Are We Ready for Precision Cosmology?

General Relativistic Effects and Gauge-Invariant Formalism

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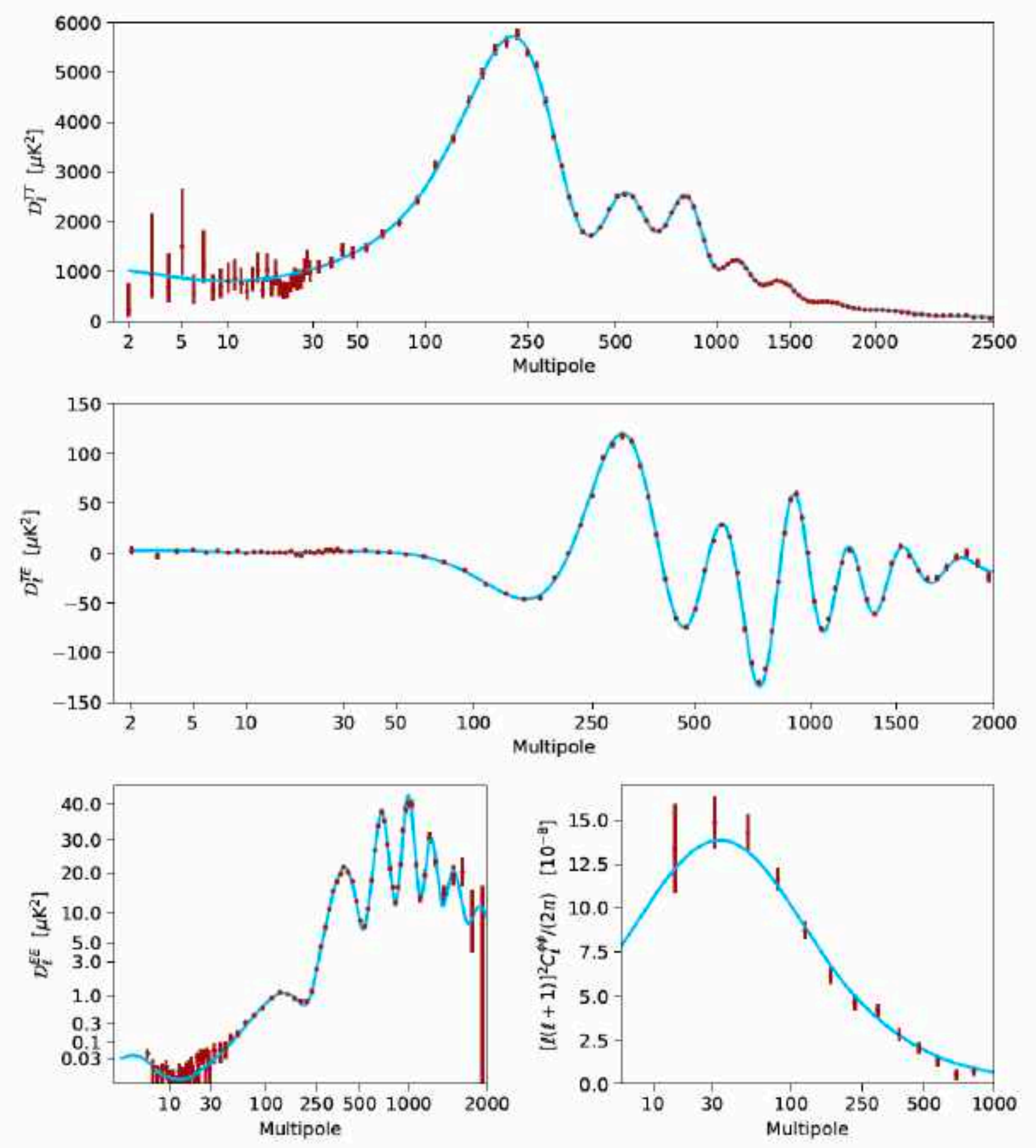
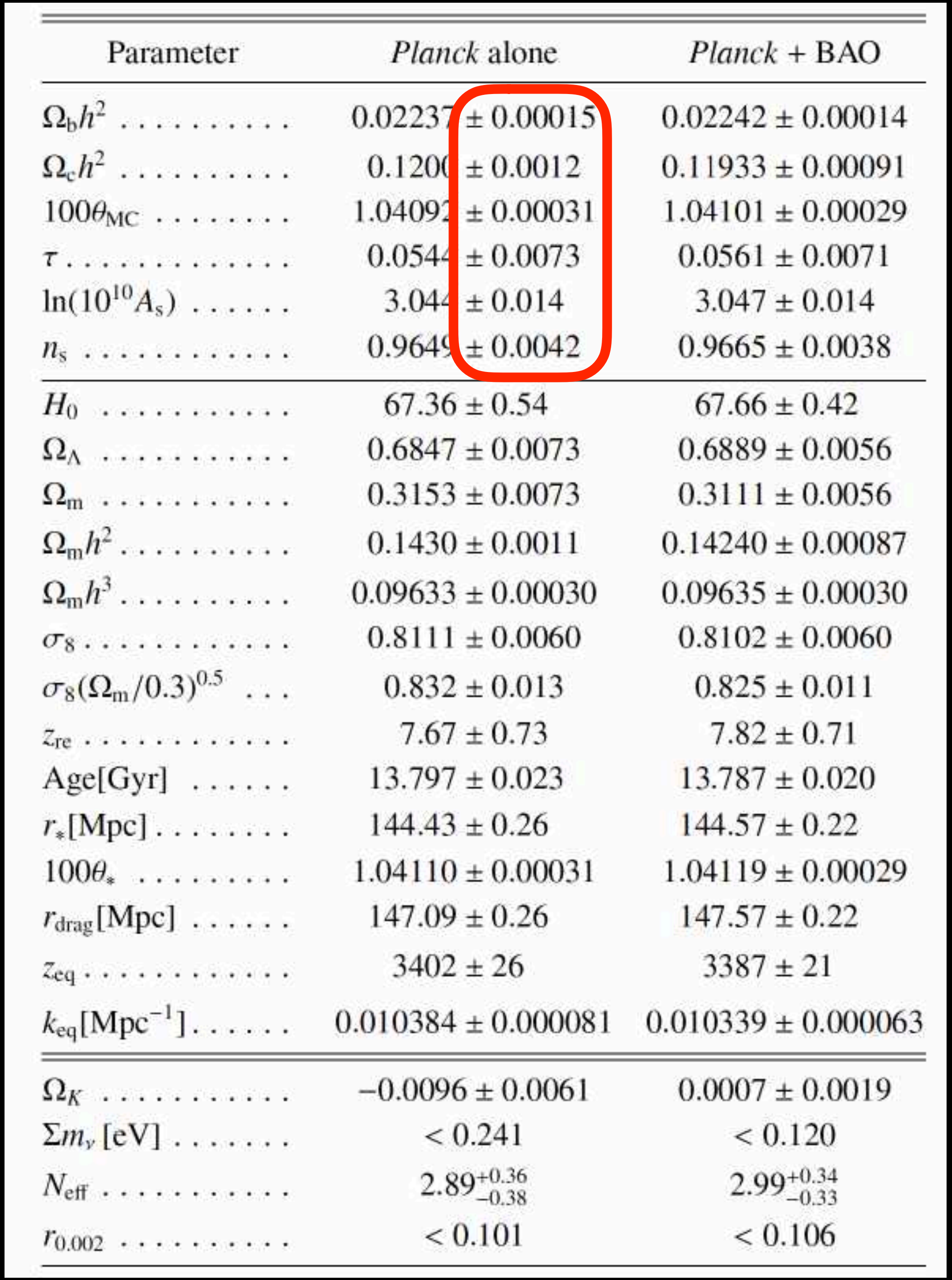
I. PRECISION COSMOLOGY:

Past, Today, and Future

I. PRECISION COSMOLOGY: PAST, TODAY, and FUTURE

- **Planck collaboration 2018:** *precision measurements!*





Parameter	*Planck* alone	*Planck* + BAO
$\Omega_b h^2$	0.02237 ± 0.00015	0.02242 ± 0.00014
$\Omega_c h^2$	0.1200 ± 0.0012	0.11933 ± 0.00091
$100\theta_{MC}$	1.04092 ± 0.00031	1.04101 ± 0.00029
τ	0.0544 ± 0.0073	0.0561 ± 0.0071
$\ln(10^{10} A_s)$	3.044 ± 0.014	3.047 ± 0.014
n_s	0.9649 ± 0.0042	0.9665 ± 0.0038
H_0	67.36 ± 0.54	67.66 ± 0.42
Ω_Λ	0.6847 ± 0.0073	0.6889 ± 0.0056
Ω_m	0.3153 ± 0.0073	0.3111 ± 0.0056
$\Omega_m h^2$	0.1430 ± 0.0011	0.14240 ± 0.00087
$\Omega_m h^3$	0.09633 ± 0.00030	0.09635 ± 0.00030
σ_8	0.8111 ± 0.0060	0.8102 ± 0.0060
$\sigma_8(\Omega_m/0.3)^{0.5}$ . . .	0.832 ± 0.013	0.825 ± 0.011
z_{re}	7.67 ± 0.73	7.82 ± 0.71
Age[Gyr]	13.797 ± 0.023	13.787 ± 0.020
r_*[Mpc]	144.43 ± 0.26	144.57 ± 0.22
$100\theta_*$	1.04110 ± 0.00031	1.04119 ± 0.00029
r_{drag}[Mpc]	147.09 ± 0.26	147.57 ± 0.22
z_{eq}	3402 ± 26	3387 ± 21
$k_{eq}[\mathrm{Mpc}^{-1}]$	0.010384 ± 0.000081	0.010339 ± 0.000063
Ω_K	-0.0096 ± 0.0061	0.0007 ± 0.0019
Σm_ν [eV]	< 0.241	< 0.120
N_{eff}	$2.89^{+0.36}_{-0.38}$	$2.99^{+0.34}_{-0.33}$
$r_{0.002}$	< 0.101	< 0.106

Standard Model of Cosmology

- **I. *inflationary epoch* in early Universe**
  - **seed fluctuations for formation of galaxies & life**

- **II. matter & energy content of Universe today**
  - **exotic particles: *dark matter* (22%)**
  - **repulsive gravity: *dark energy* (74%)**
  - **ordinary matter: only 4%!**

- **III. *general relativity*: Einstein**
  - **describe evolution of matter & energy**

Problems in Cosmology

- **I. inflationary epoch: *Not Understood***
  - **what generates initial perturbations?**

- **II. dark sector (96%): *Not Understood***
  - **dark matter (*22%*) and dark energy (*74%*)**
  - **what are the nature of dark sector?**
  - **ordinary matter (*4%*): *Understood, check!***

- **III. Einstein's general relativity: *Not Sure***
  - ***valid* on cosmological scales*?* modified gravity*?***
  - ***well tested* in Solar System**

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Problems in Cosmology

It is the *most compelling* of
all outstanding problems in physical science!
Dark Energy Task Force 2006

The Birth of the Universe: *one of the big issues for 21st century particle physicists*
Quantum Universe Report 2010

Large-Scale Surveys

- current and future ground-based surveys:
  - Baryonic Oscillation Spectroscopic Survey
  - Dark Energy Survey
  - Dark Energy Spectroscopic Instrument
  - Large Synoptic Survey Telescope

- future space missions:
  - Euclid
  - Wide-Field Infrared Survey Telescope

- sub-percent level

  precision measurements!

More Ambitious Surveys

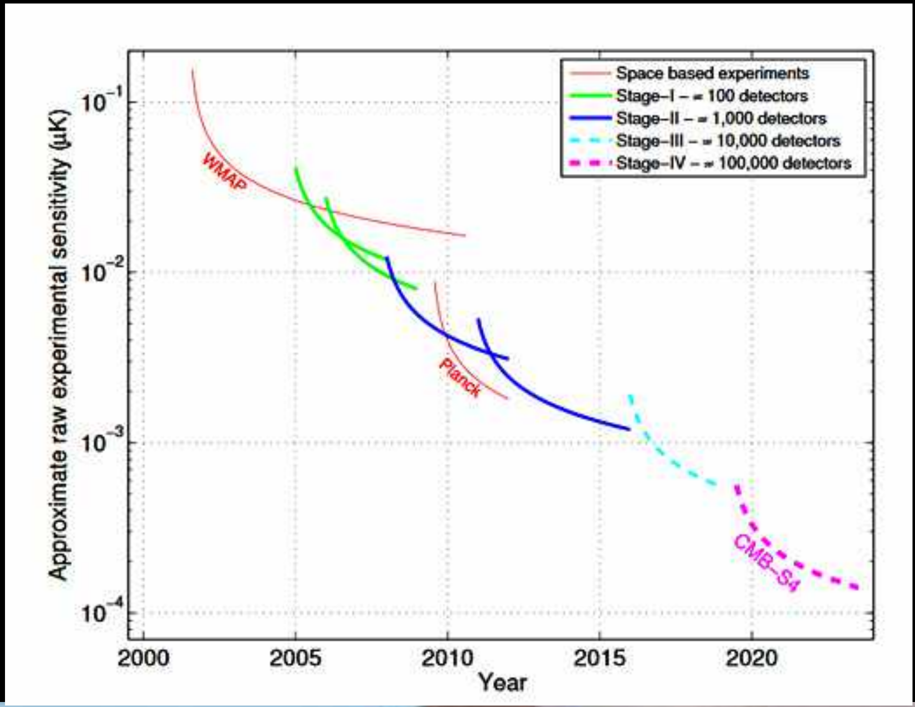
- **future radio surveys:**
  - **Murchison Wide-field Array Phase-II**
  - **Square Kilometer Array**

- **redshifted 21cm lines:**
  - **from hyperfine transition in neutral hydrogen**
  - **probe redshift 10 ~ 30**
  - ***more statistical power than CMB***

Even more *precise* measurements!

CMB Stage IV (S4)

- **next-generation CMB experiment:**
  - **dedicated telescopes**
  - **South Pole & Chile Atacama**
  - **and more telescopes?**
  - **inflation** $r < 0.002$
  - **neutrino mass** $\sum m_\nu$
  - **relativistic species** $\sigma(N_{\rm eff}) = 0.02$



South Pole station

Chile Atacama plateau

Challenges

- **precision measurements demand:**
  - ***substantial advances*** **in theoretical modeling of cosmological observables**

- **standard theoretical descriptions:**
  - **sufficiently accurate to describe precision measurements*?*** **answer: *No!***
  - **galaxy clustering, weak lensing, Boltzmann eq. etc**
  - **incomplete and limited to linear theory due to *gauge dependence* & *missing observer specification***

Research Program

- re-write theoretical descriptions of all cosmological observables:
    - in proper *relativistic framework*
    - check *gauge-invariance*
    - work out *impact of missing physics* on observables
    - relativistic effects as *novel probes* of cosmology
    - work in progress!

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warning:

They work well!

BUT not quite so at the percent level or better

Team at Zürich

- **postdoctoral fellows:**
    - Yves Dirian
    - Ermis Mitsou
    - Enea DiDio

- **PhD students:**
    - Nastassia Grimm
    - Sandra Baumgartner

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II. WHAT IS WRONG IN STD COSMOLOGY?

General Relativistic Effects & Gauge Invariance

(a) Relativistic Effects

- all cosmological observations by measuring photons:
  - well known, but often *ignored!*
  - null geodesic for light path (vs instantaneous prop.)
  - light cone observation (vs same time volume)

- missing relativistic effects:
  - gravitational redshift, lensing, frame distortion, etc
  - photon propagation over cosmological scales
  - explicit accounts of all *needed*

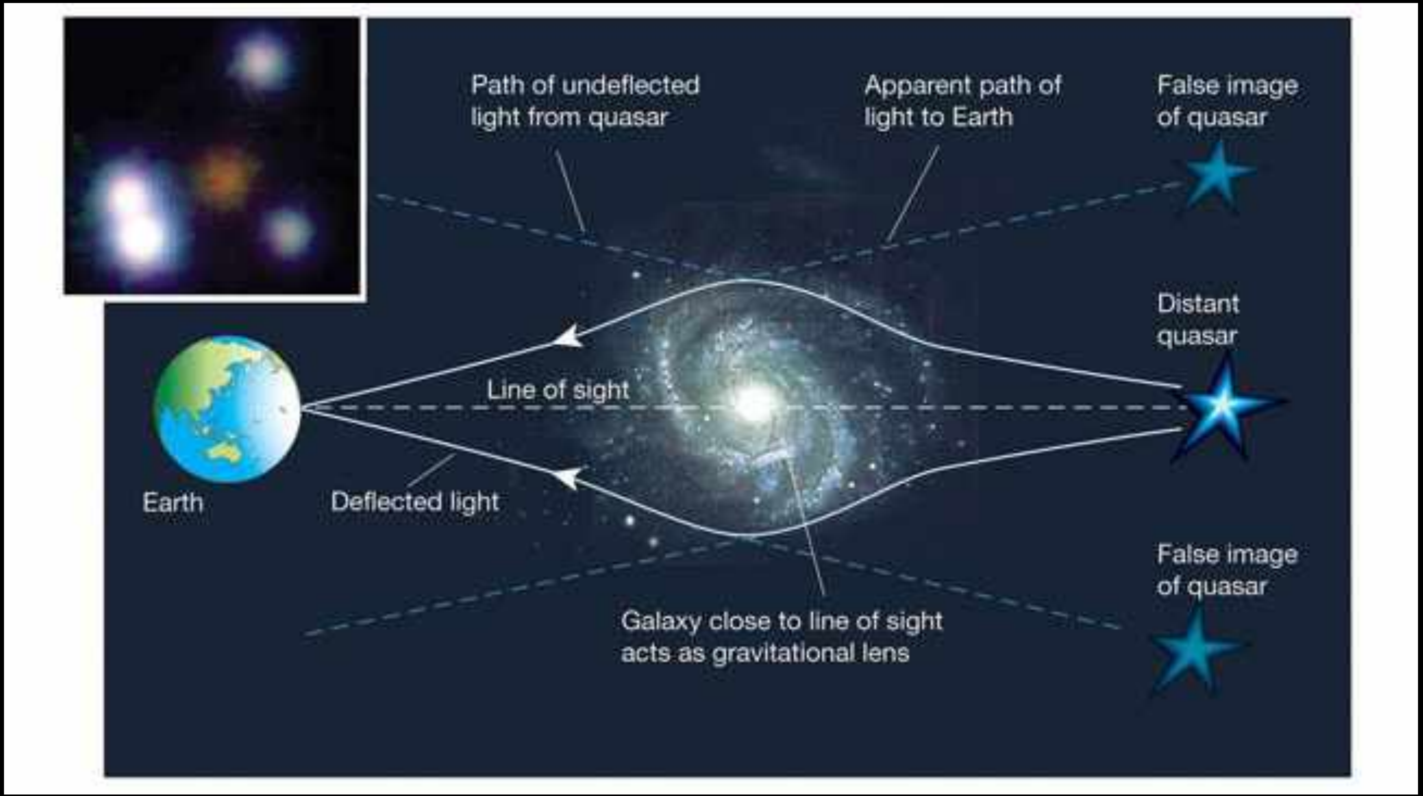
Gravitational Redshift

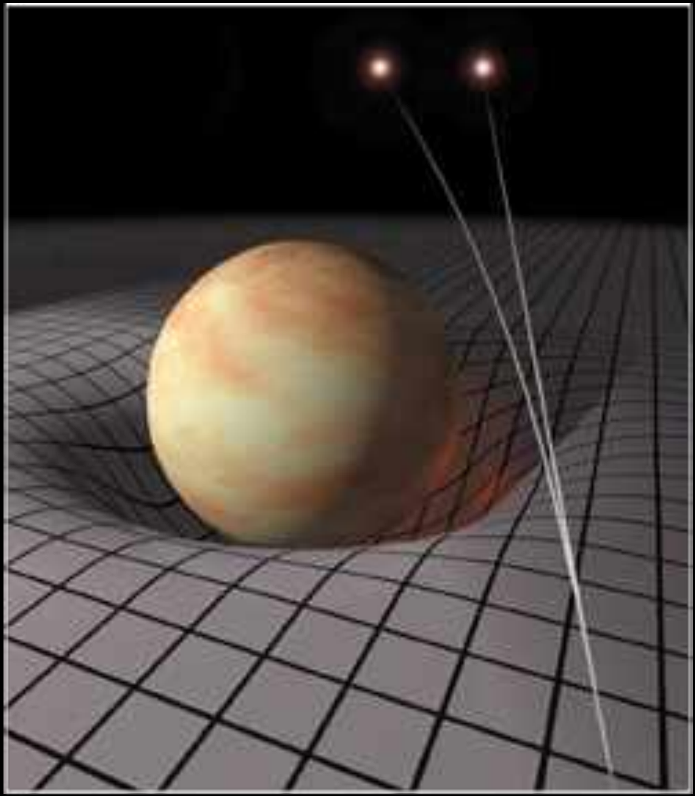
- **photon energy is affected**
- **due to gravity at source and observer (*Sachs-Wolfe*)**
- **also change in gravity during propagation (*iSW*)**

$$1 + z_{\rm obs} = (1+z)\left[1 + V(z) - V(0) - \psi(z) + \psi(0) - \int_0^r dr'\ (\dot{\psi} - \dot{\phi})\right] .$$

Gravitational Lensing

- *observed* angular position is *not* real position
- matter distribution *deflect* light propagation





$$\hat{n}_{\rm obs} = (\theta_{\rm obs},\ \phi_{\rm obs}) = \hat{n}_{\rm true} + \delta n, \qquad \delta n = (\delta\theta,\ \delta\phi)$$

$$\delta_g \propto \kappa$$

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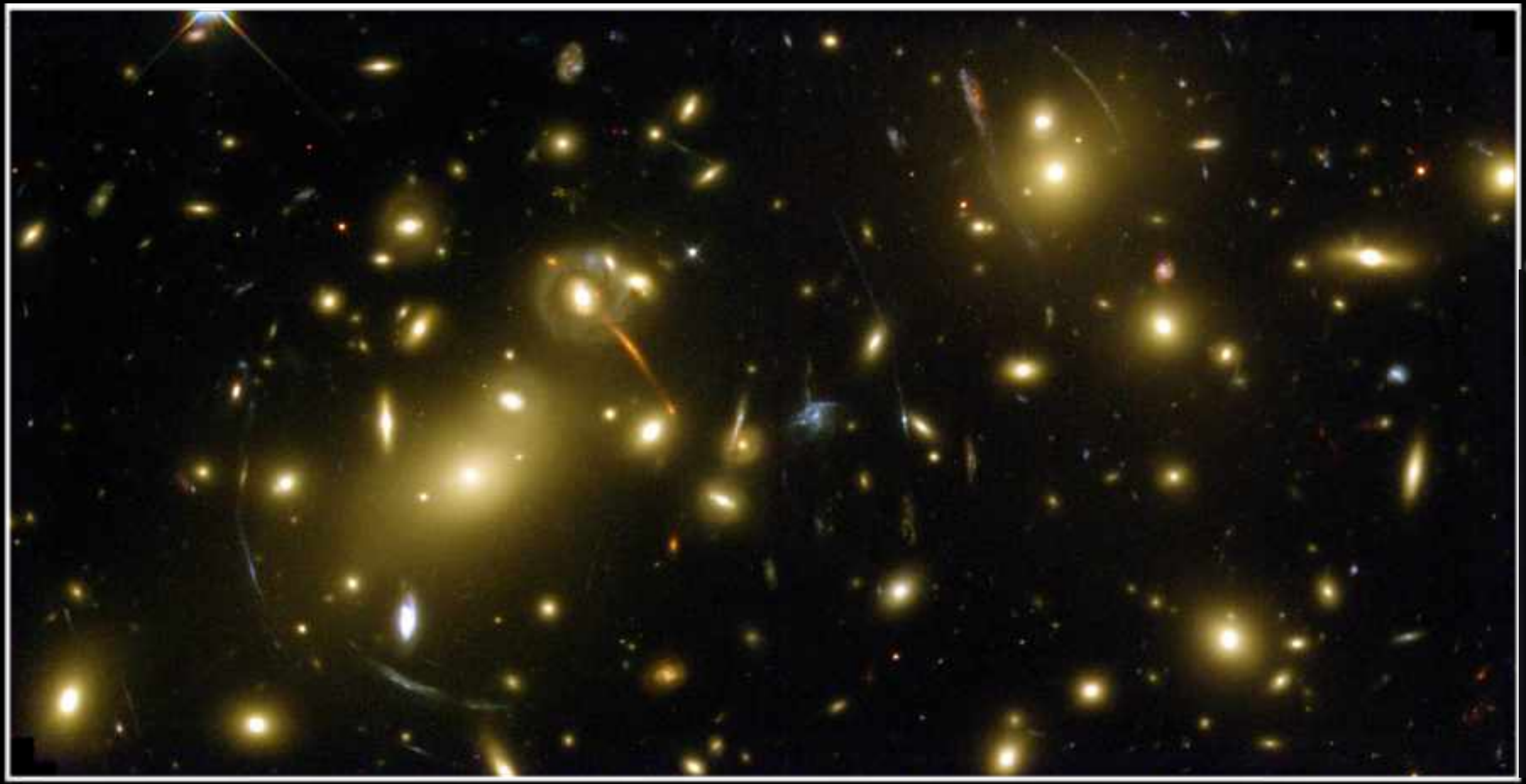

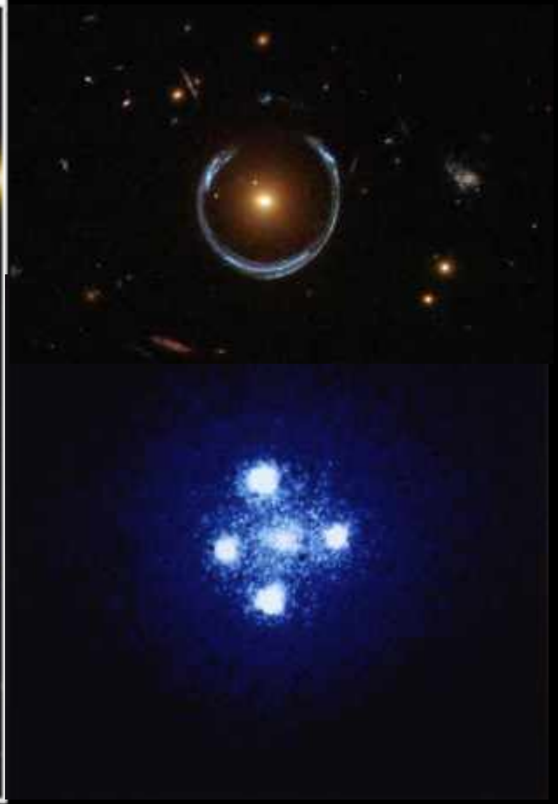

(b) Who Measures What?

- cosmological observables:
  - **photons**: frequency, polarization, flux, position
  - **derivables**: redshift, shape, luminosity, number density, lensing shear, etc.

- *observers* (*us*) in rest frame (Minkowski):
  - observer dependent, but so trivial, often ignored!
  - (FRW) **coordinate independent**! (diffeo. invariant)
  - *scalar* under diffeomorphism: FRW vectors, tensors, *all against our local basis*
  - same for physical quantities in the *source* rest frame

Symmetries

- **general relativity (in cosmology):**
  - **diffeomorphism symmetry: any coordinates work**
  - **FRW metric with any gauge choice**
  - **cosmological observables: *gauge invariant***

- **coordinates in observer rest frame (Minkowski):**
  - **(local) Lorentz symmetry (indep. of FRW coordinates)**
  - **boost is fixed, only rotational freedom**
  - **cosmological observables: *not invariant under Lorentz***
  - **direct connection to *QFT* calculations**

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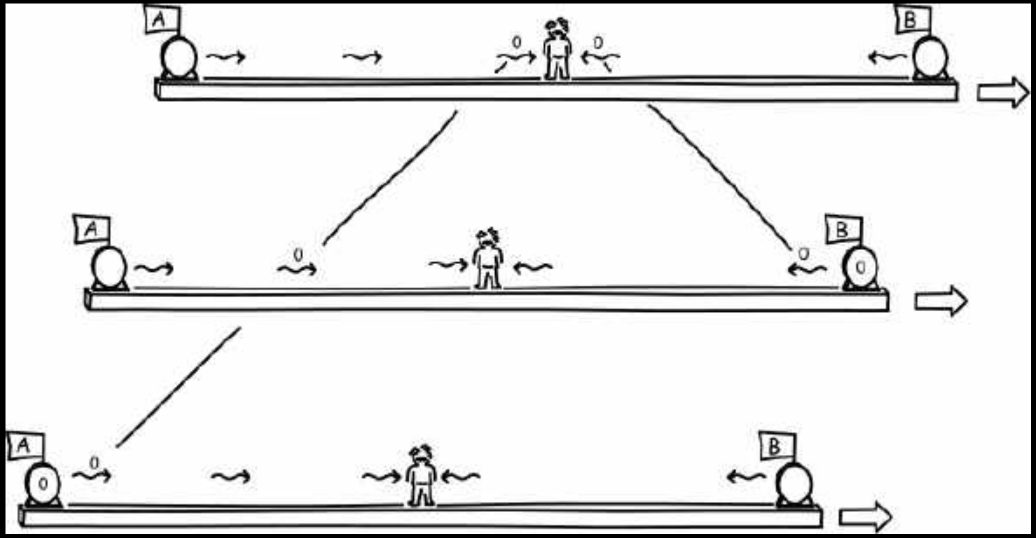
  - direct connection to *QFT* calculations

→ *tetrad* formalism (local internal gauge symmetry)

Simultaneity

- **no absolute simultaneity**
- **any choice of hypersurface is ok (*gauge freedom*)**
- **perturbations depend on *choice of hypersurface (gauge)***



simultaneity is relative!

hypersurface

Gauge Issues

- **theoretical predictions in cosmology**
  - **compute perturbations such as** $\delta_m, \ \psi, \ P_m(k), \ \cdots$
  - **compare to observations such as** $\delta_m^{\rm obs}, \ P_m^{\rm obs}(k), \ \cdots$
  - **perturbations are *gauge-dependent***
  - ***cannot* be directly associated with *observables!***

- **observable quantities:**
  - **gauge-invariance is a *necessary* condition, but *not a sufficient* condition**
  - **explicit check is *needed***

(c) Gauge-Invariance

- **cosmological observables:**
  - **should be gauge-invariant, but std description: *no!***

- **standard theoretical descriptions:**
  - **chose one gauge, e.g., conformal Newtonian gauge**
  - **complete gauge fixing: gauge-invariant, *not enough!***
  - ***gauge-dependent*: in general representations**

- **lessons learned:**
  - **gauge fixing: easier, but lose ability to verify**
  - **explicit check of gauge transformations**

Take-Home Message

- standard descriptions: incomplete
    - gauge dependent: *different values* in different gauges
    - *no* specification of observer & source
    - *nor* frames in which physical events take place

- limited to linear order:
    - rely on background FRW metric
    - similar to Minkowski metric in observer rest frame
    - *only* at 1st order, not generally valid

need to *re-write* cosmology: what we do in Zürich!

III. HOW CAN WE FIX STD COSMOLOGY?

Gauge-Invariant Formalism

Observables

- use observables, not unobservables!
- unobservables: (*gauge-dependent*)
  - most quantities in theoretical descriptions

$$\bar{z},\ \hat{s},\ \delta_m,\ \psi,\ P_m(k)\ ,\cdots$$

- observables: (*physical*)
  - photon wave vector in observer rest frame

$$k^a=\omega\left(1,-n^i\right)\ ,\qquad n^i=(\theta,\phi)\ ,\qquad \omega=2\pi\nu\ ,$$

  - *ind. FRW*, gauge-invariant, subject to *Lorentz trans.*
  - same for other observables & derivables
  - e.g., redshift, angular size, $1+z=\frac{\nu_{\rm src}}{\nu}\ ,\qquad (d\theta,d\phi)\ ,\cdots$

FRW Coordinates

- **going to coordinates: tetrad vectors at observer** $e^\mu_a(x^\nu_o)$
- **photon wave vector in FRW:**
  - ***different* from what we measure in rest frame!**

$$k^\mu = e^\mu_a k^a = \frac{\omega}{a}\left[1-\mathcal{A}-n^i\delta^\beta_i\left(\mathcal{U}_\beta-\mathcal{B}_\beta\right), -n^i\delta^\alpha_i+\mathcal{U}^\alpha+n^i\delta^\beta_i\left(\varphi\;\delta^\alpha_\beta+\mathcal{G}^\alpha{}_{,\beta}+C^\alpha_\beta\right)+\epsilon^\alpha{}_{ij}n^i\Omega^j\right]$$

  - **Doppler effect, gravitational redshift, distortion, etc.**
  - **same for other observables & derivables**

Yoo, Grimm, Mitsou, Amara, Refregier 2018 JCAP 04, 029

- **transport them to source position:**
  - **geodesic equation but with *boundary condition*!**
  - **geodesic deviation equation for shapes**
  - **check gauge dependence (yes, specific way)**

Physical Quantities at SRC

- going to *source rest frame*: **tetrad vectors** at src $e_a^\mu(x_s^\nu)$
  - **same effects** at src: Doppler effect, gravitational redshift, frame distortion, misalignment, etc.
  - src has **different** velocity, potential, etc

- **relation** bw observables & physical quantities:
  - should be **gauge invariant**, explicit check **needed**
  - e.g. src physical length **vs** observed angular size
  - emitted frequency **vs** measured frequency (redshift)
  - intrinsic luminosity **vs** apparent flux, etc.
  - also subject to LLT at observer & source

Yoo, Grimm, Mitsou, Amara, Refregier 2018 JCAP 04, 029

Boltzmann EQ & QFT

- evolution equations along trajectory:
  - LHS: propagation of any fluids in spacetime manifold
  - RHS: particle interactions in rest frame
  - S-matrix invariant under LLT, not under diffeo.

- problems in standard approach:
  - parametrize geodesic & momentum $x^\mu(\lambda) \ , \quad p^\mu = \dfrac{dx^\mu}{d\lambda}$
  - coordinate transformation

$$d\tilde{x}^\mu = \frac{\partial \tilde{x}^\mu}{\partial x^\nu}\, dx^\nu \ , \quad \tilde{p}^\mu = \frac{\partial \tilde{x}^\mu}{\partial x^\nu}\, p^\nu \qquad |\mathcal{M}(p_i^\mu)|^2 \neq |\tilde{\mathcal{M}}(\tilde{p}_i^\mu)|^2$$

  - microscopic physics in rest frame

$$p^a = e^a_\mu p^\mu \ , \quad \tilde{p}^a = \Lambda^a{}_b p^b \qquad |\mathcal{M}(p_i^a)|^2 = |\tilde{\mathcal{M}}(\tilde{p}_i^a)|^2$$

Mitsou & Yoo, arXiv:1908.10757

Tetrad Formalism

- **tetrad fields: e_a^μ**
  - **metric is fully contained in tetrad** $g_{\mu\nu} = \eta_{ab} e_\mu^a e_\nu^b$
  - **transparency: diffeo. & LLT (internal gauge symm.)**
  - **spinors: a representation of SL(2,C), not in diffeo.**
  - **already well developed in general relativity**

- **application to cosmology: natural generalization**
  - **not only at observer or src, but fields (*everywhere!*)**
  - **observer family: all possible observers everywhere**
  - **when projected, "*observables*" & *gauge invariant***
  - **natural connection to QFT in Minkowski spacetime**

Mitsou & Yoo, arXiv:1908.10757

IV. DO WE HAVE TO CARE?

Impacts of the Relativistic Effects

(a) Luminosity Distance

- **standard candle:**
  - **intrinsic luminosity: L_{SN} known**
  - **observables: flux, redshift, position** $f_{\rm obs}\,,\; z_{\rm obs}\,,\; n_{\rm obs}$
  - **luminosity distance:** $\mathcal{D}_L = \left(\frac{L_{\rm SN}}{4\pi f_{\rm obs}}\right)^{1/2}$

- **inhomogeneities:**
  - **all observables are affected**
  - **perturbations:** $\mathcal{D}_L = \bar{D}_L(z_{\rm obs})(1+\delta\mathcal{D})$
  - **LHS: observable, gauge-invariant**
  - **RHS: should be gauge-invariant**

Fluctuations in Luminosity Distance

- **linear-order calculations:**
  - **with respect to observed redshift:** $z_{\rm obs}$
  - **luminosity distance:** $\mathcal{D}_L = \bar{D}_L(z_{\rm obs})(1+\delta\mathcal{D})$

    perturbations: ***gauge invariant*** $\delta\mathcal{D} = \delta z + \frac{\delta r}{\bar{r}} - \kappa + \Xi$
  - **individual terms: gauge dependent**

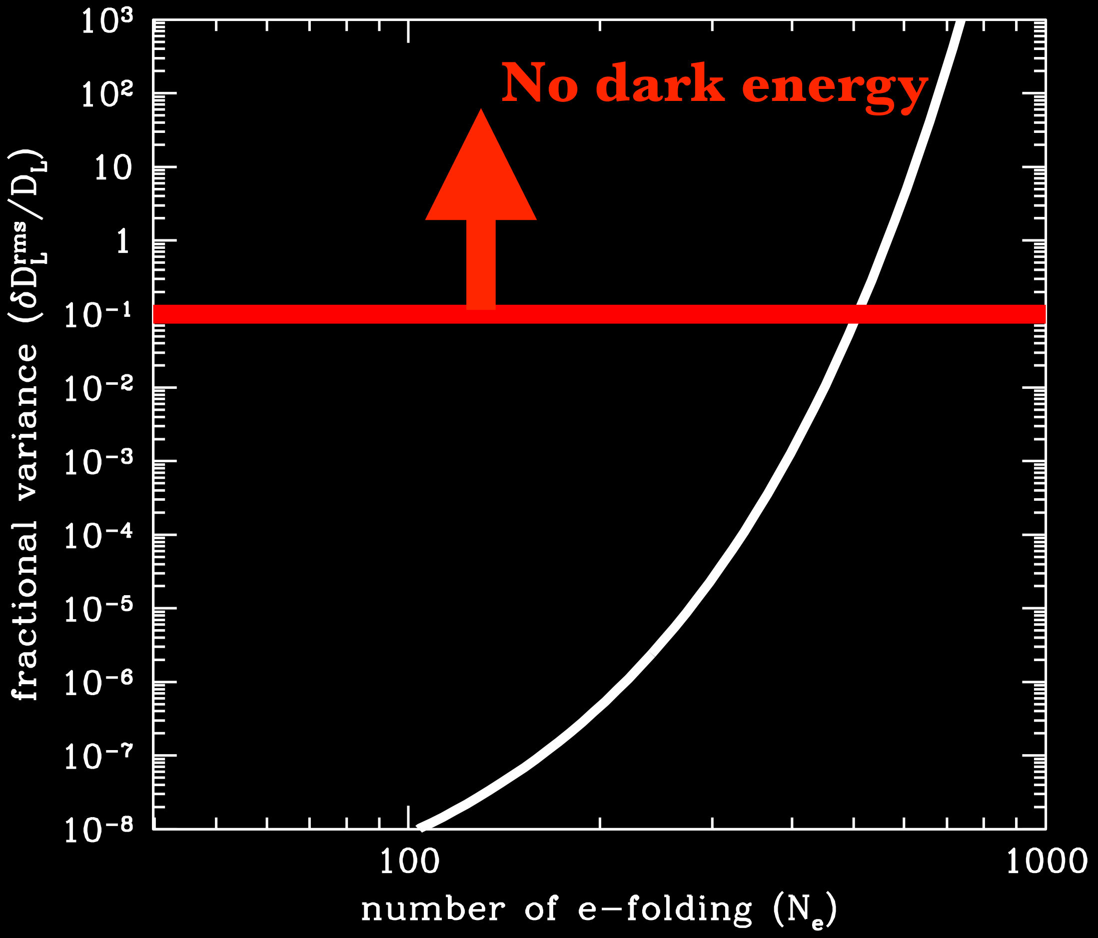
Sasaki 1987, Bonvin, Durrer, Kunz 2006, Schmidt & Jeong 2014, Biern & Yoo 2017

- **physical interpretation:**
  - **distortion in redshift:** δz $\qquad 1+z_{\rm obs} = \frac{1+\delta z}{a(\eta)}$
  - **radial & angular distortions of src position:** δr $\quad \kappa$
  - **distortion in local frame:** $\Xi = \frac{1}{2}\left(C^i_i - C_{ij}n^in^j\right)$

Infrared Divergences

- standard calculations:
    - order unity variance with $N\sim 500$ e-folding
    - *no need* for dark energy:
    - no upper limit on N
- many groups:
    - found same results
    - impose ad hoc IR cutoff $k_{IR}\sim H_0$ is imposed
    - conformal Newtonian gauge

Barausse et al. 2005, Kolb et al. 2005



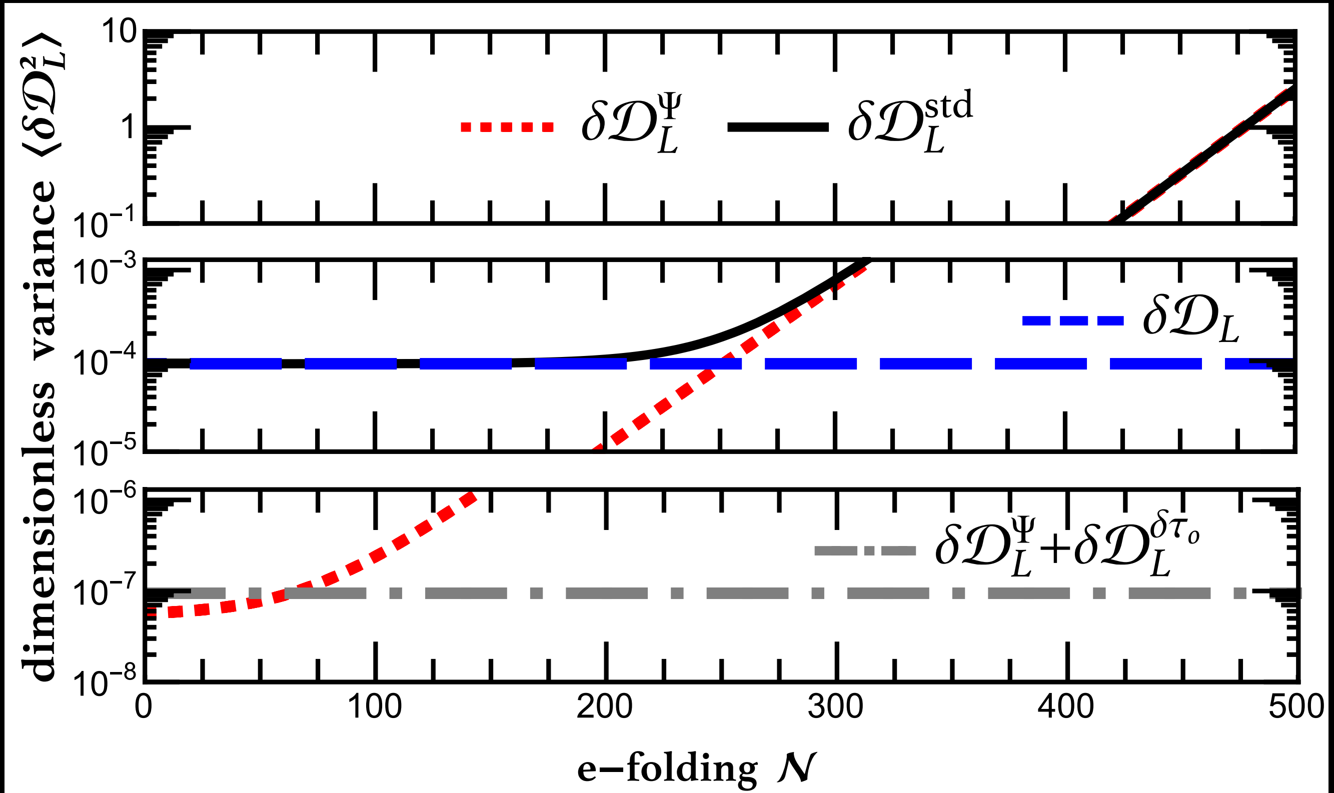
Impact on Luminosity Distance

- **linear-order expressions:**

$$\mathcal{D}_L = \bar{D}_L(z_{\rm obs})(1+\delta\mathcal{D}) \qquad \delta\mathcal{D} = \delta z + \frac{\delta r}{\bar{r}} - \kappa + \Xi$$

  - **black**: standard calculation (IR divergence)
  - **blue**: correct gauge-invariant calculation
- **grey**: missing component
- **cancellation**: potential terms balanced

Biern & Yoo 2017 JCAP



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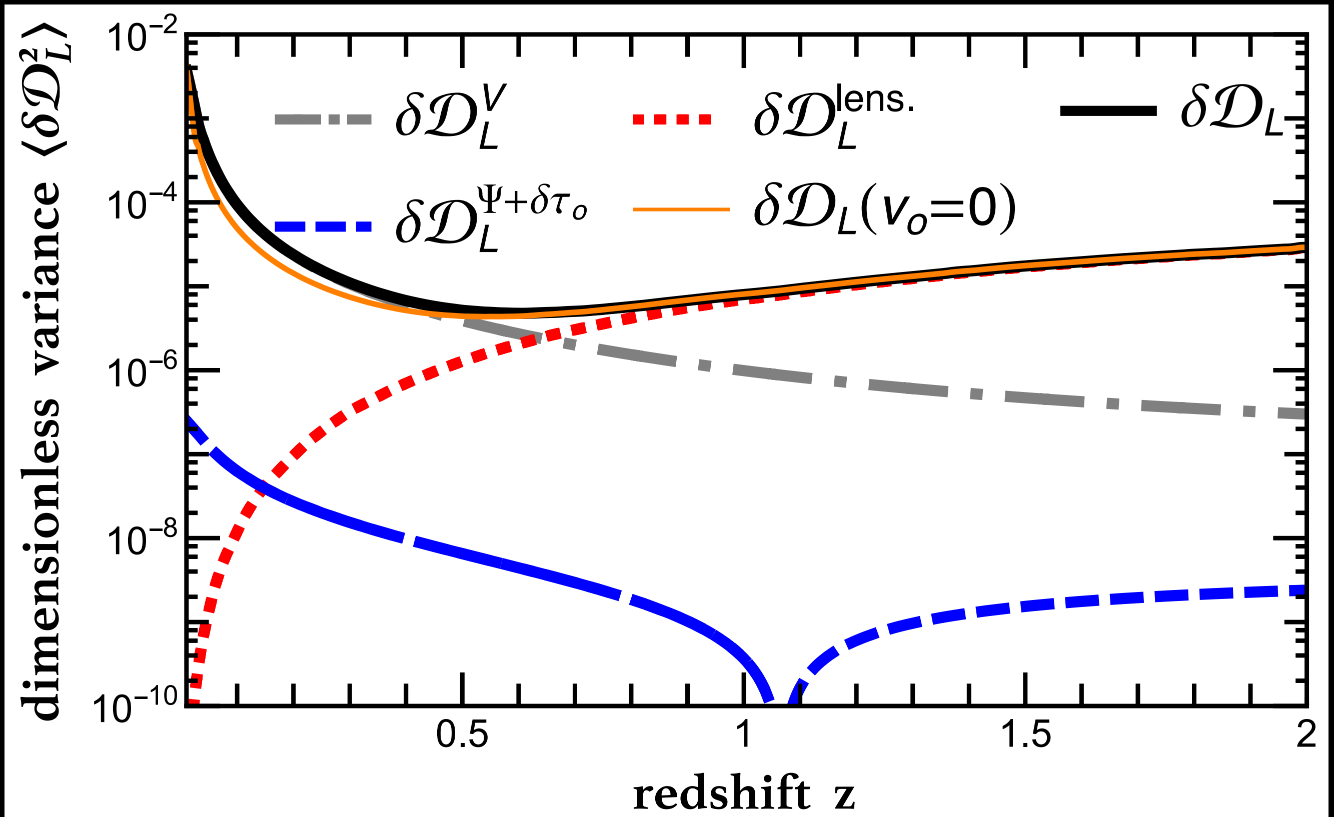


(a) Luminosity Distance

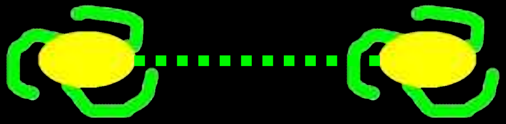
- **summary of new findings:**
  - ***no IR-divergences*** **in variance: we need dark energy!**
  - ***no ad hoc IR cutoff*** **is needed**
  - **definitive & explicit end to controversy** **Biern & Yoo 2017**

- **lessons learned:**
  - **use** ***correct gauge-invariant*** **expression**
  - **make sure to explicitly check** ***gauge-invariance***
  - **shift in mean LD from background (2nd order; in progress)**
  - **suspect more missing terms in other calculations**

(see Yoo & Scaccabarozzi 2016: compare 4 methods for computing luminosity distance)

(b) Galaxy Clustering

- measure of how galaxies are distributed:

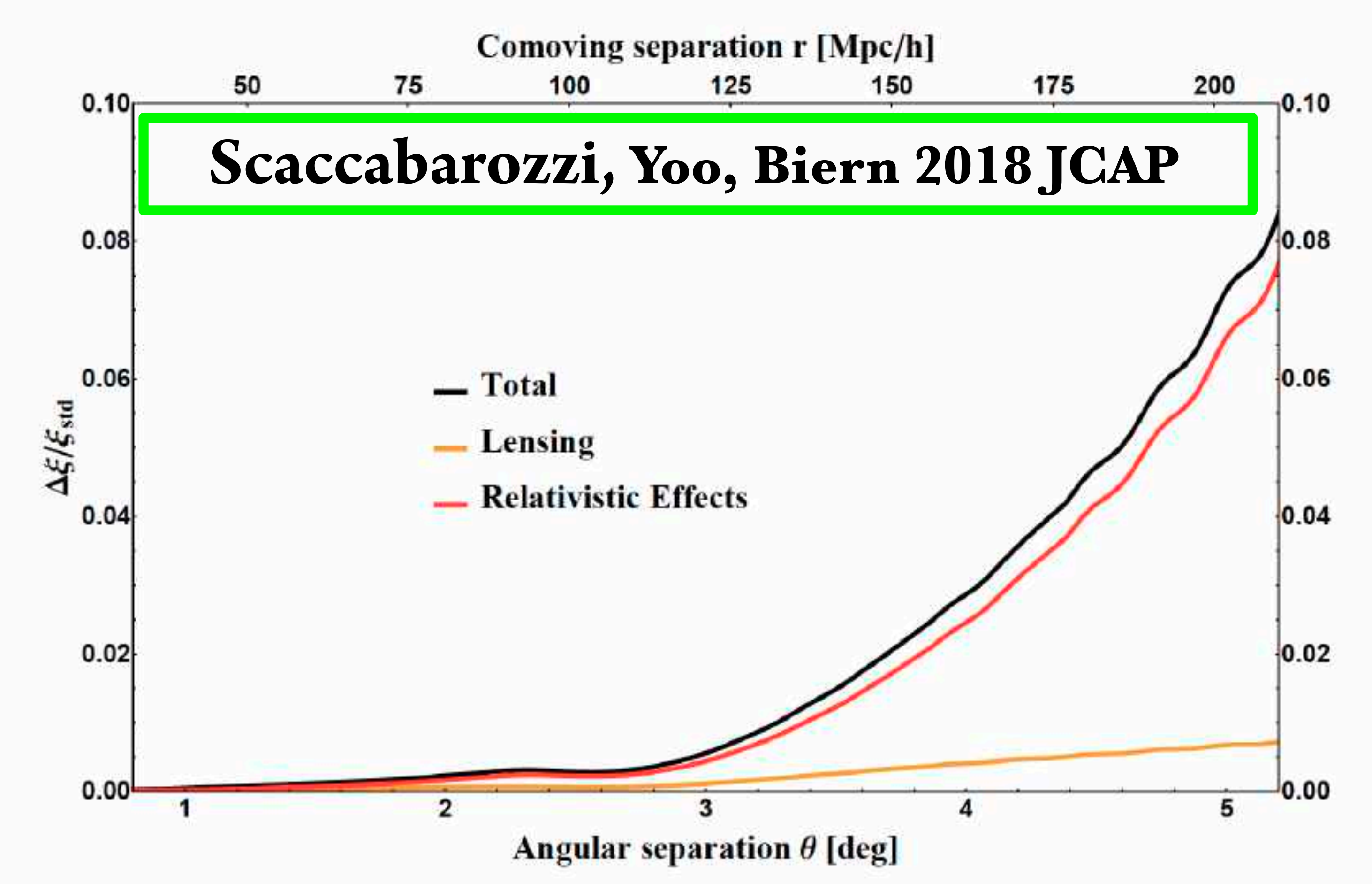


- construct fluctuation in galaxy counts:
  - total number of observed galaxies $dN_{\rm tot}$
  - observed volume $dV_{\rm obs}$ given $(z_{\rm obs}, \theta_{\rm obs}, \phi_{\rm obs})$
  - fluctuation field $\delta_g^{\rm obs} = \frac{n_g^{\rm obs}}{\langle n_g^{\rm obs} \rangle} - 1$
- relation to physical number density:
  - number conservation $dN_{\rm tot} = n_g^{\rm phy} dV_{\rm phy} = n_g^{\rm obs} dV_{\rm obs}$
  - observed number density $n_g^{\rm obs} = n_g^{\rm phy} \frac{dV_{\rm phy}}{dV_{\rm obs}}$

$$z_{\rm obs} \neq z \ , \quad f_{\rm obs} \neq f_{\rm phy} \ , \quad dV_{\rm obs} \neq dV_{\rm phy}$$

Correlation Function

- relativistic effects:
  - beyond BAO: a few percent level corrections
  - lensing and velocity contributions



(b) Galaxy Clustering

- gauge-invariant description:
  - several velocity contributions (missing)
  - relativistic effects: a few percent beyond BAO
  - gravity waves: very small contribution

- work in progress:
  - second-order calculations
  - power spectrum & bispectrum on horizon scales
  - primordial non-Gaussianity vs (late-time) relativistic effects

(c) Weak Gravitational Lensing

- **limitations in standard weak lensing:**
  - **intrinsically relativistic, but incomplete**
  - **extension beyond linear order: difficult**

- **problems in standard weak lensing:**
  - **true source angular position: *un-observable***

$$\hat{s}_{\rm true} = \hat{n}_{\rm obs} + \delta n\,, \qquad \hat{n}_{\rm obs} = (\theta,~\phi)_{\rm obs}\,, \qquad \delta n = (\delta\theta,~\delta\phi)$$

  - ***un-observable* distortion matrix: angular size** $(d\theta, d\phi)$

$$\begin{pmatrix} ds_\theta \\ \sin\theta~ds_\phi \end{pmatrix} \equiv \begin{pmatrix} \mathbb{D}_{11} & \mathbb{D}_{12} \\ \mathbb{D}_{21} & \mathbb{D}_{22} \end{pmatrix} \begin{pmatrix} d\theta \\ \sin\theta d\phi \end{pmatrix}, \quad \mathbb{D} \equiv \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2-\omega \\ -\gamma_2+\omega & 1-\kappa+\gamma_1 \end{pmatrix}$$

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observables to measure

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gauge-dependent **observables to measure**

Gauge-Invariant Formalism

- **relation to physical length & shape at src:**
  - **trace back size to src (geodesic deviation)**
  - **source position: *gauge-dependent* (still!)**
  - **rest frame: physical length** $(dL_{d\theta},\ dL_{d\phi})$ **(*coord. ind.*)**
  - ***physical* distortion matrix**

$$\begin{pmatrix} dL_{d\theta} \\ dL_{d\phi} \end{pmatrix} \equiv \bar{D}_A \begin{pmatrix} \hat{\mathbb{D}}_{11} & \hat{\mathbb{D}}_{12} \\ \hat{\mathbb{D}}_{21} & \hat{\mathbb{D}}_{22} \end{pmatrix} \begin{pmatrix} d\theta \\ \sin\theta\ d\phi \end{pmatrix} ,$$

  - **all lensing observables: *gauge-invariant***

Yoo, Grimm, Mitsou, Amara, Refregier 2018 JCAP and Grimm & Yoo 2018 JCAP

see also, Bonvin 2008 PRD, Bernardeau, Bonvin, van de Rijt, Vernizzi 2012 PRD

Lensing Convergence

- **standard formalism: (but w/ proper relativistic effects)**

$$-2\kappa = (2V_\| - 3C_\|)_o + \int_0^{\bar r_z} \frac{d\bar r}{\bar r}\left(2n_\alpha - \hat\nabla_\alpha\right) 2C^\alpha_\beta n^\beta - \frac{2n_\alpha}{\bar r_z}\left(\mathcal{G}^\alpha + \delta x^\alpha\right)_o + \frac{2n_\alpha \mathcal{G}^\alpha}{\bar r_z}$$

$$-\int_0^{\bar r_z} d\bar r \left(\frac{\bar r_z - \bar r}{\bar r_z \bar r}\right)\hat\nabla^2\left(\alpha_\chi - \varphi_\chi - C_\|\right) - \frac{1}{\bar r_z}\hat\nabla_\alpha \mathcal{G}^\alpha,$$

  - **gauge dependent due to** $\mathcal{G}^\alpha \to \mathcal{G}^\alpha - \mathcal{L}^\alpha$
  - **standard** κ **:** ***un-observable***
  - ***real lensing observable*:** $\delta\mathcal{D}$

- **gauge-invariant formalism: real lensing convergence**
  - ***angular diameter fluctuation*** $\delta\mathcal{D} = \delta z + \frac{\delta r}{\bar r} - \kappa + \Xi$
  - **velocity contributions:** $V_s - \frac{1}{\mathcal{H}\bar r_z}\left(V_s - V_o\right)$

Sasaki 1987, Bonvin, Durrer, Kunz 2006, Kaiser & Hudson 2015

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$$\boxed{- \int_0^{\bar{r}_z} d\bar{r} \left(\frac{\bar{r}_z - \bar{r}}{\bar{r}_z \bar{r}}\right) \hat{\nabla}^2 \left(\alpha_\chi - \varphi_\chi\right.} \left. - C_\parallel\right) \boxed{- \frac{1}{\bar{r}_z} \hat{\nabla}_\alpha \mathcal{G}^\alpha ,}$$

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Sasaki 1987, Bonvin, Durrer, Kunz 2006, Kaiser & Hudson 2015

Systematic Errors

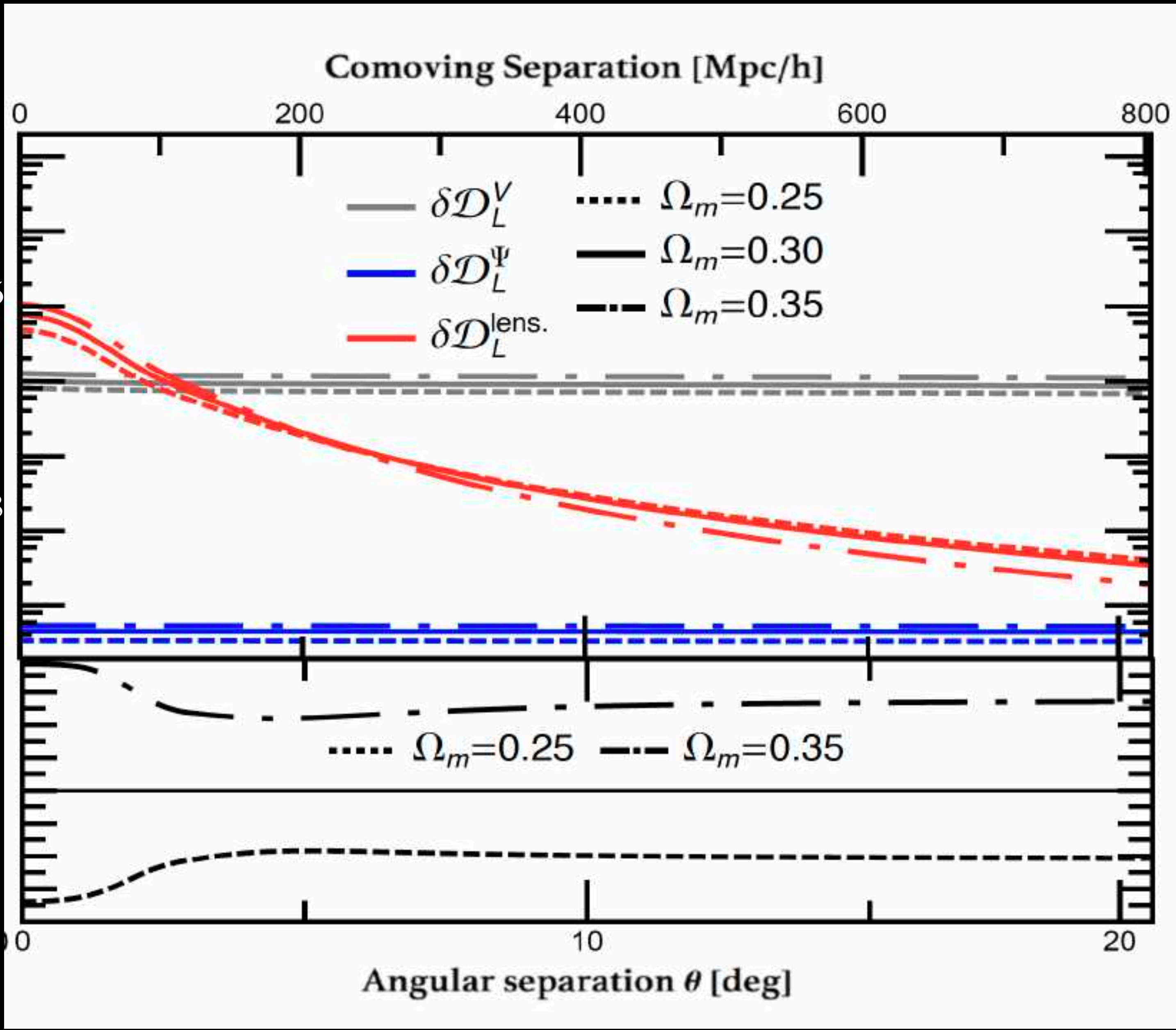
- **standard model**

  $\delta\mathcal{D}_L^{\text{lens.}} \equiv \kappa_{\text{std}}$
- **missing velocity**

  $\delta\mathcal{D}_L^V$
- **missing potential**

  $\delta\mathcal{D}_L^\Psi$
- **significant systematic errors**
- **power spectrum in progress**

Biern & Yoo 2017



Lensing Shear (scalar)

- **standard formalism:** $_{\pm 2}\gamma \equiv \gamma_1 \pm i\gamma_2 = m^\alpha_\mp m^\beta_\mp \gamma_{\alpha\beta}$

$$\gamma_{\alpha\beta} = -\left(C_{\alpha\beta}\right)_o \boxed{+\mathcal{G}_{\alpha,\beta}} + \int_0^{\bar r_z} d\bar r \left(\frac{\partial}{\partial x^\beta}\right) 2C_{\alpha\gamma}n^\gamma + \boxed{\int_0^{\bar r_z} d\bar r \left(\frac{\bar r_z - \bar r}{\bar r_z \bar r}\right)\left[\bar r^2 \left(\frac{\partial^2}{\partial x^\alpha \partial x^\beta}\right)\left(\alpha_\chi - \varphi_\chi - C_\parallel\right)\right]}$$

  - **gauge dependent due to** $\mathcal{G}^\alpha \to \mathcal{G}^\alpha - \mathcal{L}^\alpha$
  - **correct shear in cN gauge w/ scalar only (*lucky!*)**
  - **other gauges yield *different shear!***

scalar: α_χ, φ_χ **tensor:** $C_{\alpha\beta}$ **(SVT decomposition of metric)**

- **gauge-invariant formalism:**

$$\hat\gamma_{\alpha\beta} = -\left(C_{\alpha\beta o} + C_{\alpha\beta}\right) + \int_0^{\bar r_z} d\bar r \left(\frac{\partial}{\partial x^\beta}\right) 2C_{\alpha\gamma}n^\gamma + \int_0^{\bar r_z} d\bar r \left(\frac{\bar r_z - \bar r}{\bar r_z \bar r}\right)\left[\bar r^2 \left(\frac{\partial^2}{\partial x^\alpha \partial x^\beta}\right)\left(\alpha_\chi - \varphi_\chi - C_\parallel\right)\right]$$

Lensing Shear (tensor)

- **standard formalism:** $_{\pm2}\gamma \equiv \gamma_1 \pm i\gamma_2 = m^\alpha_\mp m^\beta_\mp \gamma_{\alpha\beta}$

$$\gamma_{\alpha\beta} = \boxed{-\left(C_{\alpha\beta}\right)_o} + \mathcal{G}_{\alpha,\beta} + \boxed{\int_0^{\bar r_z} d\bar r \left(\frac{\partial}{\partial x^\beta}\right) 2C_{\alpha\gamma}n^\gamma} + \int_0^{\bar r_z} d\bar r \left(\frac{\bar r_z - \bar r}{\bar r_z \bar r}\right)\left[\bar r^2 \left(\frac{\partial^2}{\partial x^\alpha \partial x^\beta}\right)\left(\alpha_\chi - \varphi_\chi \boxed{- C_\parallel}\right)\right]$$

  - **tensors: gauge invariant at 1st order**
  - **<u>incorrect</u> shear in tensor & *IR divergence!***

scalar: $\alpha_\chi,\ \varphi_\chi$ **tensor:** $C_{\alpha\beta}$ **(SVT decomposition of metric)**

- **gauge-invariant formalism:**

$$\hat\gamma_{\alpha\beta} = \boxed{-\left(C_{\alpha\beta o} + C_{\alpha\beta}\right)} + \boxed{\int_0^{\bar r_z} d\bar r \left(\frac{\partial}{\partial x^\beta}\right) 2C_{\alpha\gamma}n^\gamma} + \int_0^{\bar r_z} d\bar r \left(\frac{\bar r_z - \bar r}{\bar r_z \bar r}\right)\left[\bar r^2 \left(\frac{\partial^2}{\partial x^\alpha \partial x^\beta}\right)\left(\alpha_\chi - \varphi_\chi \boxed{- C_\parallel}\right)\right]$$

  - ***metric shear* or *FNC term*: tensor at source**

Dodelson et al. 2003, Schmidt & Jeong 2012

  - **<u>rest frame</u>: observer frame & source frame**

Lensing Rotation

- lensed images rotate!
  - *no* rotation due to scalar at 1st order
  - *rotation* by scalars beyond linear order

- Skrotsky effect
  - *rotation* by vector and tensor even at 1st order
  - probe of gravity waves

- gauge invariant formalism:
  - Skrotsky effect & difference in orientation

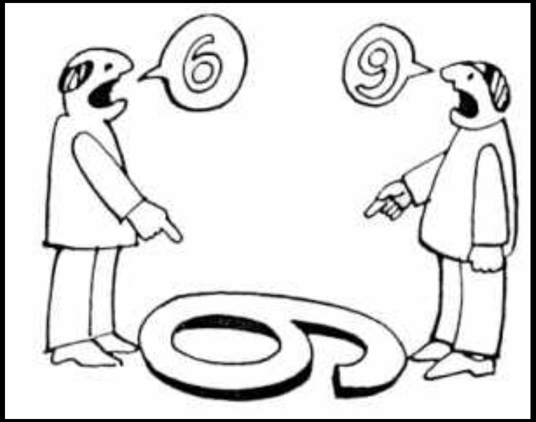
$$2\hat{\omega} = 2\left(\Omega_o^n - \Omega_s^n\right) - 2\ \cos\theta\ \Delta\phi - \int_0^{\bar{r}_z} d\bar{r}\ \mathrm{n}\cdot\nabla\times\left(\Psi^\alpha + 2C_\parallel^\alpha\right)$$

To Rotate or Not to Rotate

- how to measure *rotation*?
  - orientations should be synchronized
  - parallel transport along null path (*the only way!* path dependent in curved space)
  - orientation of src: *completely fixed* by one at obs

$$2\Omega^n_s = 2\Omega^n_o - 2\ \cos\theta\ \Delta\phi - \int_0^{\bar r} d\bar r\ n\cdot\nabla\times\left(\Psi^\alpha + 2C^\alpha_\parallel\right) \qquad \therefore \quad \hat\omega = 0$$

- bottom line:
  - *complete cancellation in rotation!*
  - *no* lensing rotation *at all* at *1st order*
  - *fictitious rotation* against FRW coordinate



Radio Jets!

- orientation of radio jets:
  - polarization: central geodesic, *parallel transported*
  - extended images: geodesic deviation, *not PTed*
- intrinsic relation:
  - jet & polarization is perpendicular in source rest frame
  - infer lensing rotation

Kronberg, Dyer, Burbidge, Junkkarinen 1991 ApJL

(c) Weak Gravitational Lensing

- summary of new findings:
  - **convergence**: fluctuation in luminosity distance (missing velocity contributions)
  - **shear**: ok with scalar in cN gauge
  - **rotation**: zero at 1st order, even with tensor

- much of the work **in progress**:
  - impact on 2pt shear correlation
  - convergence and shear cross-correlation
  - beyond linear order

Yoo, Grimm, Mitsou, Amara, Refregier 2018 JCAP and **Grimm & Yoo 2018 JCAP**

(d) Cosmic Microwave Background

- much of the work **in progress**:
  - Boltzmann equation in tetrad formalism
  - use of background metric (except **Bond & Szalay 1983**)
  - **linear order**: ok except *monopole* and *dipole* ☺
  - beyond linear order, **not ok** ☹

- future applications:
  - *CMB lensing* (2nd order): not complete
  - CMB spectral distortion
  - and more (**stay tuned!**)

CMB Temperatures $\bar{T}$ and $\langle T\rangle^{\text{obs}}$

- $\bar{T}$: **cosmological parameter**
  - **determine background evolution**
  - **defined in background universe, *unique* number**
  - **influence perturbation evolution**

- $\langle T\rangle^{\text{obs}}$: **observed CMB temperature (from FIRAS)**
  - **angle average CMB temperature over all sky**
  - **uncertainty in** $\langle T\rangle^{\text{obs}}$**:**
    - **COBE FIRAS 1996:** $\langle T\rangle^{\text{obs}} = 2.728 \pm 0.004\ \text{K}$ **(0.15%)**
    - **+ WMAP 2009:** $\langle T\rangle^{\text{obs}} = 2.7255 \pm 5.7 \cdot 10^{-4}\ \text{K}$ **(0.021%)**

Yoo, Mitsou, Dirian, Durrer 2019 PRD 100, 063510

(e) Cosmic Variance

- **standard** cosmic variance:
  - cubic box of *simultaneity*
  - number of independent modes
  - luminosity distance: no CV limit if $N_{\rm SN} = \infty$

- cosmic variance on the **light cone**:
  - **single** past-light cone
  - all cosmological observables: CV limited (luminosity distance, no exception)
  - *maximum* cosmological information

Yoo, Mitsou, Grimm, Durrer, Refregier 2019, JCAP 12, 015

(e) Cosmic Variance

- **type Ia supernova**:
  - LSST will measure approximately $N_{\rm SN}=\infty$
  - do we need to? when cos info is **saturated**?
  - what is the ***maximum cos info*** up to $z=1$

- **missing baryon & local Hubble**:
  - tip of the light cone, large cosmic variance
  - what is **CV contribution** to the problem?

Yoo, Mitsou, Grimm, Durrer, Refregier 2019, JCAP 12, 015

Maximum Cosmological Information

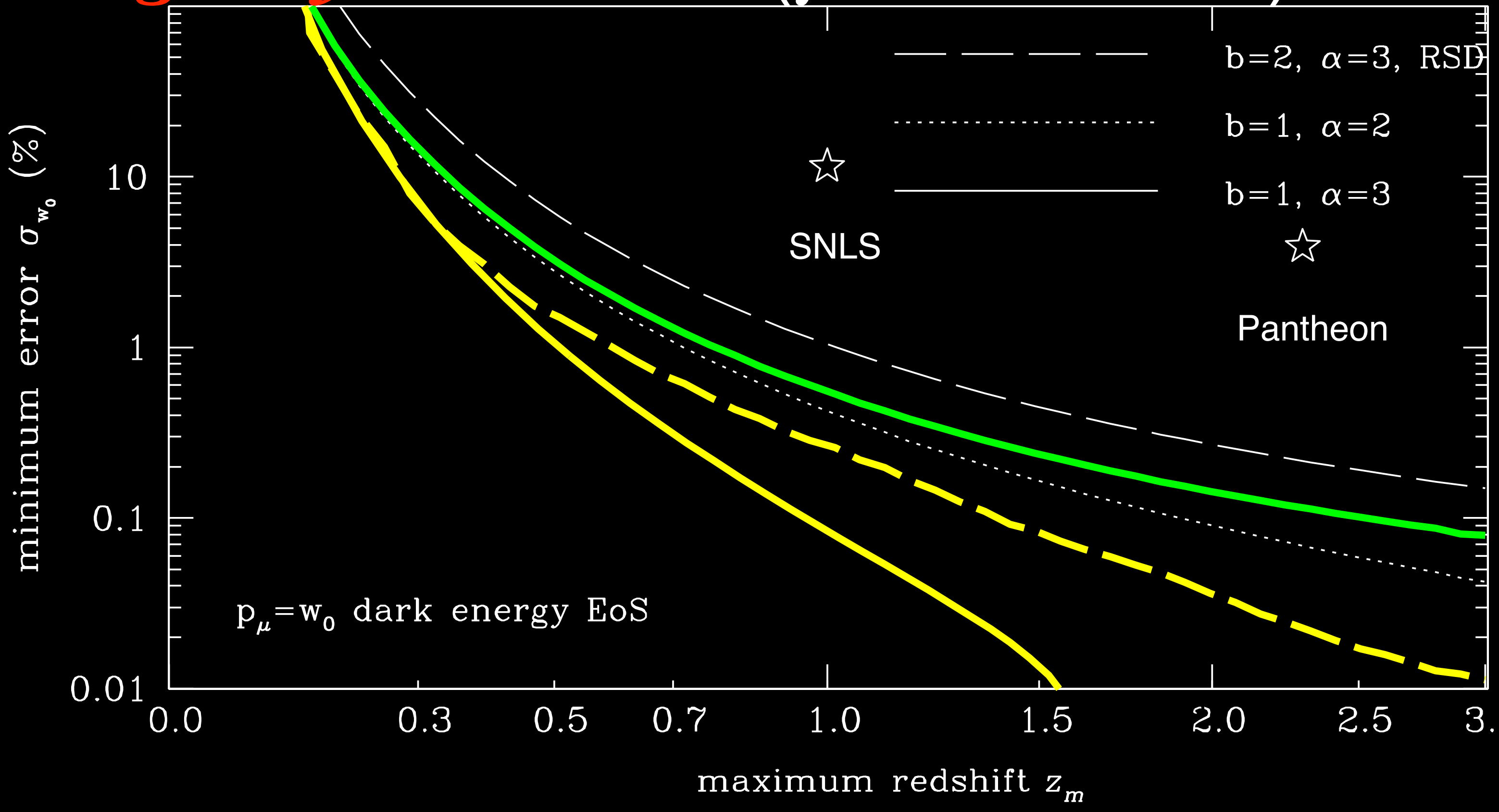
- idealized supernova observations:
  - *no* systematic errors
  - infinite number of observations (*no* stat. error)
  - all sky, all SN measurements up to maximum redshift

- cosmic variance:
  - information is *not infinite!*
  - observed flux, angular position, redshift: *correlated*
  - host galaxy fluctuations & radial correlation: *not* properly considered in literature

Yoo 2019 arXiv:1911.07869

Maximum Cosmological Information

- *two* **cosmological parameters in LCDM:** (Ω_m, w_0)
  - **imprecise forecast in yellow:**
  - **without *radial correlation* (yellow solid)**
  - **without *host galaxy correlation* (yellow dashed)**



- **b: galaxy bias**
- **α: evolution factor (3: dark matter)**

V. MORE WORK & FUTURE DIRECTION:

In the Next Five Years and the Coming Decade

What can We Learn?

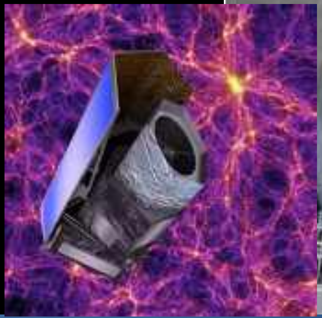
- **relativistic effects:** *small*, but *detectable!*
    - *extra* and *critical* information
    - difficult, but high gain (*new* opportunities)

- **key role:** *deviations* from standard cosmology
    - *higher-order signatures* (fossil fields, DE fluct.)
    - *not* present in Newtonian description

- **complementary role:** *enigmatic* standard cosmology
    - (better) *complementary* to CMB constraints
    - *convincing* constraints on dark energy

Future Surveys

- future ground-based surveys:
    - Dark Energy Spectroscopic Instrument
    - Large Synoptic Survey Telescope
    - Square Kilometer Array
    - CMB Stage-IV
- future space missions:
    - Euclid
    - Wide-Field Infrared Survey Telescope



Are We Ready for Precision Cosmology?

General Relativistic Effects and Gauge-Invariant Formalism

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Institutes for Computational Science & Physics, University of Zürich

24 January 2020

Universität
Zürich UZH

Executive Summary

- incomplete standard theoretical description of cosmological observables
    - *gauge dependent* & missing *relativistic effects*
    - *no* frame specification for physical events and observables
    - limited to *linear order*

- subtle relativistic effects in precision cosmology
    - a *new area* of research
    - *test* general relativity (or modified gravity)
    - *signatures* of inflationary models
    - *consistency* check & *complementary* constraints