rho_m$ i.e at larger redshift.

. Also, in all these model, (w_h, Ω_ϕ) gives α_0 and η_0 so that the local shall not be forgotten.

② In all the previous models, ϕ is light so that time variation are larger than space variations.

can we construct a model in which this is not the case

$\rightarrow$ Yes olive-peloso-Jpu.

This what I describe to finish this lecture on the theoretical $\boxed{\text{toolbox}}$.

The theory is very similar to those considered today

$$S = \int \left[\tfrac{1}{2} M_p^2 R - \tfrac{1}{2}(\partial_\mu \phi)^2 + V(\phi) + \tfrac{1}{4} B_F(\phi) F^2_{\mu\nu} - \Sigma i \bar{\psi}_j \not{D} \psi_j - B_j(\phi) m_j \bar{\psi}_j \psi_j \right] \sqrt{-g}\, d^4x.$$

$M_p^{-2} = 8\pi G$

and we choose the potential $V = \frac{1}{4}\lambda(\phi^2 - \eta^2)^2$

and the couplings $B_i(\phi) = e^{\xi_i \phi / M_*} \simeq 1 + \xi_i \frac{\phi}{M_*}$

Parameters of the model $(\lambda, M_*, \eta, \xi_F, \xi_i)$

Assume $\xi_i = 0$. Still there is a coupling to p and n

(you remember why?) $\Rightarrow \xi_n, \xi_p \neq 0$

The evolution of ϕ is dictated by the KG equation

$$\Box \phi + \frac{\partial V_{eff}}{\partial \phi} = 0$$

V_{eff} has 3 contributions

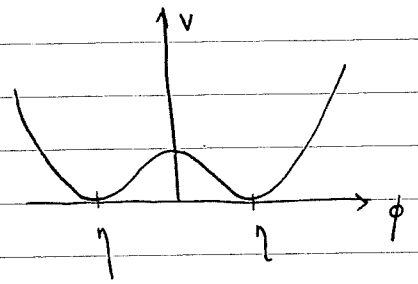
- coupling to F^2 and thus to ρ_b
- loop corrections that scale as $\xi_F^2 \phi^2 \frac{T^4}{M_*^2}$
- finite T corrections that scale as $\frac{\phi^2}{T^4} \frac{d^2V}{d\phi^2}$

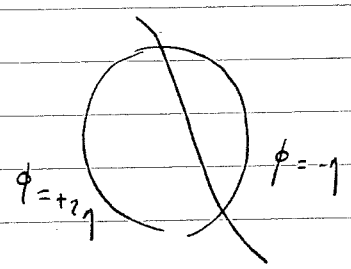
Note that there is no coupling to ρ_{rad} because $\langle F^2 \rangle = 0$

$$V_{eff} = V(\phi) + \xi_N \frac{\phi}{M_*} \rho_b + \xi_F^2 \frac{\phi^2}{M_*^2} T^4 + \frac{\lambda}{8} \phi^2 T^2$$

In order to have $\frac{\delta\alpha}{\alpha} \sim 10^{-6}$ we need

$$\frac{\delta\alpha}{\alpha} \simeq 2 \xi_F \frac{\eta}{M_*} \sim 10^{-6}$$





$\delta\alpha \propto \delta\phi \sim 2\eta$.

we assume $M_* \sim \eta$ & $\xi_F \sim 10^{-6}$

- $\Omega_{vacuum} \sim \frac{U_{vacuum} M_0}{\rho_0} \sim \left(\frac{\eta}{100\,\mathrm{MeV}}\right)^3 \Rightarrow$ assume $\eta = O(\mathrm{MeV})$

$$\Rightarrow \boxed{\begin{array}{l} \lambda \sim 1; \quad \eta \sim 1\,\mathrm{MeV}; \quad \eta \sim M_* \hat{\eta}; \quad \hat{\eta} \sim 1 \\ \xi_F \hat{\eta} \sim 10^{-6} \qquad \xi_N \sim -7 \cdot 10^{-4} \xi_F \end{array}}$$

such a theory would be a valid field theory upto $\frac{M_*}{\xi_F} \sim 10^6\,\mathrm{MeV}$

(this is the mass scale entering the only dimensionful para of the theory)

The phase transition takes place around $\eta \sim 1\,\mathrm{MeV}$. i.e. 6 order of magnitude lower.

- using the full potential one can just estimate T_c

one finds $\frac{T_c}{T_0} = 1 + z_f \sim 8.5 \cdot 10^9$

- CMB constraints : integrated SW

$$\frac{\delta T}{T} \sim G U_{rel} H_0^{-1}$$

$$\frac{\delta T}{T} \sim 10^{-6} \left(\frac{1}{1\,\mathrm{MeV}}\right)^3$$

- astrophysical constraints.

even though ϕ is heavy, $m_\phi \sim 1\,\mathrm{MeV}$ it can be produced in SN

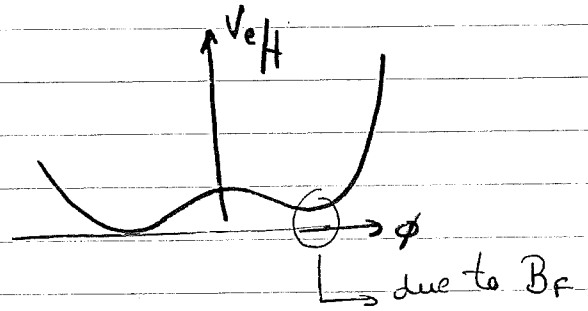
$$\Gamma_{\gamma\gamma \to \phi} \sim \frac{\xi_F^2}{M_*^2} T^3$$ this could result in an excessive energy loss

But ϕ decays in 2γ with $\Gamma \sim \xi_F^2 \frac{m^3}{M_*^2} \sim \xi_F^2 m_a$

we can show that if decay within the core exceeds

$$\frac{m_a}{1\,\mathrm{MeV}} \gtrsim \mathcal{O}(10^{-2}) \left(\frac{10^{-6}}{\xi_F}\right)$$

- <u>stability</u>



lifetime is $\gg$ age of universe.

This is a good example of all the cares one needs to take when playing this game.

SUMMARY:

- we have described a tool box to construct theories with dynamical couplings
- we have emphasized the importance of UFF
- we have described motivations from string theories
- we have some ideas about the correlations between constants in unification models.
- we have a model of pure spatial variation.

Now, we shall discuss all physical systems to put constraints on the variations of fc.