Massive Gravity on Cosmological Backgrounds

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Quadratic Massive gravity in Minkowski spacetime

The quadratic action for metric perturbations $g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu}$

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\Big[-R + \underbrace{\frac{m_g^2}{4}\left(h^2 - h_{\mu\nu}h^{\mu\nu}\right)}_{\text{Fierz-Pauli (FP) mass term (1939)}}\Big]$$

- is ghost-free
- propagates 5 massive degrees of freedom

PROBLEMS

- breaks the diffeomorphism invariance of general relativity
- leads to unacceptable observational consequences (van Dam, Veltman, Zakharov 1970)
- propagates a ghost around any other backgrounds $\tilde{g}^{\mu\nu} \neq \eta^{\mu\nu}$

SOLUTIONS

- introduction of four Stückelberg (Higgs) fields
- non-linear modifications to FP mass term (Vainshtein 1972)
- define $h^{\mu\nu} \equiv g^{\mu\nu} - \tilde{g}^{\mu\nu}$

Stückelberg trick / Higgs massive gravity

Any theory of massive gravity can be represented as Einstein gravity interacting with four scalar fields ϕ^A with $A = 0, 1, 2, 3$:

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left[-R + \frac{m_g^2}{4}\mathcal{L}_{FP}\left(\phi^A, g^{\mu\nu}\right)\right]$$

The interaction term $\mathcal{L}_{FP}$ is a function of a diffeomorphism invariant scalar

$$\bar{h}^{AB} = g^{\mu\nu}\partial_\mu\phi^A\partial_\nu\phi^B - \eta^{AB}$$

such that the quadratic action reduces to the FP mass term around the backgrounds

$$g^{\mu\nu} = \eta^{\mu\nu}, \qquad \phi^A = x^\mu\delta_\mu^A.$$

The minimal such generalization of the Fierz-Pauli action is

$$\boxed{\mathcal{L}_{FP} = \bar{h}^2 - \bar{h}^A_B\bar{h}^B_A}$$

- diffeomorphism invariant
- exhibits Vainshtein mechanism
- propagates 6 degrees of freedom
- strongly coupled at very low scale

Outline

1. Vainshtein Mechanism and Strong Coupling $\to$ Λ_3 theories
2. Cosmological Solutions with Flat Reference Metric
3. Massive Gravity with Curved Reference Metric

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Gravitational field from a massive source

We focus on the scalar metric and matter perturbations in the longitudinal gauge

METRIC

$$g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu}$$

$$h_{00} = 2\phi$$
$$h_{0i} = 0$$
$$h_{ij} = 2\psi\delta_{ij}$$

SCALAR FIELDS

$$\phi^A = x^\mu \delta^A_\mu + \chi^A$$

$$\chi^0 = \chi^0$$
$$\chi^i = \partial_i \pi = \pi_{,i}$$

Hence the metric takes the form:

$$ds^2 = (1+2\phi)\,dt^2 - (1-2\psi)\,\delta_{ik} dx^i dx^k$$

In General Relativity, in the presence of a static spherically symmetric source M_0

$$\boxed{\psi = \phi, \qquad \Delta\phi = T^{00}/2 \quad \Rightarrow \quad \phi \sim -\frac{M_0}{r}}$$

Gravitational field from a massive source

In Fierz-Pauli massive gravity

$$\mathcal{L}_{FP} = \bar{h}^2 - \bar{h}^A_B \bar{h}^B_A$$

the linearized constraints and equations of motion are:

$$\Delta\psi = \frac{m_g^2}{2}(3\psi + \Delta\pi) + \frac{T^{00}}{2} \qquad \Delta\left(\psi - \phi - m_g^2\pi\right) = 0,$$

$$\Delta\chi^0 = 0 \qquad \Delta\left(2\psi - \phi\right) = 0$$

This implies:

$$\boxed{\psi = \phi/2 \qquad \Rightarrow \qquad \phi = -\frac{4}{3}\frac{M_0}{r}e^{-m_g r}}$$

even when $m_g \to 0 \Rightarrow$ vDVZ discontinuity!

(idea of the picture: Creminelli 2011)

Vainshtein scale of the nonlinear expansion

Beyond linear order in matter perturbations the equations are modified:

$$2\psi - \phi + O(1)\,\partial^4\pi^2 = 0$$
$$\psi - \phi - m_g^2\pi = 0$$

where $\pi_{,ik}$, $\Delta\pi \to \partial^2\pi \ll 1$. In the spherically symmetric case $\partial^n \sim r^{-n}$

$$\Rightarrow \psi + m_g^2\pi + O(1)\,r^{-4}\pi^2 \simeq 0$$

At the Vainshtein scale all the terms become comparable:

$$\psi \sim m_g^2\pi \sim O(1)\,r^{-4}\pi^2.$$

For $\psi \sim -M_0/r$ this gives the well known result for the Vainshtein scale (1972)

$$\boxed{R_V \simeq \left(\frac{M_0}{M_P^2 m_g^4}\right)^{1/5}}$$

Smooth limit to General Relativity

Above the Vainshtein scale

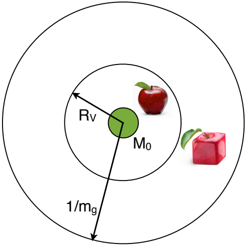
$$r \gg R_V$$

$$\psi + m_g^2 \pi + \cancel{O(1)\, r^{-4} \pi^2} \simeq 0$$

$$\psi - \phi = -\psi \left[1 - O(1) \left(\frac{R_V}{r} \right)^5 \right]$$

$$\left(\Delta - m_g^2 \right) \phi = \frac{4}{3} \left(\frac{T^{00}}{2} \right)$$

Yukawa decay!



Below the Vainshtein scale

$$r \ll R_V$$

$$\psi + \cancel{m_g^2 \pi} + O(1)\, r^{-4} \pi^2 \simeq 0$$

$$\psi - \phi = O(1) \psi \left(\frac{r}{R_V} \right)^{5/2}$$

$$\Delta \phi = \frac{T^{00}}{2}$$

GR restored up to corrections $\frac{\delta\phi}{\phi} \sim \left(\frac{r}{R_V} \right)^{5/2}$

Question remains:
Does a single
everywhere non-singular solution
matching both asymptotics exist?
YES!

Babichev, Deffayet, Ziour (2010)

Strong coupling

In terms of the canonically normalized fields $\hat{\phi}$, $\hat{\psi}$ the dominant terms in the cubic action for the field $\hat{\pi}$ become

$$S_{\hat{\pi}} \supset \int d^4x \left\{ \Delta\hat{\pi}(2\hat{\psi} - \hat{\phi}) + \frac{1}{2}\frac{1}{M_P m_g^4}\left(\Delta\hat{\pi}\hat{\pi}_{,ik}\hat{\pi}_{,ik} - \hat{\pi}_{,ik}\hat{\pi}_{,kj}\hat{\pi}_{,ji}\right) + \dots \right\}$$

$\Rightarrow$ The theory becomes strongly coupled above the scale $\Lambda_5 = (M_P m_g^4)^{1/5}$!

Arkani-Hamed, Georgi, Schwartz (2003)

$\Rightarrow$ The full quantum theory is needed to describe physics around spherically symmetric sources below the radius

$$r_* = \left(\frac{M_0}{M_P}\right)^{1/3}\frac{1}{\Lambda_5} \gg R_V = \left(\frac{M_0}{M_P}\right)^{1/5}\frac{1}{\Lambda_5}!$$

No region of applicability of the Vainshtein mechanism?!

Way out: Raise the energy cutoff by adding appropriate counterterms which eliminate the self-coupling. In this way the Vainshtein radius can be lowered order by order!

Λ_3 THEORIES

For a Langrangian with the highest self coupling $\mathcal{L} \supset (\partial^2\pi)^n$, the corresponding Vainshtein radius is

$$R_V = \left(\frac{M_0}{M_P^2 m_g^{\frac{2(n-1)}{n-2}}} \right)^{\frac{n-2}{3n-4}} , \quad \Lambda_{(n)} = \left(M_P m_g^{\frac{2(n-1)}{n-2}} \right)^{\frac{n-2}{3n-4}}$$

and the corrections to the gravitational potential within $r \ll R_V$ radius are

$$\frac{\delta\phi}{\phi} \sim \left(\frac{r}{R_V} \right)^{\frac{3n-4}{n-1}}$$

LA, Chamseddine, Mukhanov (2010)

The minimal possible scale at $n \to \infty$: $\boxed{R_V^\infty = \frac{1}{\Lambda_3} \left(\frac{M_0}{M_P} \right)^{1/3} , \; \Lambda_3 = (M_P m_g^2)^{1/3}}$

$\Rightarrow$ Λ_3 is reached after the resummation!

Strong coupling scale in Λ_3 theories: $r_* = \frac{1}{\Lambda_3} \ll R_V$

$\Rightarrow$ Vainshtein mechanism works!

dRGT Resummation of massive gravity

de Rham, Gabadadze, Tolley (2010)

Massive gravity can be resummed into infinite series

$$\mathcal{K}^\mu_\nu = \left(\sqrt{g^{-1}f}\right)^\mu_\nu - \delta^\mu_\nu$$

where $f_{\mu\nu}$ is the flat reference metric:

$$f_{\mu\nu} = \partial_\mu\phi^A\partial_\nu\phi^B\eta_{AB}$$

and the square root matrix is defined as:

$$\left(\sqrt{g^{-1}f}\right)^\mu_\lambda \left(\sqrt{g^{-1}f}\right)^\lambda_\nu = g^{\mu\lambda}f_{\lambda\nu}$$

The resulting non-linear action is

$$S_{\text{dRGT}} = -\frac{1}{2}\int d^4x\sqrt{-g}R + m^2\int d^4x\sqrt{-g}\left[\mathsf{e}_2(\mathcal{K}) + \alpha_3\mathsf{e}_3(\mathcal{K}) + \alpha_4\mathsf{e}_4(\mathcal{K})\right]$$

a finite sum of the characteristic polynomials

$$\mathsf{e}_2(\mathcal{K}) = \frac{1}{2}\left([\mathcal{K}]^2 - [\mathcal{K}^2]\right)$$

$$\mathsf{e}_3(\mathcal{K}) = \frac{1}{6}\left([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]\right)$$

$$\mathsf{e}_4(\mathcal{K}) = \frac{1}{24}\left([\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 3[\mathcal{K}^2]^2 + 8[\mathcal{K}][\mathcal{K}^3] - 6[\mathcal{K}^4]\right)$$

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Rewriting the Action

Hassan, Rosen (2011)

The action can be rewritten in a slightly simpler form by noticing

$$\det\left(\mathbb{I}+\mathcal{K}\right)=\sum_{n=0}^{4}\mathsf{e}_n(\lambda_1,\dots,\lambda_4)=\sum_{n=0}^{4}\mathsf{e}_n(\mathcal{K}),$$

where λ_i are the eigenvalues of the matrix $\mathcal{K}$.

$\Rightarrow$ The action has a structure of a deformed determinant.

Due to the definition: $\mathcal{K}=\sqrt{g^{-1}f}-\mathbb{I}$ the action can be rewritten

$$S=-\frac{1}{2}\int d^4x\,\sqrt{-g}\left[R-2m^2\sum_{n=0}^{3}\beta_n\mathsf{e}_n\left(\sqrt{g^{-1}f}\right)\right]+m^2\int d^4x\,\underbrace{\sqrt{-g}\beta_4\mathsf{e}_4\left(\sqrt{g^{-1}f}\right)}_{=\beta_4\sqrt{-\det f}}$$

The new coefficients can be expressed through the old coefficients $\beta_n=\beta_n(\alpha_i)$. In particular $\beta_0=6-4\alpha_3+\alpha_4\neq 0$.

Cosmological constant term for the reference metric: This can be dropped since the reference metric is non-dynamic

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Special Choice of Coefficients

There is a special choice of coefficients

$$\alpha_3 = \frac{2-C}{1-C}; \qquad \alpha_4 = \frac{3-3C+C^2}{(1-C)^2}$$

for which

- Schwarzschild-de Sitter solutions — Nieuwenhuizen (2011)
- static black hole solutions — Gruzinov, Mirbabayi (2011)
- de Sitter and FRW solutions — Nieuwenhuizen; Chamseddine, Volkov (2011)
- bigravity equations of motion decouple — Volkov (2011)
- the mass term takes the very simple form — (no one noticed)

$$S_{\text{mass}} = m^2 \int d^4x \sqrt{-g} \left[(1-C) + \frac{1}{C(1-C)^2} \det\left(\sqrt{g^{-1}f} - C\mathbb{I} \right) \right]$$

Square Root of the Isotropic Ansatz

We consider the isotropic metric

$$ds^2 = N^2(t,\, r)dt^2 - a^2(t,\, r)\left(dr^2 + r^2 d\Omega^2\right)$$

and take the most general spherically symmetric scalar field configuration

$$\phi^0 = f(r,\, t),$$
$$\phi^i = g(r,\, t)n^i, \qquad n^i \equiv (\sin\theta\,\cos\phi,\, \sin\theta\,\sin\phi,\, \cos\theta)$$

D'Amico et al. (2011)

where ϕ^A are the Stückelberg scalars of the reference metric $f_{\mu\nu} = \partial_\mu\phi^A\partial_\nu\phi^B\eta_{AB}$

What is a square root of a matrix?
For a diagonalizable $n \times n$ matrix A
it holds that $\boxed{A^k = PD^kP^{-1}}$
where P is the matrix of eigenvectors
and $D = \mathrm{diag}(\lambda_1, \ldots, \lambda_n)$.

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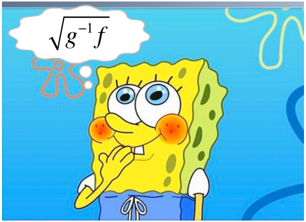

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Square Root for the Isotropic Ansatz

In this case the square root can be taken explicitly:

$$\left(\sqrt{g^{-1}f}\right)^{\mu}{}_{\nu} = \begin{pmatrix} \frac{1}{\sqrt{X}}\left[\frac{\dot f^2-\dot g^2}{N^2}+\sqrt{\lambda_1\lambda_2}\right] & -\frac{1}{\sqrt{X}}\frac{1}{N^2}\left(\dot g g' - \dot f f'\right) & 0 & 0 \\ \frac{1}{\sqrt{X}}\frac{1}{a^2}\left(\dot g g' - \dot f f'\right) & \frac{1}{\sqrt{X}}\left[\frac{-f'^2+g'^2}{a^2}+\sqrt{\lambda_1\lambda_2}\right] & 0 & 0 \\ 0 & 0 & g/ar & 0 \\ 0 & 0 & 0 & g/ar \end{pmatrix}$$

where

$$X = \left(\frac{\dot f}{N} + \mu\frac{g'}{a}\right)^2 - \left(\frac{\dot g}{N} + \mu\frac{f'}{a}\right)^2 \quad \text{with} \quad \mu = \mathrm{sgn}(g'\dot f - f'\dot g)$$

and $\lambda_{1,2}$ denote the eigenvalues of the upper 2×2 matrix and can be expressed as

$$\sqrt{\lambda_1} = \frac{1}{2}\left(\sqrt{X_+} + \sqrt{X_-}\right)$$
$$\sqrt{\lambda_2} = \frac{1}{2}\mu\left(\sqrt{X_+} - \sqrt{X_-}\right), \qquad X_\pm \equiv X_{\mu=\pm 1}$$

Equations of Motion

Varying the action with respect to the Stückelberg fields f and g yields the equations of motion

of the field f

$$\partial_t \left[a^2 r^2 \left(\frac{g}{ar} - C \right)^2 \left\{ \frac{1}{\sqrt{X}} (C - \sqrt{X}) \mu g' + \frac{C}{\sqrt{X}} \frac{a \dot{f}}{N} \right\} \right] -$$
$$-\partial_r \left[a N r^2 \left(\frac{g}{ar} - C \right)^2 \left\{ \frac{1}{\sqrt{X}} (C - \sqrt{X}) \mu \frac{a \dot{g}}{N} + \frac{C}{\sqrt{X}} f' \right\} \right] = 0,$$

and of the field g

$$\partial_t \left[a^2 r^2 \left(\frac{g}{ar} - C \right)^2 \left\{ \frac{1}{\sqrt{X}} (C - \sqrt{X}) \mu f' + \frac{C}{\sqrt{X}} \frac{a \dot{g}}{N} \right\} \right] -$$
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$$-2a^2 N r \left[C(C - \sqrt{X}) + \frac{\mu}{aN} \left(g' \dot{f} - f' \dot{g} \right) \right] = 0$$

The equation of motion for the field f is satisfied if $g = Car$.

The equation of g gives an equation for f which is especially simple on the solution.
However an explicit solution is not required for computing the $T^{(\phi)}_{\mu\nu}$.

Equations of Motion

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$$-\partial_r \left[a N r^2 \left(\frac{g}{ar} - C\right)^2 \left\{\frac{1}{\sqrt{X}}(C - \sqrt{X})\mu \frac{a\dot{g}}{N} + \frac{C}{\sqrt{X}} f'\right\}\right] = 0,$$

and of the field g

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Energy-Momentum Tensor

Variation of the action with respect to the metric leads to the *modified*

$$G_{\mu\nu} = T^{(\phi)}_{\mu\nu} + T^{(\mathrm{m})}_{\mu\nu}, \qquad 8\pi G_N \equiv 1$$

where $G_{\mu\nu}$ is the Einstein tensor, $T^{(\phi)}_{\mu\nu}$ is the effective stress tensor due to the mass term, and $T^{(\mathrm{m})}_{\mu\nu}$ is the *usual* matter tensor.

For any isotropic distribution of the matter the effective energy-momentum tensor on the solution $g = Car$ takes the form of a cosmological constant independently on the solution of the field f!

$$T^{(\phi)}_{tt} = -m^2 N^2 (1 - C)$$
$$T^{(\phi)}_{rr} = m^2 a^2 (1 - C)$$
$$T^{(\phi)}_{\theta\theta} = \frac{T^{(\phi)}_{\phi\phi}}{\sin^2\theta} = m^2 a^2 r^2 (1 - C)$$

for $C = \frac{3}{2}$ see D'Amico et al. (2011);
for arbitrary α_3, α_4 see Gratia, Hu, Wyman (2012)

Vanishing of Kinetic Terms

By careful inspection of the square root matrix in the case of isotropic ansatz

$$\left(\sqrt{g^{-1}f}\right)^{\mu}_{\ \nu} = \begin{pmatrix} a_{11}(\dot{f},\dot{g},\dots) & a_{12}(\dot{f},\dot{g},\dots) & 0 & 0 \\ a_{21}(\dot{f},\dot{g},\dots) & a_{22}(\dot{f},\dot{g},\dots) & 0 & 0 \\ 0 & 0 & g/ar & 0 \\ 0 & 0 & 0 & g/ar \end{pmatrix}$$

we see that all the time derivatives of the Stückelberg fields f and g enter only in the upper-left 2×2 matrix!

It is useful to split the action for the spherically symmetric scalar fields

$$S_{\rm mass} = m^2 \int d^4x \sqrt{-g} \left[(1-C) + \frac{1}{C(1-C)^2} \underbrace{\det\left(\sqrt{g^{-1}f} - C\mathbb{I}\right)}_{=\left(\frac{g}{ar}-C\right)^2 \det A} \right]$$

where $A \equiv \left(\sqrt{g^{-1}f} - C\mathbb{I}\right)_{2\times 2}$ contains all the time derivatives!

Vanishing of Kinetic Terms

The isotropic ansatz allows also for arbitrary isotropic scalar field perturbations

$$f(t,\,r) = f_0(t,\,r) + \delta f(t,\,r), \quad g(t,\,r) = g_0(t,\,r) + \delta g(t,\,r)$$

and spherically symmetric perturbations for example around the Friedmann solution

$$N^2(t,\,r) = 1 + 2\phi(t,\,r)$$
$$a^2(t,\,r) = a^2(t)\,[1 - 2\psi(t,\,r)]$$

Gratia, Hu, Wyman (2012)

We see however that on the scalar field solution $g_0 = Car$ the action for the perturbations $\delta g,\, \delta f$ becomes schematically

$$S_{\text{mass}} = m^2 \int d^4x\,\sqrt{-g}\left[(1-C) + \frac{1}{C(1-C)^2}\left(\frac{\delta g}{ar}\right)^2 \det A\left(\dot f_0 + \delta\dot f, \dot g_0 + \delta\dot g, \ldots\right)\right]$$

$\Rightarrow$ The isotropic scalar field perturbations have vanishing kinetic terms!

for more details: Koyama et al. (2011); Mukohyama et al. (2012)

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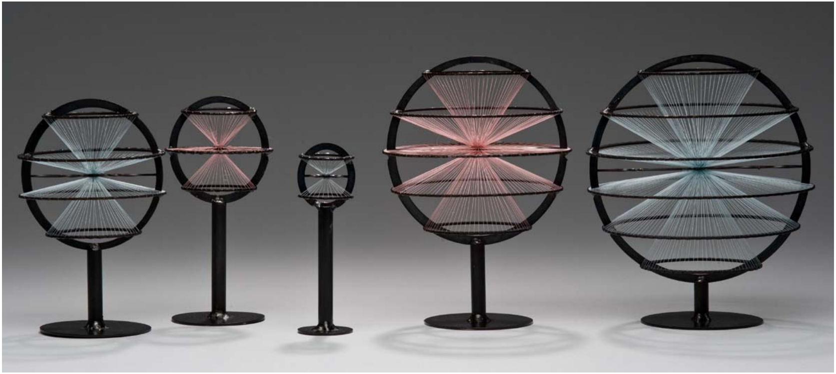
"Massive spin-2 particle" on curved background

What is it?



"Spin family", Julian Voss-Andreae (2009)

"Massive spin-2 particle" on curved background

- In Lorentz invariant spacetime there is a rigid definition of the mass of the particle:
$$m^2 \equiv -\eta_{\mu\nu} p^\mu p^\nu = E^2 - \vec{p}^2$$
$\Rightarrow$ Under the little group transformations a particle with spin j belongs to a representation of dimension $2j+1$.
$\Rightarrow$ Spin-2 particle has 5 degrees of freedom.

- On the other hand consider a free scalar field obeying $(\Box + m^2)\phi = 0$. Expanded in the Fourier series $\sum_k \phi_k e^{ikx}$ its modes satisfy the dispersion relation $-\omega^2 + \vec{k}^2 + m^2 = 0$.
$\Rightarrow$ "Alternative" definition of mass!

- Combine the two! $\Rightarrow$ Define a massive spin-2 particle on curved background such that
  - it has 5 degrees of freedom and
  - they all obey the same equation of motion $(\Box_g + m^2)\phi = 0$ and thus have equal dispersion relations

Non-Minkowski solutions of dRGT

We have seen that the Stückelberg scalars ϕ^A have their own effective EMT

$$T^{(\phi)}_{\mu\nu} \equiv \frac{2}{\sqrt{-g}}\frac{\delta S_{\text{mass}}}{\delta g^{\mu\nu}} = -\frac{m^2}{2} g_{\mu\nu}\left([\mathcal{K}]^2 - [\mathcal{K}^2]\right) + \frac{m^2}{2}\mathcal{K}^\alpha_\beta \frac{\delta \mathcal{K}^\lambda_\rho}{\delta g^{\mu\nu}}\left[\delta^\beta_\alpha \delta^\rho_\lambda - \delta^\rho_\alpha \delta^\beta_\lambda\right].$$

$\Rightarrow$ a whole zoo of different non-trivial solutions

$$\left\{\tilde{g}_{\mu\nu} \neq \eta_{\mu\nu}, \tilde{\phi}^A\right\}!$$

How do the metric excitations around these solutions behave?

Split the field $\mathcal{K}^\mu_\nu$ as $\mathcal{K}^\mu_\nu = \tilde{\mathcal{K}}^\mu_\nu + \delta\mathcal{K}^\mu_\nu$ with

$$\tilde{\mathcal{K}}^\mu_\nu = \delta^\mu_\nu - \sqrt{\tilde{g}^{\mu\lambda}\partial_\lambda\tilde{\phi}^A\partial_\nu\tilde{\phi}^B\eta_{AB}}$$

and observe that if $\tilde{\mathcal{K}}^\mu_\nu \neq 0 \Rightarrow$ the Fierz-Pauli structure is lost!

Demand that

$$\tilde{\mathcal{K}}^\mu_\nu = 0 \quad \Leftrightarrow \quad \tilde{g}^{\mu\nu}(x)\frac{\partial\tilde{\phi}^A}{\partial x^\mu}\frac{\partial\tilde{\phi}^B}{\partial x^\nu} = \eta^{AB}$$

This is a coordinate transformation: curved $\rightarrow$ flat

Generalization of the Stückelberg trick

LA (2011)

Replace the Minkowski metric by a set of scalar functions as

$$\eta^{AB} \to \bar{f}^{AB}(\phi)$$

Then define

$$\bar{h}^{AB} \equiv g^{\mu\nu}(x)\partial_\mu\phi^A\partial_\nu\phi^B - \bar{f}^{AB}(\phi)$$

with

$$\bar{f}^{AB}(\phi) \equiv \tilde{g}^{\mu\nu}(\phi)\delta^A_\mu\delta^B_\nu .$$

$$\Rightarrow \text{the old story:} \qquad \mathcal{L}_{FP} = \bar{h}^2 - \bar{h}^A_B\bar{h}^B_A!$$

- The scalar field "distances" are now measured by the "metric" $\bar{f}^{AB}$, and the indices in the scalar field space have to be raised and lowered as

$$\phi_B \equiv \bar{f}_{AB}\phi^A$$

- Lagrangian is invariant under the isometries of the metric $\bar{f}^{AB}$!
- The nonlinear dRGT completion can be written in terms of

$$\mathcal{K}^\mu_\nu = \delta^\mu_\nu - \underbrace{\sqrt{g^{\mu\lambda}\partial_\lambda\phi^A\partial_\nu\phi^B\bar{f}_{AB}(\phi)}}_{=\sqrt{g^{\mu\lambda}f_{\lambda\nu}}} .$$

Example: massive graviton on de Sitter spacetime

Consider spatially flat de Sitter metric $\tilde{g}^{\mu\nu} = a^{-2}(\eta)\eta^{\mu\nu}$ with $a(\eta) = -1/(H\eta)$.

$\Rightarrow$ The scalar field metric has to be chosen as $\bar{f}^{AB}(\phi^0) = (H\phi^0)^2\eta^{AB}$

$\Rightarrow$ The FP mass term

$$\mathcal{L}_{FP} = g^{\mu\nu}g^{\alpha\beta}\partial_\mu\phi^A\partial_\nu\phi^B\partial_\alpha\phi^C\partial_\beta\phi^D\left[\eta_{AB}\eta_{CD} - \eta_{BC}\eta_{AD}\right] - \\ - 6(H\phi^0)^2 g^{\mu\nu}\partial_\mu\phi^A\partial_\nu\phi^B\eta_{AB} + 12(H\phi^0)^4$$

- diffeomorphism invariant
- NOT invariant under the shifts $\phi^A \to \phi^A + \lambda^A$
- invariant under $\phi^A \to \hat{\Lambda}^A_B\phi^B$ such that $\hat{\Lambda}^A_C\hat{\Lambda}^B_D\bar{f}_{AB}(\phi) \to \bar{f}_{CD}(\phi)$
- gives 5 degrees of freedom q_i for massive graviton each of which satisfy

$$(\Box_{dS} + m^2)q_i = 0 \quad \Leftrightarrow \quad (\partial_\eta^2 + 2\mathcal{H}\partial_\eta + m^2)q_i = 0$$

In terms of physical time $dt = a(\eta)d\eta$:

$$\ddot{q}_i - \frac{\Delta}{a^2}q_i + m_{eff}^2 q_i = 0, \qquad m_{eff}^2 = m^2 - \frac{9}{4}H^2$$

Deser and Waldron (2001)

Conclusions

- Λ_3 / dRGT theories are non-linear massive gravity theories with working Vainshtein mechanism and with the highest achievable strong coupling scale
- General relativity can be restored around static spherically symmetric massive sources up to corrections to the Newtons potential $\delta\phi/\phi \sim (r/R_V)^3$

- In dRGT massive gravity with flat reference metric and an external isotropic matter distribution, the effective energy-momentum tensor due to the mass term takes the form of a cosmological constant
- The isotropic perturbations of the scalar fields have vanishing quadratic kinetic terms and signal towards strong coupling

- A diffeomorphism invariant Fierz-Pauli mass term can be constructed on arbitrary background by the use of four scalar fields
- BUT: on each background the generally covariant theory is a fundamentally different theory with different symmetries
- No single unified theory of massive gravity exists such that the graviton always behaves as a massive spin-2 particle around any background metric

Thank you for your attention!

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