Quantum loop effects during inflation

and

a possible connection to nonlocal gravity

Sohyun Park (CERN)

Geneva Cosmology and Astroparticle physics group

31 January 2020

Plan

- Motivation: Inflation and enhancement of quantum effects during inflation

- An example: Quantum loop corrections to Newtonian potential in flat space and in an inflationary background (de Sitter space)

- Discussions: A lot of open problems; Hope to continue discussing afterwards

I am local, living in Geneva!

Inflationary Scenario tells us:

"Inflation amplified quantum fluctuations into initial density perturbations, which grew to form the structure of the universe."

How could inflation do that?

Inflation

- On large scales, our universe is homogeneous, isotropic and spatially flat.

$$ds^2 = -dt^2 + a^2(t)d\vec{x} \cdot d\vec{x}$$

- Hubble parameter: $H(t) = \frac{\dot{a}}{a}$
- First slow-roll parameter: $\epsilon(t) = -\frac{\dot{H}}{H^2} = -\frac{a\ddot{a}}{\dot{a}^2} + 1$
- Inflation: $\dot{a} > 0 \;\&\; \ddot{a} > 0$ or $H > 0 \;\&\; \epsilon < 1$
- Data (Planck, …): $H_I \simeq 10^{14} GeV, \quad \epsilon_I \simeq 0.013$
- de Sitter: $a(t) = e^{Ht}, \quad H = \text{const} > 0, \quad \epsilon = 0$

Quantum effects during inflation → **Quantum field theory in de Sitter**

Quantum effects as a response to virtual particles

- Stronger quantum effects: more virtual particles produced and live longer.
- Virtual pair production from the vacuum: conservation of energy violated!

	before	**after**
flat space	$E = 0$	$E = 2\sqrt{m^2 + k^2}$
expanding universe	$E = 0$	$E = 2\sqrt{m^2 + \frac{k^2}{a^2(t)}}$

- In flat space: $\Delta E \Delta t \geq \hbar$ allows the violation during $\Delta t \leq \frac{\hbar}{\Delta E} = \frac{\hbar}{2\sqrt{m^2 + k^2}}$
- In expanding universe $k_{phys} = \frac{k}{a(t)} \rightarrow \int_t^{t+\Delta t} dt' 2\sqrt{m^2 + \frac{k^2}{a^2(t')}} \leq \hbar$
- Maximum lifetime obtained if $m = 0$ & de Sitter $a(t) = e^{Ht}$

$$2k \int_t^{t+\Delta t} dt' \frac{1}{a(t')} \lesssim 1 \rightarrow \frac{2k}{Ha(t)}(1 - e^{-H\Delta t}) \lesssim 1$$

- Massless particles persist forever in de Sitter if emerge with $k \lesssim Ha(t)$

Conformal invariance and emergence rate

- In flat space: the emergence rate $\Gamma_{flat} = \text{const.}$ (Poincare invariance)
- In an expanding universe: $\Gamma(t)$ (no time translation invariance)
- For particles with conformal invariance

1. Take $dt = a(t)d\eta$ then the FRW metric becomes:
   $ds^2 = -dt^2 + a^2(t)d\vec{x}\cdot d\vec{x} = a^2(t)(-d\eta^2 + d\vec{x}\cdot d\vec{x})$
2. Do conformal transf. for a theory possessing conformal symmetry with $\Omega = \frac{1}{a(t)}$:
   in $D=4$, $g_{\mu\nu} \to \Omega^2(x)g_{\mu\nu}$, $A_\mu \to A_\mu$, $\psi \to \psi\Omega^{-3/2}$, $\phi \to \phi\Omega^{-1}$

- In $(\eta, \vec{x})$ coords. $\frac{dN}{d\eta} = \Gamma_{flat}$
- Back to physical time $\Gamma(t) = \frac{dN}{dt} = \frac{dN}{d\eta}\frac{d\eta}{dt} = \frac{\Gamma_{flat}}{a(t)}$
- In an expanding universe, the emergence rate of virtual particles possessing conformal symmetry is suppressed by a factor of $1/a$.

Biggest quantum effects: Maximum particle production

- Maximum quantum effects are obtained if:
  - inflation: holds virtual particles apart longer
  - massless particles: can live forever during inflation
  - No conformal invariance: the rate of emergence NOT suppressed
- Only two types of particles with no mass and no conformal symmetry:
  - gravitons
  - massless, minimally coupled (MMC) scalar
- Particle production in an expanding universe:
  - Schrödinger, Physica 6 (1939) 899 "The proper vibrations of the expanding universe"
  - Parker 1968 - 1971 "Quantized fields and particle creation in expanding universes", ...
  - Grishchuk 1974 "Amplification of gravitational waves in an isotropic universe"
  - ...

Number of particles produced

- MMC scalar $\varphi(t,\vec{x})$: $\mathcal{L} = -\frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi g^{\mu\nu}\sqrt{-g}$
- In FLRW, $L = \int \frac{d^3k}{(2\pi)^3}\left[\frac{1}{2}a^3\dot{\tilde{\varphi}}(t,\vec{k})\dot{\tilde{\varphi}}^*(t,\vec{k}) - \frac{1}{2}ak^2\tilde{\varphi}(t,\vec{k})\tilde{\varphi}^*(t,\vec{k})\right]$
- No coupling between different $\vec{k}$'s, pick one mode with $\vec{k}$ and call it q(t):
  $L = \frac{1}{2}a^3\dot{q}^2 - \frac{1}{2}k^2aq^2$: a harmonic oscillator $m(t) \sim a^3(t)$ & $\omega(t) = \frac{k}{a(t)}$
- Hamiltonian: $E(t) = \frac{1}{2}m(t)\dot{q}^2 + \frac{1}{2}m(t)\omega^2(t)q^2$ with EoM: $\ddot{q} + 3H\dot{q} + \frac{k^2}{a^2}q = 0$
- Solution for de Sitter $q(t) = u(t)\alpha + u^*(t)\alpha^\dagger$, $\quad u(t,k) = \frac{H}{\sqrt{2k^3}}\left[1 - \frac{ik}{Ha(t)}\right]e^{\frac{ik}{Ha(t)}}$
- The expectation value of the energy of the Bunch-Davis vacuum, $|\Omega\rangle$

$$< \Omega|E(t)|\Omega > = \frac{k}{a}\left[\frac{1}{2} + \left(\frac{Ha}{2k}\right)^2\right] = \frac{k}{a}\left[\frac{1}{2} + \text{“}N(t)\text{”}\right]$$

- Number of particles with wave number k: $N(k,t) = \left(\frac{Ha(t)}{2k}\right)^2$ Parker 1968
- Number grows significantly for infrared $k \lesssim Ha$.

Using perturbative general relativity as an effective field theory

- Quantum gravity is not perturbatively renormalizable.
- The loop counting parameter GE^2 is extremely small, making GR a perfectly good perturbation theory and we are interested in the IR regime.

"General Relativity as an effective field theory: The leading quantum corrections", Donoghue gr-qc/9405057

- Reliable predictions for long range quantum gravitational phenomena can be made using perturbative GR as a low energy effective field theory.

- UV divergences can be absorbed by counterterms at any finite order.

BPHZ procedure: a guide how to construct counterterms in a given order in perturbation theory

- The finite terms from primitive diagrams are nonlocal while the counterterms are local; i.e., they are distinct from each other.
- The finite, nonlocal terms dominate over the finite part of local counterterms at IR.

Bogoliubov, Parasiuk, Hepp, Zimmermann (BPHZ) renormalization scheme

Example: Quantum scalar correction to Einstein equations

- How a loop of MMC scalars changes dynamical gravitons (quanta of GWs) and the force of gravity?

$$\mathcal{L} = \frac{1}{16\pi G}\Big[R - 2\Lambda\Big]\sqrt{-g} - \frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi g^{\mu\nu}\sqrt{-g}$$

- Graviton: Perturbation around the open conformal patch of de Sitter space

Define the graviton field $h_{\mu\nu}$ as $g_{\mu\nu} = \overline{g}_{\mu\nu} + \kappa h_{\mu\nu}$,

where $\overline{g}_{\mu\nu} = a^2\eta_{\mu\nu}, \quad \kappa^2 = 16\pi G, \quad a = -\frac{1}{H\eta}, \quad H = \sqrt{\frac{1}{3}\Lambda}$

- The quantum effective field equations formally derived from the quantum effective action

$$\mathcal{D}^{\mu\nu\rho\sigma}h_{\rho\sigma}(x) - \int d^4x' \left[{}^{\mu\nu}\Sigma^{\rho\sigma}\right](x;x')h_{\rho\sigma}(x') = \frac{\kappa}{2}\mathcal{T}^{\mu\nu}_{\text{lin}}(x)$$

Linearized Einstein equation: Classical

Quantum correction

- $\mathcal{D}^{\mu\nu\rho\sigma}$: Lichnerowicz operator s.t. $\mathcal{D}^{\mu\nu\rho\sigma}h_{\rho\sigma}$: Linearized Einstein tensor
- $-i\left[{}^{\mu\nu}\Sigma^{\rho\sigma}\right](x;x')$: graviton self-energy = 1PI graviton 2-point function

1PI: 1 Particle Irreducible

A three-step procedure

1. Compute and renormalize the one-loop contribution to the graviton self-energy from a MMC scalar on de Sitter background

   SP & Woodard 1101.5804

   Leonard, SP, Prokopec, Woodard 1403.0896

$$-i\left[{}^{\mu\nu}\Sigma^{\rho\sigma}\right](x;x') \to \left[{}^{\mu\nu}\Sigma^{\rho\sigma}_{\rm Ren}\right](x;x')$$

2. Convert the in-out self-energy to the retarded one of the Schwinger-Keldysh formalism (the in-in self-energy)

$$\left[{}^{\mu\nu}\Sigma^{\rho\sigma}_{\rm Ren}\right](x;x') \to \left[{}^{\mu\nu}\Sigma^{\rho\sigma}_{\rm Ren,\ Ret}\right](x;x')$$

3. Solve the quantum corrected, linearized Einstein field equation

$$\mathcal{D}^{\mu\nu\rho\sigma}h_{\rho\sigma}(x) - \underbrace{\int d^4x' \left[{}^{\mu\nu}\Sigma^{\rho\sigma}_{\rm Ren,\ Ret}\right](x;x')h_{\rho\sigma}(x')}_{\text{quantum: nonlocal}} = \frac{\kappa}{2}\mathcal{T}^{\mu\nu}_{\rm lin}(x)$$

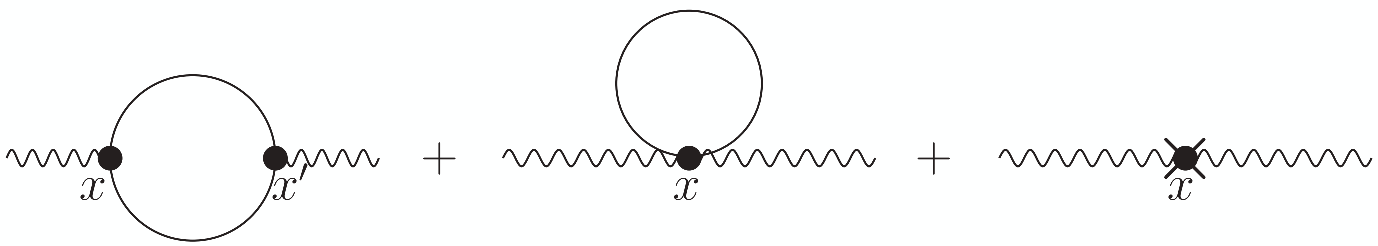
quantum: nonlocal

SP & Woodard 1109.4187

SP, Prokopec, Woodard 1510.03352

Step 1: One Loop Scalar Contributions to Graviton Self-Energy

- One loop contribution to the graviton self-energy from MMC scalars consists of 3 Feynman diagrams



$$-i[^{\mu\nu}\Sigma^{\rho\sigma}](x;x') = \frac{1}{2}\sum_{I=1}^{2} T_I^{\mu\nu\alpha\beta}(x) \sum_{J=1}^{2} T_J^{\rho\sigma\gamma\delta}(x') \times \partial_\alpha \partial'_\gamma i\Delta(x;x') \times \partial_\beta \partial'_\delta i\Delta(x;x')$$
$$+\frac{1}{2}\sum_{I=1}^{4} F_I^{\mu\nu\rho\sigma\alpha\beta}(x) \times \partial_\alpha \partial'_\beta i\Delta(x;x') \times \delta^D(x-x') + 2\sum_{I=1}^{2} C_I^{\mu\nu\rho\sigma}(x) \times \delta^D(x-x')$$

- The 3-pt and 4-pt interaction (between MMC scalars and gravitons) vertices derive from expanding the Lagrangian

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi g^{\mu\nu}\sqrt{-g} = -\frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi \bar{g}^{\mu\nu}\sqrt{-\bar{g}} - \frac{\kappa}{2}\partial_\mu\varphi\partial_\nu\varphi\Big(\frac{1}{2}h\bar{g}^{\mu\nu} - h^{\mu\nu}\Big)\sqrt{-\bar{g}}$$
$$-\frac{\kappa^2}{2}\partial_\mu\varphi\partial_\nu\varphi\left\{\Big[\frac{1}{8}h^2 - \frac{1}{4}h^{\rho\sigma}h_{\rho\sigma}\Big]\bar{g}^{\mu\nu} - \frac{1}{2}hh^{\mu\nu} + h^\mu{}_\rho h^{\rho\nu}\right\}\sqrt{-\bar{g}} + O(\kappa^3)$$

Step1: MMC scalar propagator

- The MMC scalar propagator obeys

$$\partial_\mu \left[\sqrt{-\bar{g}}\ \bar{g}^{\mu\nu}\partial_\nu\right] i\triangle(x;x') = i\delta^D(x-x')$$

- No de Sitter invariant solution for the propagator (Allen & Folacci 1987) but there is a solution preserving the homogeneity and isotropy

$$i\triangle(x;x') = A\Big(y(x;x')\Big) + \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}}\frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})}\ln(aa')$$

= de Sitter invariant function of y + de Sitter breaking term

$$\text{where}\quad A(y) \equiv \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}}\Bigg\{\frac{\Gamma(\frac{D}{2})}{\frac{D}{2}-1}\left(\frac{4}{y}\right)^{\frac{D}{2}-1} + \frac{\Gamma(\frac{D}{2}+1)}{\frac{D}{2}-2}\left(\frac{4}{y}\right)^{\frac{D}{2}-2} - \pi\cot\left(\frac{\pi D}{2}\right)\frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} + \sum_{n=1}^{\infty}\left[\frac{1}{n}\frac{\Gamma(n+D-1)}{\Gamma(n+\frac{D}{2})}\left(\frac{y}{4}\right)^n - \frac{1}{n-\frac{D}{2}+2}\frac{\Gamma(n+\frac{D}{2}+1)}{\Gamma(n+2)}\left(\frac{y}{4}\right)^{n-\frac{D}{2}+2}\right]\Bigg\}$$

- The de Sitter breaking term drops differentiated by $\partial_\alpha\partial'_\beta$

- $y(x;x') \equiv aa'H^2\Delta x^2$ the de Sitter invariant length function

De Sitter transformations

De Sitter space has the maximum number of space-time symmetries in a given dimension. For our D-dimensional conformal coordinates the $\frac{1}{2}D(D+1)$ de Sitter transformations can be decomposed as follows:

- Spatial transformations - $(D-1)$ transformations.

$$\eta' = \eta \, , \quad x'^i = x^i + \epsilon^i \, . \tag{12}$$

- Rotations - $\frac{1}{2}(D-1)(D-2)$ transformations.

$$\eta' = \eta \, , \quad x'^i = R^{ij} x^j \, . \tag{13}$$

- Dilation - 1 transformation.

$$\eta' = k\eta \, , \quad x'^i = k x^j \, . \tag{14}$$

- Spatial special conformal transformations - $(D-1)$ transformations.

$$\eta' = \frac{\eta}{1 - 2\vec{\theta}\cdot\vec{x} + \|\vec{\theta}\|^2 \, x\cdot x} \, , \quad x' = \frac{x^i - \theta^i x \cdot x}{1 - 2\vec{\theta}\cdot\vec{x} + \|\vec{\theta}\|^2 \, x\cdot x} \, . \tag{15}$$

Taken from 1101.5804

Step 1: The two primitive diagrams

- **Contribution from 4-point vertices**

$$-i\Big[{}^{\mu\nu}\Sigma^{\rho\sigma}\Big]_{\rm 4pt}(x;x') \equiv \frac{1}{2}\sum_{I=1}^{4} F_I^{\mu\nu\rho\sigma\alpha\beta}(x) \times \partial_\alpha \partial'_\beta i\triangle(x;x') \times \delta^D(x-x')$$
$$= \Big(\frac{D-4}{4}\Big)\frac{i\kappa^2 H^D}{(4\pi)^{\frac{D}{2}}}\frac{\Gamma(D)}{\Gamma(\frac{D}{2}+1)}\sqrt{-\overline{g}}\left\{\frac{1}{2}\overline{g}^{\mu\nu}\overline{g}^{\rho\sigma} - \overline{g}^{\mu(\rho}\overline{g}^{\sigma)\nu}\right\}\delta^D(x-x') = 0 \text{ for } D=4$$

- **Contribution from 3-point vertices**

$$-i\Big[{}^{\mu\nu}\Sigma^{\rho\sigma}\Big]_{\rm 3pt}(x;x') = \sqrt{-\overline{g}}\sqrt{-\overline{g}'}\Bigg\{\frac{\partial^2 y}{\partial x_\mu \partial x'_{(\rho}}\frac{\partial^2 y}{\partial x'_{\sigma)}\partial x_\nu} \times \alpha(y) + \frac{\partial y}{\partial x_{(\mu}}\frac{\partial^2 y}{\partial x_{\nu)}\partial x'_{(\rho}}\frac{\partial y}{\partial x'_{\sigma)}} \times \beta(y)$$
$$+\frac{\partial y}{\partial x_\mu}\frac{\partial y}{\partial x_\nu}\frac{\partial y}{\partial x'_\rho}\frac{\partial y}{\partial x'_\sigma} \times \gamma(y) + \overline{g}^{\mu\nu}\overline{g}'^{\rho\sigma}H^4 \times \delta(y) + \Big[\overline{g}^{\mu\nu}\frac{\partial y}{\partial x'_\rho}\frac{\partial y}{\partial x'_\sigma} + \frac{\partial y}{\partial x_\mu}\frac{\partial y}{\partial x_\nu}\overline{g}'^{\rho\sigma}\Big]H^2 \times \epsilon(y)\Bigg\}$$
$$\propto \frac{1}{y^4} \sim \frac{1}{\Delta x^8} \text{ in } D=4 \quad \longrightarrow \int d^4x' \frac{1}{\Delta x^8} \quad \text{quartically divergent}$$

Correspondence with flat space limit

- **Flat space limit** $H \to 0$

$$\Delta x^0 \longrightarrow t - t' \;,\;\; y(x;x') \longrightarrow H^2 \Delta x^2 \;,\;\; \frac{\partial y}{\partial x_\mu} \longrightarrow 2H^2 \Delta x^\mu \;\;,\;\; \frac{\partial y}{\partial x'_\nu} \longrightarrow -2H^2 \Delta x^\nu \;\;,\;\; \frac{\partial y^2}{\partial x_\mu \partial x'_\nu} \longrightarrow -2H^2 \eta^{\mu\nu}$$

$$-i\Big[{}^{\mu\nu}\Sigma^{\rho\sigma}\Big]_{\rm flat}(x;x') = \frac{\kappa^2 \Gamma^2(\frac{D}{2})}{16\pi^D} \Bigg\{ \eta^{\mu(\rho}\eta^{\sigma)\nu} \times \Big[-\frac{2}{\Delta x^{2D}}\Big] + \Delta x^{(\mu}\eta^{\nu)(\rho}\Delta x^{\sigma)} \times \Big[\frac{4D}{\Delta x^{2D+2}}\Big]$$
$$+ \Delta x^\mu \Delta x^\nu \Delta x^\rho \Delta x^\sigma \times \Big[-\frac{2D^2}{\Delta x^{2D+4}}\Big] + \eta^{\mu\nu}\eta^{\rho\sigma} \times \Big[-\frac{1}{2}\frac{(D^2-D-4)}{\Delta x^{2D}}\Big]$$
$$+ \Big[\eta^{\mu\nu}\Delta x^\rho \Delta x^\sigma + \Delta x^\mu \Delta x^\nu \eta^{\rho\sigma}\Big] \times \Big[\frac{D(D-2)}{\Delta x^{2D+2}}\Big] \Bigg\}$$

- **Agrees with 't Hooft and Veltman, Ann. Inst. Henri Poincare XX (1974) 69.**

Correspondence with stress tensor correlators

- The graviton self-energy is related to the 2-point correlator of the stress tensor as

$$-i\left[{}^{\mu\nu}\Delta^{\rho\sigma}\right](x;x') = -\frac{1}{4}\kappa^2\sqrt{-\bar{g}(x)}\,\sqrt{-\bar{g}(x')}\left\langle\Omega\middle|\delta\mathcal{T}^{\mu\nu}(x)\delta\mathcal{T}^{\rho\sigma}(x')\middle|\Omega\right\rangle + O(\kappa^4)$$

- The stress tensor correlator obtained by Perez-Nadal, Roura and Verdaguer (2010) agrees with out result.

$$\left\langle\Omega\middle|\delta\mathcal{T}^{\mu\nu}(x)\delta\mathcal{T}^{\rho\sigma}(x')\middle|\Omega\right\rangle = F_{\mu\nu\rho\sigma} = P(\mu)n_\mu n_\nu n_\rho n_\sigma + Q(\mu)(n_\mu n_\nu g_{\rho\sigma} + n_\rho n_\sigma g_{\mu\nu})$$
$$+R(\mu)(n_\mu n_\rho g_{\nu\sigma} + n_\nu n_\sigma g_{\mu\rho} + n_\mu n_\sigma g_{\nu\rho} + n_\nu n_\rho g_{\mu\sigma}) + S(\mu)(g_{\mu\rho}g_{\nu\sigma} + g_{\nu\rho}g_{\mu\sigma}) + T(\mu)g_{\mu\nu}g_{\rho\sigma}$$

- Note: their 5 basis tensors are converted into ours as

$$n_a n_b n_{c'} n_{d'} = \frac{1}{H^4(4y-y^2)^2}\frac{\partial y}{\partial x^a}\frac{\partial y}{\partial x^b}\frac{\partial y}{\partial x'^{c'}}\frac{\partial y}{\partial x'^{d'}}\,,$$
$$n_a n_b g_{c'd'} + n_{c'} n_{d'} g_{ab} = \frac{1}{H^2(4y-y^2)}\left[g_{ab}\frac{\partial y}{\partial x'^{c'}}\frac{\partial y}{\partial x'^{d'}} + \frac{\partial y}{\partial x^a}\frac{\partial y}{\partial x^b}g_{c'd'}\right],$$
$$4n_{(a}g_{b)(c'}n_{d')} = -\frac{2}{H^4(4y-y^2)}\frac{\partial y}{\partial x^{(a}}\frac{\partial^2 y}{\partial x^{b)}\partial x'^{(c'}}\frac{\partial y}{\partial x'^{d')}} - \frac{2}{H^4(4y-y^2)(4-y)}\frac{\partial y}{\partial x^a}\frac{\partial y}{\partial x^b}\frac{\partial y}{\partial x'^{c'}}\frac{\partial y}{\partial x'^{d'}}\,,$$
$$2g_{a(c'}g_{d')b} = \frac{1}{2H^4}\frac{\partial^2 y}{\partial x^a\partial x'^{(c'}}\frac{\partial^2 y}{\partial x^{d')}\partial x'^b} + \frac{1}{H^4(4-y)}\frac{\partial y}{\partial x^{(a}}\frac{\partial^2 y}{\partial x^{b)}\partial x'^{(c'}}\frac{\partial y}{\partial x'^{d')}} + \frac{1}{2H^4}\frac{1}{(4-y)^2}\frac{\partial y}{\partial x^a}\frac{\partial y}{\partial x^b}\frac{\partial y}{\partial x'^{c'}}\frac{\partial y}{\partial x'^{d'}}\,,$$
$$g_{ab}g_{c'd'} = g_{ab}g_{c'd'}$$

One loop counterterms

- For quantum gravity at one loop order the necessary counterterms are R^2 and C^2 first derived by t Hooft and Veltman, 1974
  Graviton 2-point function $\rightarrow$ 2 graviton fields
  S.D.D. $= 4 \rightarrow 4\partial$'s,
  with general coord. invariance 3 possibilities: $R^2, R^{\mu\nu}R_{\mu\nu}, R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}$
  with the Gauss-Bonet relation, only 2 are linearly indep.

BPHZ procedure

Two projection operators

- We define two 2nd order differential operators by expanding the scalar and Weyl curvatures around de Sitter background

$$\begin{aligned} R - D(D-1)H^2 &\equiv \mathcal{P}^{\mu\nu}\kappa h_{\mu\nu} + O(\kappa^2 h^2) \, , \\ C_{\alpha\beta\gamma\delta} &\equiv \mathcal{P}^{\mu\nu}_{\alpha\beta\gamma\delta}\kappa h_{\mu\nu} + O(\kappa^2 h^2) \, . \end{aligned}$$

- Spin zero projection operator:

$$\mathcal{P}^{\mu\nu} = D^\mu D^\nu - \overline{g}^{\mu\nu}\Big[D^2 + (D-1)H^2\Big] \, ,$$

- Spin two projection operator

$$\mathcal{P}^{\mu\nu}_{\alpha\beta\gamma\delta} = \mathcal{D}^{\mu\nu}_{\alpha\beta\gamma\delta} + \frac{1}{D-2}\Big[\overline{g}_{\alpha\delta}\mathcal{D}^{\mu\nu}_{\beta\gamma} - \overline{g}_{\beta\delta}\mathcal{D}^{\mu\nu}_{\alpha\gamma} - \overline{g}_{\alpha\gamma}\mathcal{D}^{\mu\nu}_{\beta\delta} + \overline{g}_{\beta\gamma}\mathcal{D}^{\mu\nu}_{\alpha\delta}\Big] + \frac{1}{(D-1)(D-2)}\Big[\overline{g}_{\alpha\gamma}\overline{g}_{\beta\delta} - \overline{g}_{\alpha\delta}\overline{g}_{\beta\gamma}\Big]\mathcal{D}^{\mu\nu} \, ,$$

where we define,

$$\begin{aligned} \mathcal{D}^{\mu\nu}_{\alpha\beta\gamma\delta} &\equiv \frac{1}{2}\Big[\delta^{(\mu}_{\alpha}\delta^{\nu)}_{\delta} D_\gamma D_\beta - \delta^{(\mu}_{\beta}\delta^{\nu)}_{\delta} D_\gamma D_\alpha - \delta^{(\mu}_{\alpha}\delta^{\nu)}_{\gamma} D_\delta D_\beta + \delta^{(\mu}_{\beta}\delta^{\nu)}_{\gamma} D_\delta D_\alpha\Big] \, , \\ \mathcal{D}^{\mu\nu}_{\beta\delta} &\equiv \overline{g}^{\alpha\gamma}\mathcal{D}^{\mu\nu}_{\alpha\beta\gamma\delta} = \frac{1}{2}\Big[\delta^{(\mu}_{\delta} D^{\nu)} D_\beta - \delta^{(\mu}_{\beta}\delta^{\nu)}_{\delta} D^2 - \overline{g}^{\mu\nu} D_\delta D_\beta + \delta^{(\mu}_{\beta} D_\delta D^{\nu)}\Big] \, , \\ \mathcal{D}^{\mu\nu} &\equiv \overline{g}^{\alpha\gamma}\overline{g}^{\beta\delta}\mathcal{D}^{\mu\nu}_{\alpha\beta\gamma\delta} = D^{(\mu}D^{\nu)} - \overline{g}^{\mu\nu} D^2 \, . \end{aligned}$$

Counterterms in terms of two projection operators

- The counterterms are expressed in terms of these two operators:

$$\frac{i\delta\Delta S_1}{\delta h_{\mu\nu}(x)\delta h_{\rho\sigma}(x')}\Big|_{h=0} = 2c_1\kappa^2\sqrt{-g}\,\mathcal{P}^{\mu\nu}\mathcal{P}^{\rho\sigma}\,i\delta^D(x-x') \longrightarrow 2c_1\kappa^2\Pi^{\mu\nu}\Pi^{\rho\sigma}\,i\delta^D(x-x'),$$

flat space limit

$$\frac{i\delta\Delta S_2}{\delta h_{\mu\nu}(x)\delta h_{\rho\sigma}(x')}\Big|_{h=0} = 2c_2\kappa^2\sqrt{-g}\,g^{\alpha\kappa}g^{\beta\lambda}g^{\gamma\theta}g^{\delta\phi}\mathcal{P}^{\mu\nu}_{\alpha\beta\gamma\delta}\mathcal{P}^{\rho\sigma}_{\kappa\lambda\theta\phi}\,i\delta^D(x-x')$$

$$\longrightarrow 2c_2\kappa^2\Big(\frac{D-3}{D-2}\Big)\Big[\Pi^{\mu(\rho}\Pi^{\sigma)\nu} - \frac{\Pi^{\mu\nu}\Pi^{\rho\sigma}}{D-1}\Big]i\delta^D(x-x')$$

flat space limit

where we define $\Pi^{\mu\nu} \equiv \partial^\mu\partial^\nu - \eta^{\mu\nu}\partial^2$ in flat space limit.

Renormalizing the Flat Space Result (a guide for de Sitter)

- Reorganize the primitive terms in the terms of two projection operators so as to be in the form of counterterms:

$$-i\left[{}^{\mu\nu}\Sigma^{\rho\sigma}\right]_{\rm flat}(x;x') = \Pi^{\mu\nu}\Pi^{\rho\sigma}F_0(\Delta x^2) + \left[\Pi^{\mu(\rho}\Pi^{\sigma)\nu} - \frac{\Pi^{\mu\nu}\Pi^{\rho\sigma}}{D-1}\right]F_2(\Delta x^2)\ .$$

- Find the structure functions F_0 and F_2 comparing this with the previous primitive result:

$$F_0(\Delta x^2) = \frac{\kappa^2\Gamma^2(\frac{D}{2})}{16\pi^D} \times -\frac{1}{8(D-1)^2}\left(\frac{1}{\Delta x^2}\right)^{D-2}$$

$$F_2(\Delta x^2) = \frac{\kappa^2\Gamma^2(\frac{D}{2})}{16\pi^D} \times -\frac{1}{4(D-2)^2(D-1)(D+1)}\left(\frac{1}{\Delta x^2}\right)^{D-2}$$

- Note: $\Pi^{\mu\nu}\Pi^{\rho\sigma} \sim \partial^4$ are w.r.t x Extract these outside the integral w.r.t x'. Now the factor of $1/\Delta x^{2D-4}$ is logarithmically divergent. Then extract one more d'Alembertian

$$\left(\frac{1}{\Delta x^2}\right)^{D-2} = \frac{\partial^2}{2(D-3)(D-4)}\left(\frac{1}{\Delta x^2}\right)^{D-3}.$$

- Now the integrand converges, however, we still cannot take the $D=4$ limit owing to the factor of $1/(D-4)$.
  The solution is to add zero in the form of the identity

$$\partial^2\left(\frac{1}{\Delta x^2}\right)^{\frac{D}{2}-1} - \frac{4\pi^{\frac{D}{2}}\, i\delta^D(x-x')}{\Gamma(\frac{D}{2}-1)} = 0\ .$$

Renormalizing the Flat Space Result (a guide for de Sitter)

- Rewrite it by adding zero:

$$\left(\frac{1}{\Delta x^2}\right)^{D-2} = \frac{\partial^2}{2(D-3)(D-4)}\left\{\frac{1}{\Delta x^{2D-6}} - \frac{\mu^{D-4}}{\Delta x^{D-2}}\right\} + \frac{4\pi^{\frac{D}{2}}\mu^{D-4} i\delta^D(x-x')}{2(D-3)(D-4)\Gamma(\frac{D}{2}-1)}$$

$$= -\frac{1}{4}\partial^2\left\{\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2} + O(D-4)\right\} + \frac{4\pi^{\frac{D}{2}}\mu^{D-4} i\delta^D(x-x')}{2(D-3)(D-4)\Gamma(\frac{D}{2}-1)} .$$

: nonlocal finite term : local divergent term

- The divergence now segregated on the delta function: remove them with counterterms:

$$-i\left[{}^{\mu\nu}\Delta\Sigma^{\rho\sigma}\right]_{\rm flat}(x;x') = \Pi^{\mu\nu}\Pi^{\rho\sigma}\left\{2c_1\kappa^2 i\delta^D(x-x')\right\} + \left[\Pi^{\mu(\rho}\Pi^{\sigma)\nu} - \frac{\Pi^{\mu\nu}\Pi^{\rho\sigma}}{D-1}\right]\left\{2\left(\frac{D-3}{D-2}\right)c_2\kappa^2 i\delta^D(x-x')\right\}$$

- by choosing the constants c_1 and c_2 as ,

$$c_1 = \frac{\mu^{D-4}\Gamma(\frac{D}{2})}{2^8\pi^{\frac{D}{2}}}\,\frac{(D-2)}{(D-1)^2(D-3)(D-4)} , \quad c_2 = \frac{\mu^{D-4}\Gamma(\frac{D}{2})}{2^8\pi^{\frac{D}{2}}}\,\frac{2}{(D+1)(D-1)(D-3)^2(D-4)} .$$

- The fully renormalized graviton self-energy for flat space background is,

$$-i\left[{}^{\mu\nu}\Sigma^{\rho\sigma}\right]_{\substack{\rm ren\\ \rm flat}} = \lim_{D\to 4}\left\{-i\left[{}^{\mu\nu}\Sigma^{\rho\sigma}\right]_{\rm flat}(x;x') - i\left[{}^{\mu\nu}\Delta\Sigma^{\rho\sigma}\right]_{\rm flat}(x;x')\right\} ,$$

$$= \Pi^{\mu\nu}\Pi^{\rho\sigma}\partial^2\left\{\frac{\kappa^2}{2^9 3^2\pi^4}\,\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\right\} + \left[\Pi^{\mu(\rho}\Pi^{\sigma)\nu} - \frac{1}{3}\Pi^{\mu\nu}\Pi^{\rho\sigma}\right]\partial^2\left\{\frac{\kappa^2}{2^{10}\cdot 3\cdot 5\pi^4}\,\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\right\}$$

Again, this agrees with 't Hooft and Veltman

Renormalizing de Sitter result

- Reorganize the primitive result in terms of the projection operators as for flat space:

$$-i\big[{}^{\mu\nu}\Sigma^{\rho\sigma}\big](x;x') = \sqrt{-\overline{g}(x)}\,\mathcal{P}^{\mu\nu}(x)\sqrt{-\overline{g}(x')}\,\mathcal{P}^{\rho\sigma}(x')\Big\{\mathcal{F}_0(y)\Big\}$$
$$+\sqrt{-\overline{g}(x)}\,\mathcal{P}^{\mu\nu}_{\alpha\beta\gamma\delta}(x)\sqrt{-\overline{g}(x')}\,\mathcal{P}^{\rho\sigma}_{\kappa\lambda\theta\phi}(x')\Big\{\mathcal{T}^{\alpha\kappa}\mathcal{T}^{\beta\lambda}\mathcal{T}^{\gamma\theta}\mathcal{T}^{\delta\phi}\Big(\frac{D-2}{D-3}\Big)\mathcal{F}_2(y)\Big\},$$

where the bitensor is $\mathcal{T}^{\alpha\kappa}(x;x') \equiv -\frac{1}{2H^2}\frac{\partial^2 y(x;x')}{\partial x_\alpha \partial x'_\kappa}$. Note: $\mathcal{T}^{\alpha\kappa}(x;x') \quad \leftarrow \quad \eta^{\alpha\kappa}$ in flat space

- Find the structure functions F_0 and F_2 comparing this with the previous primitive result:

$$\mathcal{F}_0(y) = \frac{\kappa^2 H^{2D-4}\Gamma^2(\frac{D}{2})}{(4\pi)^D}\Big\{\frac{-1}{8(D-1)^2}\Big(\frac{4}{y}\Big)^{D-2} + \dots\Big\}$$
$$\mathcal{F}_2(y) = \frac{\kappa^2 H^{2D-4}\Gamma^2(\frac{D}{2})}{(4\pi)^D}\Big\{\frac{-1}{4(D-3)(D-2)(D-1)(D+1)}\Big(\frac{4}{y}\Big)^{D-2} + \dots\Big\}$$

- Add zero in the form of the identity

$$\Big[\Box - \frac{D}{2}\Big(\frac{D}{2}-1\Big)H^2\Big]\Big(\frac{4}{y}\Big)^{\frac{D}{2}-1} - \frac{(4\pi)^{\frac{D}{2}}\, i\delta^D(x-x')}{\Gamma(\frac{D}{2}-1)H^{D-2}\sqrt{-\overline{g}}} = 0\ .$$

- Then

$$\Big(\frac{4}{y}\Big)^{D-2} = -\Big[\frac{\Box}{H^2}-2\Big]\Big\{\frac{4}{y}\ln\Big(\frac{y}{4}\Big)\Big\} - \frac{4}{y} + O(D-4) + \frac{2(4\pi)^{\frac{D}{2}}\, i\delta^D(x-x')/\sqrt{-\overline{g}}}{(D-4)(D-3)\Gamma(\frac{D}{2}-1)H^D}$$

nonlocal finite term — local divergent term

Renormalizing de Sitter result

- Add the counterterms to subtract the divergences off:

$$-i\Big[{}^{\mu\nu}\Delta\Sigma^{\rho\sigma}\Big](x;x') = \sqrt{-g}\Big[2c_1\kappa^2\mathcal{P}^{\mu\nu}\mathcal{P}^{\rho\sigma} + 2c_2\kappa^2\overline{g}^{\alpha\kappa}\overline{g}^{\beta\lambda}\overline{g}^{\gamma\theta}\overline{g}^{\delta\phi}\mathcal{P}^{\mu\nu}_{\alpha\beta\gamma\delta}\mathcal{P}^{\rho\sigma}_{\kappa\lambda\theta\phi}$$
$$-c_3\kappa^2H^2\mathcal{D}^{\mu\nu\rho\sigma} + c_4\kappa^2H^4\sqrt{-g}\left[\frac{1}{4}\overline{g}^{\mu\nu}\overline{g}^{\rho\sigma} - \frac{1}{2}\overline{g}^{\mu(\rho}\overline{g}^{\sigma)\nu}\right]\Big]i\delta^D(x-x')\ .$$

- The fully renormalized graviton self-energy for de Sitter is :

$$-i\Big[{}^{\mu\nu}\Sigma^{\rho\sigma}_{\rm ren}\Big](x;x') = \lim_{D\to4}\Big\{-i\Big[{}^{\mu\nu}\Sigma^{\rho\sigma}\Big](x;x') - i\Big[{}^{\mu\nu}\Delta\Sigma^{\rho\sigma}\Big](x;x')\Big\}\ ,$$
$$= \sqrt{-\overline{g}(x)}\,\mathcal{P}^{\mu\nu}(x)\sqrt{-\overline{g}(x')}\,\mathcal{P}^{\rho\sigma}(x')\Big[\mathcal{F}_{0R}(y)\Big]$$
$$+2\sqrt{-\overline{g}(x)}\,\mathcal{P}^{\mu\nu}_{\alpha\beta\gamma\delta}(x)\sqrt{-\overline{g}(x')}\mathcal{P}^{\rho\sigma}_{\kappa\lambda\theta\phi}(x')\Big[\mathcal{T}^{\alpha\kappa}\mathcal{T}^{\beta\lambda}\mathcal{T}^{\gamma\theta}\mathcal{T}^{\delta\phi}\mathcal{F}_{2R}(y)\Big]\ .$$

where $\mathcal{F}_{0R} = \frac{\kappa^2H^4}{(4\pi)^4}\Big\{\frac{\Box}{H^2}\Big[\frac{1}{72}\times\frac{4}{y}\ln\Big(\frac{y}{4}\Big)\Big]+\cdots\Big\}$, $\mathcal{F}_{2R} = \frac{\kappa^2H^4}{(4\pi)^4}\Big\{\frac{\Box}{H^2}\Big[\frac{1}{240}\times\frac{4}{y}\ln(\Big(\frac{y}{4}\Big)\Big]+\cdots\Big\}$

next pages

Note 1: The leading terms agree with the corresponding flat results.

Note 2: $\mathcal{F}_{0R}$ and $\mathcal{F}_{2R}$ are the first fully renormalized results for the graviton structure functions on de Sitter.

SP & Woodard 1101.5804

Spin zero structure function

$$
\begin{aligned}
\mathcal{F}_{0R} = \frac{\kappa^2 H^4}{(4\pi)^4} \Bigg\{ & \frac{\square}{H^2}\left[\frac{1}{72} \times \frac{4}{y} \ln\Big(\frac{y}{4}\Big)\right] - \frac{1}{12} \times \frac{4}{y} \ln\Big(\frac{y}{4}\Big) + \frac{1}{72} \times \frac{4}{y} + \frac{1}{6} \ln^2\Big(\frac{y}{4}\Big) \\
& + \frac{1}{45} \times \frac{4}{4-y} \ln(\frac{y}{4}) - \frac{1}{45} \ln(\frac{y}{4}) + \frac{43}{216} \times \frac{4}{4-y} - \frac{5}{6} \times \frac{y}{4} \ln(1 - \frac{y}{4}) \\
& + \frac{7}{90} \times \frac{4}{y} \ln(1 - \frac{y}{4}) - \frac{1}{20} \ln(1 - \frac{y}{4}) - \frac{7(12\pi^2 + 265)}{540} \times \frac{y}{4} \\
& + \frac{84\pi^2 - 131}{1080} - \frac{1}{3} \times \frac{y}{4} \ln^2\Big(\frac{y}{4}\Big) + \frac{4}{9} \times \frac{y}{4} \ln\Big(\frac{y}{4}\Big) \\
& - \frac{1}{30}(2-y)\left[7\mathrm{Li}_2(1 - \frac{y}{4}) - 2\mathrm{Li}_2(\frac{y}{4}) + 5\ln(1 - \frac{y}{4})\ln(\frac{y}{4})\right] \Bigg\} .
\end{aligned}
$$

Spin two structure function

$$
\begin{aligned}
\mathcal{F}_{2R} = \frac{\kappa^2 H^4}{(4\pi)^4}\Bigg\{ & \frac{\square}{H^2}\Big[\frac{1}{240}\times\frac{4}{y}\ln\big(\big(\frac{y}{4}\big)\Big] + \frac{3}{40}\times\frac{4}{y}\ln\big(\frac{y}{4}\big) - \frac{11}{48}\times\frac{4}{y} + \frac{1}{4}\ln^2\big(\frac{y}{4}\big) - \frac{119}{60}\ln\big(\frac{y}{4}\big) \\
& + \frac{4096}{(4y-y^2-8)^4}\Big[\Big[-\frac{47}{15}\big(\frac{y}{4}\big)^8 + \frac{141}{10}\big(\frac{y}{4}\big)^7 - \frac{2471}{90}\big(\frac{y}{4}\big)^6 + \frac{34523}{720}\big(\frac{y}{4}\big)^5 \\
& - \frac{132749}{1440}\big(\frac{y}{4}\big)^4 + \frac{38927}{320}\big(\frac{y}{4}\big)^3 - \frac{10607}{120}\big(\frac{y}{4}\big)^2 + \frac{22399}{720}\big(\frac{y}{4}\big) - \frac{3779}{960}\Big]\frac{4}{4-y} \\
& + \Big[\frac{193}{30}\big(\frac{y}{4}\big)^4 - \frac{131}{10}\big(\frac{y}{4}\big)^3 + \frac{7}{20}\big(\frac{y}{4}\big)^2 + \frac{379}{60}\big(\frac{y}{4}\big) - \frac{193}{120}\Big]\ln(2-\frac{y}{2}) \\
& + \Big[-\frac{14}{15}\big(\frac{y}{4}\big)^5 - \frac{1}{5}\big(\frac{y}{4}\big)^4 + \frac{19}{2}\big(\frac{y}{4}\big)^3 - \frac{889}{60}\big(\frac{y}{4}\big)^2 + \frac{143}{20}\big(\frac{y}{4}\big) - \frac{13}{20} - \frac{7}{60}\big(\frac{4}{y}\big)\Big]\ln(1-\frac{y}{4}) \\
& + \Big[-\frac{476}{15}\big(\frac{y}{4}\big)^9 + 160\big(\frac{y}{4}\big)^8 - \frac{5812}{15}\big(\frac{y}{4}\big)^7 + \frac{8794}{15}\big(\frac{y}{4}\big)^6 - \frac{18271}{30}\big(\frac{y}{4}\big)^5 + \frac{54499}{120}\big(\frac{y}{4}\big)^4 \\
& \quad - \frac{59219}{240}\big(\frac{y}{4}\big)^3 + \frac{1917}{20}\big(\frac{y}{4}\big)^2 - \frac{1951}{80}\big(\frac{y}{4}\big) + \frac{367}{120}\Big]\frac{4}{4-y}\ln(\frac{y}{4}) \\
& + \Big[4\big(\frac{y}{4}\big)^7 - 12\big(\frac{y}{4}\big)^6 + 20\big(\frac{y}{4}\big)^5 - 20\big(\frac{y}{4}\big)^4 + 15\big(\frac{y}{4}\big)^3 - 7\big(\frac{y}{4}\big)^2 + \big(\frac{y}{4}\big)\Big]\frac{4-y}{4}\ln^2(\frac{y}{4}) \\
& + \Big[\frac{367}{30}\big(\frac{y}{4}\big)^4 - \frac{4121}{120}\big(\frac{y}{4}\big)^3 + \frac{237}{16}\big(\frac{y}{4}\big)^2 + \frac{1751}{240}\big(\frac{y}{4}\big) - \frac{367}{120}\Big]\ln(\frac{y}{2}) \\
& + \frac{1}{64}(y^2-8)\Big[4(2-y)-(4y-y^2)\Big]\Big[\frac{1}{5}\mathrm{Li}_2(1-\frac{y}{4}) + \frac{7}{10}\mathrm{Li}_2(\frac{y}{4})\Big]\Big]\Bigg\}\ .
\end{aligned}
$$

Solving the quantum-corrected linearized Einstein equation

- Use the renormalized self-energy for the quantum correction term:

$$\sqrt{-g}\,\mathcal{D}^{\mu\nu\rho\sigma}h_{\rho\sigma}(x) - \int d^4x' \left[{}^{\mu\nu}\Sigma^{\rho\sigma}_{\rm ren}\right](x;x')h_{\rho\sigma}(x') = \frac{1}{2}\kappa\sqrt{-g}\,T^{\mu\nu}_{\rm lin}(x)\ ,$$

- Only know the self-energy at one loop order (at order $\kappa^2 = 16\pi G$), solve it perturbatively:

$$h_{\mu\nu}(x) = h^{(0)}_{\mu\nu}(x) + \kappa^2 h^{(1)}_{\mu\nu}(x) + O(\kappa^4)\ , \quad \left[{}^{\mu\nu}\Sigma^{\rho\sigma}_{\rm ren}\right](x;x') = \kappa^2\left[{}^{\mu\nu}\Sigma^{\rho\sigma}_{1}\right](x;x') + O(\kappa^4)\ .$$

- The corresponding one loop correction is

$$\int d^4x' \left[{}^{\mu\nu}\Sigma^{\rho\sigma}_{1}\right](x;x')h^{(0)}_{\rho\sigma}(x') = i\int d^4x'\sqrt{-g(x)}\,\mathcal{P}^{\mu\nu}(x)\sqrt{-g(x')}\,\mathcal{P}^{\rho\sigma}(x')\Big\{\mathcal{F}_0\Big\}h^{(0)}_{\rho\sigma}(x')$$
$$+2i\int d^4x'\sqrt{-g(x)}\,\mathcal{P}^{\mu\nu}_{\alpha\beta\gamma\delta}(x)\sqrt{-g(x')}\,\mathcal{P}^{\rho\sigma}_{\kappa\lambda\theta\phi}(x')\Big\{\mathcal{T}^{\alpha\kappa}\mathcal{T}^{\beta\lambda}\mathcal{T}^{\gamma\theta}\mathcal{T}^{\delta\phi}\mathcal{F}_2\Big\}h^{(0)}_{\rho\sigma}(x')\ .$$

Solving the quantum-corrected linearized Einstein eqn for dynamical gravitons

- For dynamical gravitons, that is for zero stress-energy $T^{\mu\nu}_{\rm lin}(x) = 0$:

$$h^{(0)}_{\rho\sigma}(x) = \epsilon_{\rho\sigma}(\vec{k})a^2 u(\eta, k)e^{i\vec{k}\cdot\vec{x}} \ , \quad u(\eta, k) = \frac{H}{\sqrt{2k^3}}\Big[1 - \frac{ik}{Ha}\Big]\exp\Big[\frac{ik}{Ha}\Big], \quad 0 = \epsilon_{0\mu} = k_i\epsilon_{ij} = \epsilon_{jj} \quad \text{and} \quad \epsilon_{ij}\epsilon^*_{ij} = 1$$

- The result turns out to be zero!

$$\int d^4x' \left[{}^{\mu\nu}\Sigma^{\rho\sigma}_1\right](x; x')h^{(0)}_{\rho\sigma}(x') \longrightarrow 0$$

"Inflationary Scalars Don't Affect Gravitons at One Loop," SP and Woodard, arXiv: 1109.4187

- Gravitons interact with MMC scalar only through their kinetic energies which are redshifted. (Gravitons couple minimally only to differentiated scalars.)

Noncovariant representation of the graviton self-energy

- A noncovariant representation of the conformally rescaled graviton field
  - Noncovariant Rep includes de Sitter breaking basis vectors in terms of $u(x;x') \equiv \ln(aa')$
  - Covariant Rep: $g_{\mu\nu} = \bar{g}_{\mu\nu} + \kappa\chi_{\mu\nu}$ vs Noncovariant Rep: $g_{\mu\nu} = a^2(\eta_{\mu\nu} + \kappa h_{\mu\nu})$
  - Covariant Rep: 5 basis tensors - 3 relations = 2 structure ftns vs Noncovariant Rep: 14 - 10 = 4

$$
\begin{aligned}
-i\Big[{}^{\mu\nu}\Sigma^{\rho\sigma}\Big](x;x') &= \mathcal{F}^{\mu\nu}(x) \times \mathcal{F}^{\rho\sigma}(x')\Big[F_0(x;x')\Big] \\
&+\mathcal{G}^{\mu\nu}(x) \times \mathcal{G}^{\rho\sigma}(x')\Big[G_0(x;x')\Big] + \mathcal{F}^{\mu\nu\rho\sigma}\Big[F_2(x;x')\Big] + \mathcal{G}^{\mu\nu\rho\sigma}\Big[G_2(x;x')\Big]
\end{aligned}
$$

Leonard, SP, Prokopec and Woodard, arXiv: 1403.0896

- Confirmed the previous result: no effect on dynamical gravitons from MMC scalars

- Much simpler than the de Sitter covariant representation, so can be easily employed to study the force of gravity

Structure functions in the noncovariant representation

$$
\begin{aligned}
F_{0R}(x;x') &= \frac{\kappa^2 (aa'H^2)^2}{2304\pi^4}\Big\{\frac{\partial^2}{2(aa'H^2)^2}\Big[\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\Big] - \frac{6}{y} + 6 + \Big[-\frac{2}{y} + 6 - \frac{2}{4-y}\Big]\ln\Big(\frac{y}{4}\Big) + \frac{3}{2}(2-y)\Psi(y)\Big\} \\
F_{2R}(x;x') &= \frac{\kappa^2 (H^2aa')^2}{(4\pi)^4}\Big\{\frac{\partial^2}{30(H^2aa')^2}\Big[\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\Big] + \frac{2}{3}\Big[\frac{1}{y} - \frac{1}{4-y}\Big]\ln\Big(\frac{y}{4}\Big) - \frac{1}{3}\Psi(y)\Big\} \\
G_0(x;x') &= 0 \\
G_2(x;x') &= \frac{\kappa^2 (H^2aa')^2}{(4\pi)^4}\Big\{-2 + \frac{8}{3}\frac{\ln(\frac{y}{4})}{(4-y)} + \frac{2}{3}\Psi(y)\Big\}
\end{aligned}
$$

where

$$
\Psi(y) \equiv \frac{1}{2}\ln^2\Big(\frac{y}{4}\Big) - \ln\Big(1-\frac{y}{4}\Big)\ln\Big(\frac{y}{4}\Big) - \mathrm{Li}_2\Big(\frac{y}{4}\Big)
$$

Schwinger-Keldysh (in-in = closed time path integral) formalism

In-Out

- free vacuum → ends up the same way
- future contributes; not causal
- gives complex results for the matrix elements of Hermitian operators
- suitable for scattering experiments in flat space

In-In

- start from an initial state
  e.g., Bunch-Davies vacuum
  (free vacuum at a finite time)
  →
  unknown state in the asymptotic future
- gives causal & real results
- answers the question:

  What happens when the universe is released from a prepared state at a finite time and allowed to evolve as it will?

Convert the in-out self-energy to the in-in self-energy

- The linearized Schwinger-Keldysh effective field equation is obtained by replacing the in-out self-energy with its retarded counterpart

$$\left[{}^{\mu\nu}\Sigma^{\rho\sigma}\right](x;x') \rightarrow \left[{}^{\mu\nu}\Sigma^{\rho\sigma}_{\rm Ret}\right](x;x') \equiv \left[{}^{\mu\nu}\Sigma^{\rho\sigma}_{++}\right](x;x') + \left[{}^{\mu\nu}\Sigma^{\rho\sigma}_{+-}\right](x;x') \; .$$

For reviews see: Chou, Su, Hao and Yu, Phys. Rept. 118, 1 (1985); Jordan PRD 33, 444 (1986)

Structure functions in the Schwinger-Keldysh formalism

This converts the nonzero structure functions in to the retarded ones of the Schwinger-Keldysh formalism,

$$F_0^{(1)}(x;x') = \frac{i\kappa^2}{576\pi^3}\Bigg\{\frac{\partial^4 - 4H^2 aa'\partial^2}{16}\Bigg[\Big[\ln\Big(\frac{-y}{4aa'}\Big) - 1\Big]\Theta\Bigg] - \frac{1}{4}H^2 aa' \ln(aa')\partial^2\Theta$$
$$+ H^4 a^2 a'^2 \Big[3 - \frac{1}{4-y} + \frac{3}{4}(2-y)\ln\Big(\frac{-y}{4-y}\Big)\Big]\Theta\Bigg\},$$

$$F_2^{(1)}(x;x') = \frac{i\kappa^2}{64\pi^3}\Bigg\{\frac{\partial^4 + 20H^2 aa'\partial^2}{240}\Bigg[\Big[\ln\Big(\frac{-y}{4aa'}\Big) - 1\Big]\Theta\Bigg] + \frac{H^2 aa' \ln(aa')}{12}\,\partial^2\Theta$$
$$+ H^4 a^2 a'^2 \Big[\frac{-\frac{1}{3}}{4-y} - \frac{1}{6}\ln\Big(\frac{-y}{4-y}\Big)\Big]\Theta\Bigg\},$$

$$G_2^{(1)}(x;x') = \frac{i\kappa^2}{64\pi^3}\Bigg\{H^4 a^2 a'^2 \Big[\frac{\frac{4}{3}}{4-y} + \frac{1}{3}\ln\Big(\frac{-y}{4-y}\Big)\Big]\Theta\Bigg\}.$$

(Note that $G_0^{(1)}(x;x')$ is zero for the MMC scalar at one loop.)

One loop corrections from MMC scalar to the Newtonian potential

- For the linearized response to a static point mass M

$$h_{00}^{(0)}(x) = \frac{2GM}{a\|\vec{x}\|} = -2\Phi^{(0)} \ , \ h_{0i}^{(0)}(x) = 0 \ , \ h_{ij}^{(0)}(x) = \frac{2GM}{a\|\vec{x}\|}\delta_{ij} = -2\Psi^{(0)}\delta_{ij} \ , \quad T^{\mu\nu} = Ma\delta^3(\vec{x})\delta_\mu^0\delta_\nu^0$$

in flat space $h_{00}^{(0)}(x) = \frac{2GM}{\|\vec{x}\|} \ , \ h_{0i}^{(0)}(x) = 0 \ , \ h_{ij}^{(0)}(x) = \frac{2GM}{\|\vec{x}\|}\delta_{ij} \ , \quad T_{\mu\nu} = M\delta^3(\vec{x})\delta_\mu^0\delta_\nu^0$

- One loop corrections

$$h_{00}^{(1)}(x) \equiv f_1 \ , \ h_{0i}^{(1)}(x) = 0 \ , \ h_{ij}^{(1)}(x) \equiv f_3\delta_{ij}$$

The solutions are

$$\begin{aligned} f_1(x) &= -\frac{\kappa^2 M}{2a^2} S_0^1(x) + \frac{\kappa^2 M}{a^2}\Big[-\frac{2}{3} + \nabla^{-2}(\partial_0^2 - aH\partial_0)\Big] S_2^1(x) = -2\Phi^{(1)} \\ f_3(x) &= \frac{\kappa^2 M}{2a^2} S_0^1(x) + \frac{\kappa^2 M}{a^2}\Big[-\frac{1}{3} - \nabla^{-2} aH\partial_0\Big] S_2^1(x) = -2\Psi^{(1)} \end{aligned}$$

where $\nabla^{-2} f(\eta, \vec{x}) = -[1/(4\pi)] \int d^3x' f(\eta, \vec{x}')/\|\vec{x} - \vec{x}'\|$

$$\begin{aligned} S_0^1(x) &= \int \frac{d\eta'}{a(\eta')} [iF_0^1(x, x')]_{\vec{x}'=0} \ , \\ S_2^1(x) &= \int \frac{d\eta'}{a(\eta')} \Big[F_2^1(x; x') + \frac{1}{2} G_2^1(x; x')\Big]_{\vec{x}'=0} \end{aligned}$$

One loop corrections from MMC scalar to the Newtonian potential

- In flat space

$$\Phi_{flat} = -\frac{GM}{r}\left\{1 + \frac{\hbar}{20\pi c^3}\frac{G}{r^2} + O\Big(G^2\Big)\right\}$$
$$\Psi_{flat} = -\frac{GM}{r}\left\{1 - \frac{\hbar}{60\pi c^3}\frac{G}{r^2} + O\Big(G^2\Big)\right\}$$

SP and Woodard, arXiv:1007.2662, Marunovic and Prokopec, arXiv: 1101.5059

Not the first for this result, but the first to solve the effective field eqns using the Schwinger-Keldysh or in-in formalism. The previous calculations e.g. Radkowski, 1970, Donoghue 1993, ... were done by the scattering amplitude technique.

- In de Sitter space

$$\Phi_{dS} = -\frac{GM}{ar}\left\{1 + \frac{\hbar}{20\pi c^3}\frac{G}{(ar)^2} + \frac{\hbar G H^2}{\pi c^5}\Big[-\frac{1}{30}\ln(a) - \frac{3}{10}\ln\Big(\frac{Har}{c}\Big)\Big] + O\Big(G^2H^4\Big)\right\}$$
$$\Psi_{dS} = -\frac{GM}{ar}\left\{1 - \frac{\hbar}{60\pi c^3}\frac{G}{(ar)^2} + \frac{\hbar G H^2}{\pi c^5}\Big[-\frac{1}{30}\ln(a) - \frac{3}{10}\ln\Big(\frac{Har}{c}\Big) + \frac{2}{3}\frac{Har}{c}\Big] + O\Big(G^2H^4\Big)\right\}$$

the de Sitterized version of the flat space correction + intrinsic de Sitter correction

SP, Prokopec and Woodard, arXiv:1510.03352

- One loop correction to the gravitational slip differs from zero in both flat and de Sitter space: $\Phi^{(1)} - \Psi^{(1)} \neq 0$

Another example of IR log correction

• Another example of ln(a) correction: One loop corrected conformally coupled scalar mode functions in de Sitter

$$u_{\rm CC} \sim \frac{1}{\sqrt{2k}}\left\{\frac{1}{a} + GH^2\left[\frac{42323}{2^5\cdot 15\pi}\ \ln(a)\ -\ \frac{530953}{2^6\cdot 15\pi}\ -\left(\Delta c_4 - \frac{3}{4}\right)\frac{\ln(a)}{a}\right] + \mathcal{O}(G^2H^4)\right\}$$

1708.01831 Boran, Kahya and SP

• Even though the loop counting parameter GH^2 is extremely small, the *ln(a)* will grow and eventually overcome it, then perturbation theory will break down; Need a nonperturbative resummation method such as Starobinsky's stochastic technique, but not available yet…

Open problems

Doable though tedious…

- Compute quantum corrections to graviton self-energy from the non-minimally coupled scalar (regarding Higgs inflation, etc.)
- Extend the computations to a more general FLRW background, not just for de Sitter
- Computerise the computations; Develop algebraic computer codes; Machine learning for the loop calculations?

Challenging…

- How to re-sum the IR logs?
- How to connect to observations: e.g., Is the size of anisotropy (given in the difference of the two potentials) detectable?
- Hint towards deriving nonlocal gravity (nonlocal quantum effective action) from first principles?

Thank you for listening!