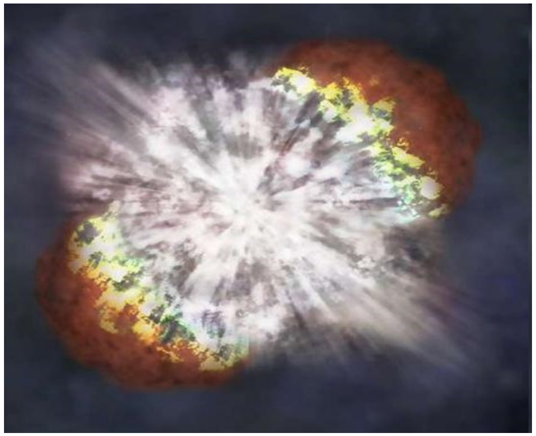
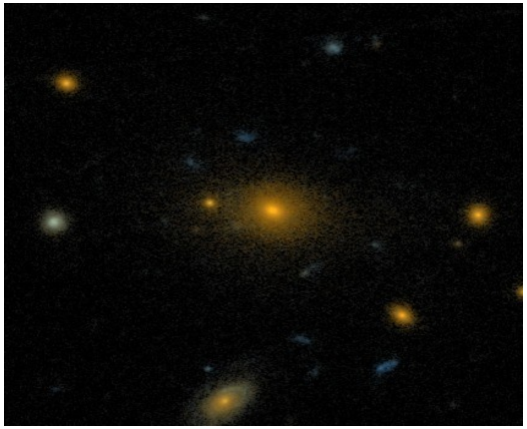
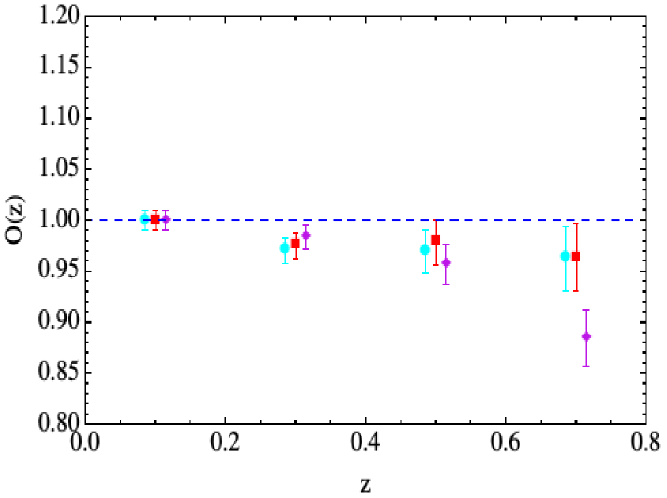
Reconstruction of the null-test for the matter density perturbations







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arXiv: 1409.3697, 1410.0338

Main points of the talk

- Introduction
- What is a null test?
- The null test for the growth-rate data
- The data
- Results:
  - i) Binning
  - ii) Modeling
- Conclusions

The Standard Cosmological model

Einstein equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

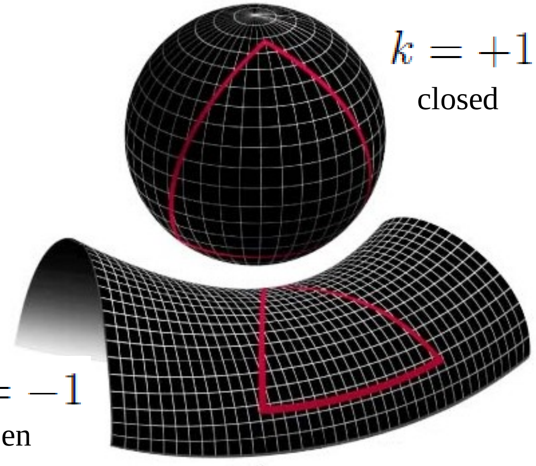
$G_{\mu\nu}$

Cosmological Constant

$$T^{\mu}_{\nu} = P g^{\mu}_{\nu} + (\rho + P) U^{\mu} U_{\nu}$$

Robertson-Walker metric:

$$ds^2 = c^2 dt^2 - \alpha(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + sin(\theta)^2 d\phi^2) \right)$$



$k = +1$

closed

$k = -1$

open

Friedmann equations:

$$H^2(\alpha) = \left(\frac{\dot{\alpha}}{\alpha} \right)^2 = \frac{8\pi G}{3} \rho(\alpha) - \frac{k}{\alpha^2}$$

$$\frac{\ddot{\alpha}}{\alpha} = -\frac{4\pi G}{3} (\rho(\alpha) + P(\alpha))$$

Continuity:

(from Bianchi identities)

$$\nabla_{\nu} T^{\mu\nu} = 0 \implies \dot{\rho} + 3H(\rho + P) = 0$$

The Standard Cosmological model

Hubble (1929): Universe is expanding

From redshift of distant galaxies

Riess et al. (1998): … and actually is accelerating

from supernovae type Ia

2^{nd} Friedmann equation: $\frac{\ddot{\alpha}}{\alpha} = -\frac{4\pi G}{3}\left(\rho(\alpha) + 3P(\alpha)\right) \Rightarrow P < -\frac{\rho}{3}$

Equation of state $P = w\,\rho$

- $w = 0$ Non-relativistic matter $P << \rho$
- $w = \frac{1}{3}$ Radiation $P = \frac{1}{3}\rho$

$$P < -\frac{\rho}{3} \Rightarrow w < -\frac{1}{3}$$

Known kinds of matter cannot explain the accelerated expansion of the universe!

Cosmological perturbations and structure formation

To understand structure formation, we need to study the metric (scalar) perturbations

$$ds^2 = -(1+2\Phi)dt^2 + a^2(t)(1-2\Psi)\delta_{ij}dx^i dx^j$$

Newtonian potentials

And the matter density perturbations: $T^0_0 = -(\rho_m + \delta\rho_m)\,,\qquad T^0_i = -\rho_m v_{m,i}$

Density perturbation

Velocity potential

Join the two:

$$\delta''(a) + \left(\frac{3}{a} + \frac{H'(a)}{H(a)}\right)\delta'(a) - \frac{3}{2}\frac{\Omega_m G_{\text{eff}}(a)/G_N}{a^5 H(a)^2/H_0^2}\,\delta(a) = 0$$

$$\delta_m \equiv \frac{\delta\rho_m}{\rho_m} + 3Hv$$

Effective Newton's constant in Modified Gravity. For GR, $G_{eff}/G_N=1$

Disclaimer: Valid only in subhorizon scales (k^2>>H^2 a^2) and in (certain) metric theories with an FLRW metric, eg f(R), ~f(G), f(T), Scalar-Tensor etc. Other terms and conditions may apply.

The usual way to do things...

Test a variety of models, eg

Model 1: ΛCDM (w=-1)

Model 2: $w(a)=w_0+w_1(1-a)$

Model 3: f(R)

...

Model n: ???

The usual way to do things...

- Eg, SnIa data are given in terms of the distance modulus:

$$\mu_{obs}(z_i) \equiv m_{obs}(z_i) - M$$

- DE is described by w(z)

$w(z) \equiv \frac{P}{\rho}$

$$w(z) = -1 + \frac{1}{3}(1+z)\frac{d\ln(\delta H(z)^2)}{d\ln z}$$

$$\delta H(z)^2 = H(z)^2/H_0^2 - \Omega_{0\mathrm{m}}(1+z)^3$$

- Theoretical prediction:

(flat universe)

$$D_L(z) = (1+z)\int_0^z dz' \frac{H_0}{H(z';\Omega_{0\mathrm{m}}, w_0, w_1)}$$

$\mu_0 = 42.38 - 5log_{10}h$

- Minimize to find the best fit parameters:

$$\chi^2_{SnIa}(\Omega_{0\mathrm{m}}, w_0, w_1) = \sum_{i=1}^{N} \frac{(\mu_{obs}(z_i) - \mu_{th}(z_i))^2}{\sigma^2_{\mu\ i}}$$

The usual way to do things...

+

Bayesian Voodoo

And the winner is...

Everything else

ΛCDM

Problems:

1) Model bias (interpretation of the results depends on chosen models+assumptions)

2) Limited number of tested models (finite number of theories, impossible to test everything)

Can we find a more general and straightforward way to test the fundamental assumptions of our theories???

What is a null test?

A null test is a consistency relation that has to be true for all z and usually equal to zero or some constant value! Examples:

1) The ΩK test of Clarkson et al:

(arXiv:0712.3457)

$$d_L(z) = \frac{(1+z)}{H_0\sqrt{-\Omega_k}} \sin\left(\sqrt{-\Omega_k}\int_0^z \mathrm{d}z' \frac{H_0}{H(z')}\right)$$

$$\Omega_k = \frac{[H(z)D'(z)]^2 - 1}{[H_0 D(z)]^2}$$

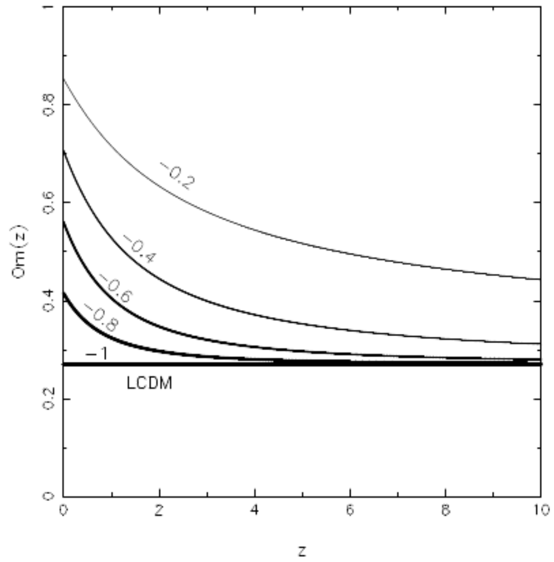
$$D = (1+z)d_A$$

$$\mathscr{C}(z) = 1 + H^2\left(DD'' - D'^2\right) + HH'DD'$$

2) The Om statistic of Shafieloo et al:

(arXiv: 0807.3548, 1004.0960)

$$Om(x) \equiv \frac{h^2(x) - 1}{x^3 - 1}, \qquad x = 1+z\,, \qquad h(x) = H(x)/H_0$$



Can we find a null test for the growth rate???

The null test for the growth rate

The answer is yes!!!

$$\mathcal{O}(z) = \frac{a^2 H(a) f(a)}{a_0^2 H(a_0) f(a_0)} \; e^{\int_{a_0}^{a}\left(\frac{f(x)}{x} - \frac{3\Omega_m}{2x^4 H(x)^2 f(x)}\right)dx} = 1$$

$$\mathcal{O}(z) = \frac{a^2 H(a) f\sigma_8(a)}{a_0^2 H(a_0) f\sigma_8(a_0)} \times$$

$$\times \; e^{-\frac{3}{2}\Omega_m \int_{a_0}^{a} \frac{\sigma_8(a=1)\frac{\delta(a_0)}{\delta(1)} + \int_{a_0}^{x} \frac{f\sigma_8(y)}{y} dy}{x^4 H(x)^2 / H_0^2 f\sigma_8(x)} dx} = 1$$

$$G_{\rm eff}/G_N = 1$$

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arXiv: 1409.3697, 1410.0338

Notes:

i) O(z) should be 1 at all z!!!!

ii) Independent of the DE model used (GR)

iii) Doesn't contain derivatives! **(derivatives of noisy data are bad!!!)**

iv) Deviations from unity could be: New physics (MoGs, DE perts), deviation from FRLW or tension in the data (H and fσ8).

The Lagrangian formalism

Goal: Find a conserved quantity for the growth-rate.

Hint: create a Lagrangian that produces the ODE for growth and check for symmetries

$$\delta''(a) + \left(\frac{3}{a} + \frac{H'(a)}{H(a)}\right)\delta'(a) - \frac{3}{2}\frac{\Omega_m G_{\rm eff}(a)/G_N \delta(a)}{a^5 H(a)^2/H_0^2} = 0$$

Start with: $\mathcal{L} = \mathcal{L}(a, \delta(a), \delta'(a))$

"velocity"

"time"

"position"

Euler-Lagrange equations:

$$\frac{\partial \mathcal{L}}{\partial \delta} - \frac{d}{da}\frac{\partial \mathcal{L}}{\partial \delta'} = 0$$

The Lagrangian formalism

Ansatz: $\mathcal{L} = T - V$

$$T = \frac{1}{2} f_1(a, H(a)) \delta'(a)^2$$ **"kinetic term"**

$$V = \frac{1}{2} f_2(a, H(a)) \delta(a)^2$$ **"potential"**

Use E-L eqs:

$$\delta''(a) + \left(\frac{\partial_a f_1(a,H)}{f_1(a,H)} + \frac{H'(a) \partial_H f_1(a,H)}{f_1(a,H)} \right) \delta'(a) + \frac{f_2(a,H)}{f_1(a,H)} \delta(a) = 0$$

Compare to ODE:

$$\delta''(a) + \left(\frac{3}{a} + \frac{H'(a)}{H(a)} \right) \delta'(a) - \frac{3}{2} \frac{\Omega_m G_{\rm eff}(a)/G_N \delta(a)}{a^5 H(a)^2 / H_0^2} = 0$$

The Lagrangian formalism

Find f1, f2

$$f_1(a, H(a)) = a^3 H(a)/H_0$$
$$f_2(a, H(a)) = -\frac{3\Omega_{\rm m} G_{\rm eff}(a)/G_N}{2a^2 H(a)/H_0}$$

Lagrangian: $$\mathcal{L} = T - V = \frac{1}{2} a^3 H(a)/H_0 \delta'(a)^2 + \frac{3\Omega_{\rm m} G_{\rm eff}(a)/G_N}{4a^2 H(a)/H_0} \delta(a)^2$$

Hamiltonian: $$\mathcal{H} = T + V = \frac{1}{2} a^3 H(a)/H_0 \delta'(a)^2 - \frac{3\Omega_{\rm m} G_{\rm eff}(a)/G_N}{4a^2 H(a)/H_0} \delta(a)^2$$

Depends on "time", not conserved!

Use the above to find conserved quantities for the ODE!

Noether's theorem and conserved quantities

Symmetries ⟷ Conserved quantities

Examples:

1) Lagrangian independent of time, ie t->t+δt → **Hamiltonian is conserved (constant energy)**

2) Lagrangian independent of position, ie x->x+δx → **Momentum px is conserved**

Noether's theorem and conserved quantities

In general, given an infinitesimal transformation X such that

$$\mathbf{X} = \alpha(\delta)\frac{\partial}{\partial\delta} + \frac{d\alpha(\delta)}{da}\frac{\partial}{\partial\delta'} \quad \Rightarrow \quad L_X\mathcal{L} = 0$$

Then, the quantity Σ is conserved:

$$\Sigma = \alpha(a)\frac{\partial\mathcal{L}}{\partial\delta'}$$

Easy to prove with the EL equations as well!

Goal:Find α(a), such that Σ is conserved, ie demand the existence of a conserved quantity and determine the symmetry! **(standard method used to "guess" analytical solutions)**

SN, S. Capozziello and L. Perivolaropoulos arXiv: 0705.3586

Noether's theorem and the growth rate

Solve the equations:

$$L_X \mathcal{L} = 0 \quad \Rightarrow \quad \alpha'(a) a^3 H(a)/H_0 \delta'(a) + \frac{3\Omega_{\rm m} G_{\rm eff}(a)/G_N \delta(a)\alpha(a)}{2a^2 H(a)/H_0} = 0$$

$$\Sigma = a^3 H(a)/H_0 \alpha(\delta)\delta'(a)$$

Solution: $\alpha(a) = c\, e^{-\int_{a_0}^{a} \frac{3\Omega_{\rm m} G_{\rm eff}(x)/G_N \delta(x)}{2x^5 H(x)^2/H_0^2 \delta'(x)} dx}$

$$\Sigma = a^3 H(a)/H_0 \delta'(a)\, e^{-\int_{a_0}^{a} \frac{3\Omega_{\rm m} G_{\rm eff}(x)/G_N \delta(x)}{2x^5 H(x)^2/H_0^2 \delta'(x)} dx} \quad \Rightarrow \quad \Sigma = a_0^3 H(a_0)\delta'(a_0)$$

Introduce growth rate:

$$f(a) \equiv \frac{d ln \delta}{d ln a} \quad \Rightarrow \quad \frac{a^2 H(a) f(a)}{a_0^2 H(a_0) f(a_0)}\, e^{\int_{a_0}^{a} \left(\frac{f(x)}{x} - \frac{3\Omega_{\rm m} G_{\rm eff}(x)/G_N}{2x^4 H(x)^2 f(x)}\right) dx} = 1$$

Noether's theorem and the growth rate

Introduce fσ8(a):

$$f\sigma_8(a) \equiv f(a)\sigma_8(a) = \xi a \delta'(a)$$

$$\delta(a) = \delta(a_0) + \int_{a_0}^{a} \frac{f\sigma_8(x)}{\xi x} dx$$

$$\sigma_8(a) = \sigma_8(a=1)\frac{\delta(a)}{\delta(a=1)}$$

$$\xi \equiv \frac{\sigma_8(a=1)}{\delta(a=1)}$$

New version: $$\frac{a^2 H(a) f\sigma_8(a)}{a_0^2 H(a_0) f\sigma_8(a_0)} \cdot e^{-\frac{3}{2}\Omega_m \int_{a_0}^{a} \frac{G_{\rm eff}(x)}{G_N} \frac{\sigma_8(a=1)\frac{\delta(a_0)}{\delta(1)} + \int_{a_0}^{x} \frac{f\sigma_8(y)}{y} dy}{x^4 H(x)^2/H_0^2 f\sigma_8(x)} dx} = 1$$

i) 100% equivalent

ii) Necessary as surveys measure fσ8 form **(Euclid: f(a)???)**

The null test for the growth rate

Finally, the null test:

$$G_{\rm eff}/G_N = 1$$

$$\mathcal{O}(z) = \frac{a^2 H(a) f(a)}{a_0^2 H(a_0) f(a_0)} \, e^{\int_{a_0}^{a}\left(\frac{f(x)}{x} - \frac{3\Omega_{\rm m}}{2x^4 H(x)^2 f(x)}\right)dx} = 1$$

$$\mathcal{O}(z) = \frac{a^2 H(a) f\sigma_8(a)}{a_0^2 H(a_0) f\sigma_8(a_0)} \times$$

$$\times \; e^{-\frac{3}{2}\Omega_m \int_{a_0}^{a} \frac{\sigma_8(a=1)\frac{\delta(a_0)}{\delta(1)} + \int_{a_0}^{x} \frac{f\sigma_8(y)}{y} dy}{x^4 H(x)^2/H_0^2 f\sigma_8(x)} dx} = 1$$

Notes:

i) O(z) should be 1 at all z!!!!

ii) Independent of the DE model used (GR)

iii) Doesn't contain derivatives! **(derivatives of noisy data are bad!!!)**

iv) Deviations from unity could be: New physics (MoGs, DE perts), deviation from FRLW or tension in the data (H and fσ8).

Strategy

Reconstruct the null test with :

- mock data (H(z) & fσ8(z))
- real data (H(z) & fσ8(z))

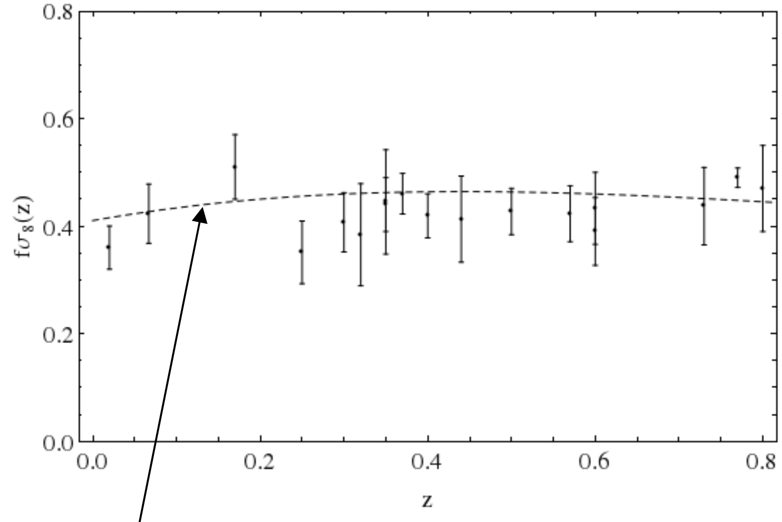
The reconstruction can be done with :

- binning
- Modeling (ΛCDM)

Does the null test really work? How well???

The real data: fσ8(z)

Index	z	$f\sigma_8(z)$	Refs.
1	0.02	0.360 ± 0.040	[38]
2	0.067	0.423 ± 0.055	[39]
3	0.25	0.3512 ± 0.0583	[40]
4	0.37	0.4602 ± 0.0378	[40]
5	0.30	0.407 ± 0.055	[41]
6	0.40	0.419 ± 0.041	[41]
7	0.50	0.427 ± 0.043	[41]
8	0.60	0.433 ± 0.067	[41]
9	0.17	0.510 ± 0.060	[42]
10	0.35	0.440 ± 0.050	[42]
11	0.77	0.490 ± 0.018	[42] [43]
12	0.44	0.413 ± 0.080	[44]
13	0.60	0.390 ± 0.063	[44]
14	0.73	0.437 ± 0.072	[44]
15	0.80	0.470 ± 0.080	[45]
16	0.35	0.445 ± 0.097	[46]
17	0.32	0.384 ± 0.095	[47]
18	0.57	0.423 ± 0.052	[48]



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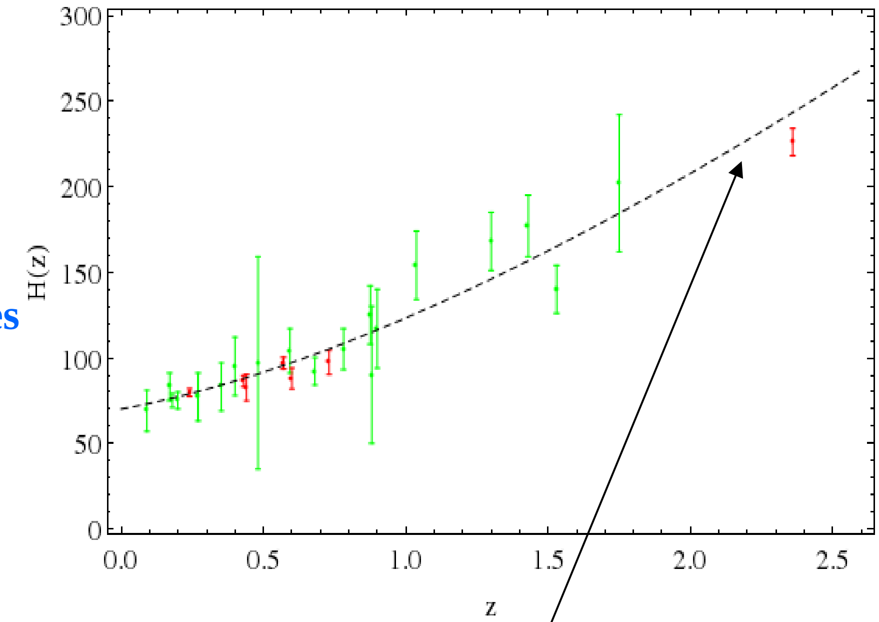
The real data: H(z)

Index	z	$H(z)$	Refs.
1	0.090	69 ± 12	[49]
2	0.170	83 ± 8	[49]
3	0.179	75 ± 4	[49]
4	0.199	75 ± 5	[49]
5	0.270	77 ± 14	[49]
6	0.352	83 ± 14	[49]
7	0.400	95 ± 17	[49]
8	0.480	97 ± 62	[49]
9	0.593	104 ± 13	[42]
10	0.680	92 ± 8	[49]
11	0.781	105 ± 12	[49]
12	0.875	125 ± 17	[49]
13	0.880	90 ± 40	[49]
14	0.900	117 ± 23	[49]
15	1.037	154 ± 20	[49]
16	1.300	168 ± 17	[49]
17	1.430	177 ± 18	[49]
18	1.530	140 ± 14	[49]
19	1.750	202 ± 40	[49]
20	0.240	79.69 ± 2.32	[50]
21	0.430	86.45 ± 3.27	[50]
22	0.440	82.60 ± 7.80	[51]
23	0.570	96.80 ± 3.40	[52]
24	0.600	87.90 ± 6.10	[51]
25	0.730	97.30 ± 7.00	[51]
26	2.36	226.0 ± 8.00	[53]

Passively evolving galaxies

Radial BAO



ΛCDM, Ω_m=0.3, H0=70 km/s/Mpc

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The models (mocks and modeling)

1) ΛCDM (w=-1), Ω_m=0.3, H0=70 km/s/Mpc, σ8=0.8

$$H(a)^2/H_0^2 = \Omega_{\rm m} a^{-3} + (1-\Omega_{\rm m}) a^{-3(1+w)}$$

2) wCDM (w=-0.8), Ω_m=0.3, H0=70 km/s/Mpc, σ8=0.8

$$H(a)^2/H_0^2 = \Omega_{\rm m} a^{-3} + (1-\Omega_{\rm m}) a^{-3(1+w)}$$

$$G_{\rm eff}(a)/G_N \equiv Q(a) = 1 + \frac{1-\Omega_{m_0}}{\Omega_{m_0}} \frac{1+w}{1-3w} a^{-3w}$$

D. Sapone, M. Kunz and L. Amendola, Phys. Rev. D **82**, 103535 (2010). arXiv:1007.2188 [astro-ph.CO]].

3) f(R) Ω_m=0.3, H0=70 km/s/Mpc, σ8=0.8 (exactly ΛCDM at background)

$$f(R) = R - 2\Lambda + \alpha H_0^2 \left(\frac{\Lambda}{R-3\Lambda}\right)^b {}_2F_1\left(b, \frac{3}{2}+b, \frac{13}{6}+2b, \frac{\Lambda}{R-3\Lambda}\right)$$

$$b = \frac{1}{12}\left(-7+\sqrt{73}\right)$$

$$H(a)^2 = H_0^2\left(\Omega_{\rm m} a^{-3} + 1 - \Omega_{\rm m}\right)$$

$$\alpha = (0.002, 0.2)$$

$$G_{\rm eff}/G_N = \frac{1}{F}\frac{1+4\frac{k^2}{a^2}m}{1+3\frac{k^2}{a^2}m},$$

$$m \equiv \frac{F_{,R}}{F},$$

$$F \equiv f_{,R} = \frac{\partial f}{\partial R}.$$

S. Nesseris, Phys. Rev. D **88**, 123003 (2013) arXiv:1309.1055 [astro-ph.CO]].

The models (mocks and modeling)

4) f(G) Ω_m=0.3, H0=70 km/s/Mpc, σ8=0.8 (exactly ΛCDM at background)

S. Nesseris, Phys. Rev. D **88**, 123003 (2013) arXiv:1309.1055 [astro-ph.CO]].

$$f(\mathcal{G}) = -3H_0^2(1-\Omega_{\rm m}) + \alpha\, H_0^2 \mathcal{G} \int \frac{a(\mathcal{G})H(\mathcal{G})/H_0}{\mathcal{G}^2} d\mathcal{G}$$

$$H(a)^2 = H_0^2\left(\Omega_{\rm m} a^{-3} + 1 - \Omega_{\rm m}\right)$$

$$\mathcal{G} \equiv R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho}$$

$$\ddot{\delta}_m + C_1(k,a)\dot{\delta}_m + C_2(k,a)\delta_m \simeq 0$$

5) LTB model (void)

$$ds^2 = -dt^2 + X^2(r,t)\, dr^2 + A^2(r,t)\, d\Omega^2$$

$$X(r,t) = A'(r,t)/\sqrt{1-k(r)}$$

$$\Omega_M(r) = 1 + (\Omega_M^{(0)} - 1)\frac{1-\tanh[(r-r_0)/2\Delta r]}{1+\tanh[r_0/\Delta r]}$$

$$H_0(r) = H_0\left[\frac{1}{1-\Omega_M(r)} - \frac{\Omega_M(r)}{(1-\Omega_M(r))^{3/2}} \times \right.$$
$$\left. \times \mathrm{arcsinh}\sqrt{\frac{1-\Omega_M(r)}{\Omega_M(r)}}\right],$$

$$r_0 = 3.0\ \mathrm{Gpc}\,,\ \Delta r = r_0\,,\ h_0 = 0.71\,,\ \Omega_M^{(0)} = 0.19$$

The models (mocks and modeling)

5) LTB model (void) **(continues...)**

Different expansion rates in different directions:

1) Longitudinal $H_L = \dot{A}'/A'$

2) Transverse $H_T \equiv \dot{A}/A$

$$H_T^2(t,r) = H_0^2(r)\left[\Omega_M(r)\frac{A_0^3(r)}{A^3(t,r)} + (1-\Omega_M(r))\frac{A_0^2(r)}{A^2(t,r)}\right]$$

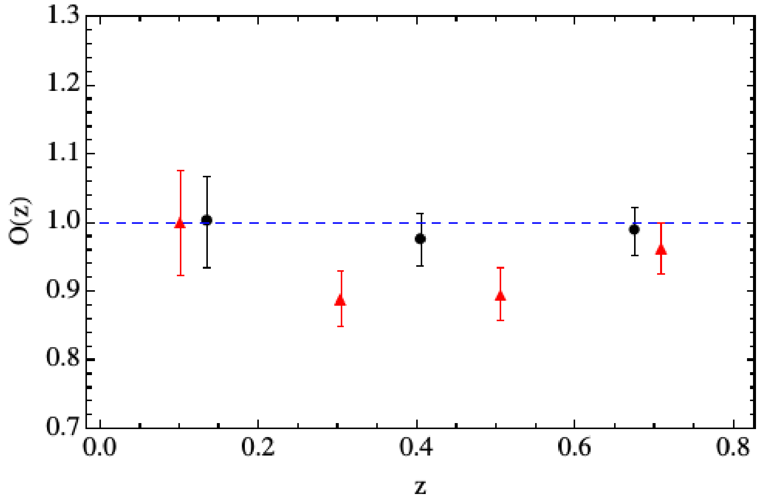
Matter density profile: important parameters for the model

Finally:

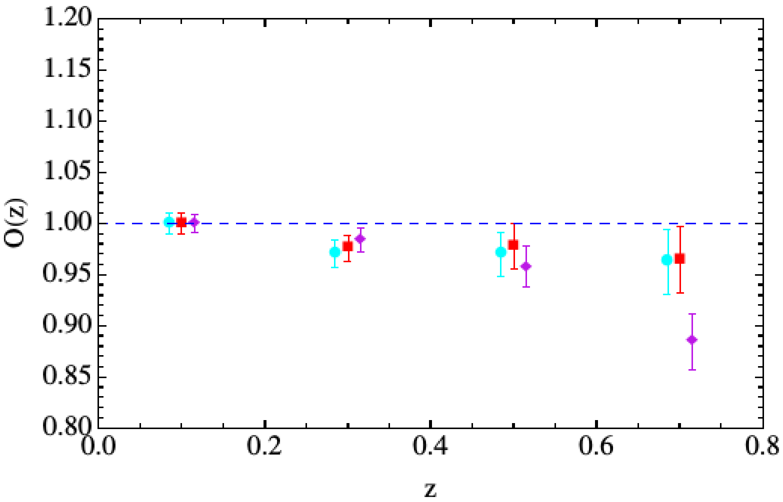
Mocks: z->[0,2], dz=0.1, errors according to a Euclid-like and LSST-like survey

Results: Binning

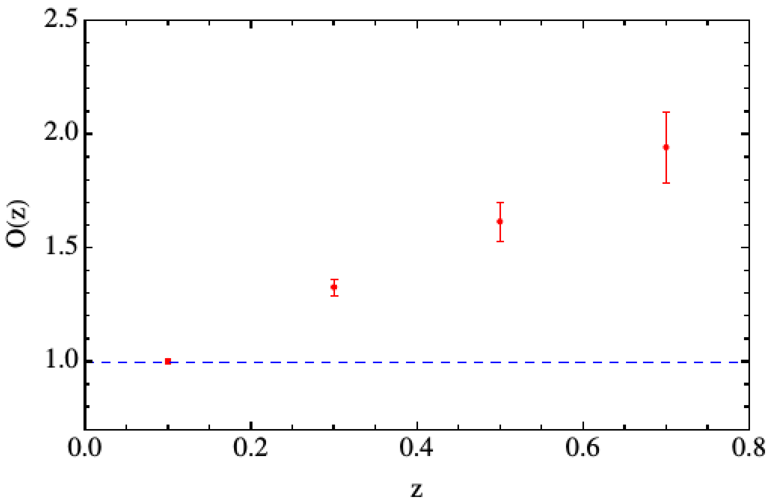
Real data: 3 or 4 bins



Mock data: DE, f(R), f(G)



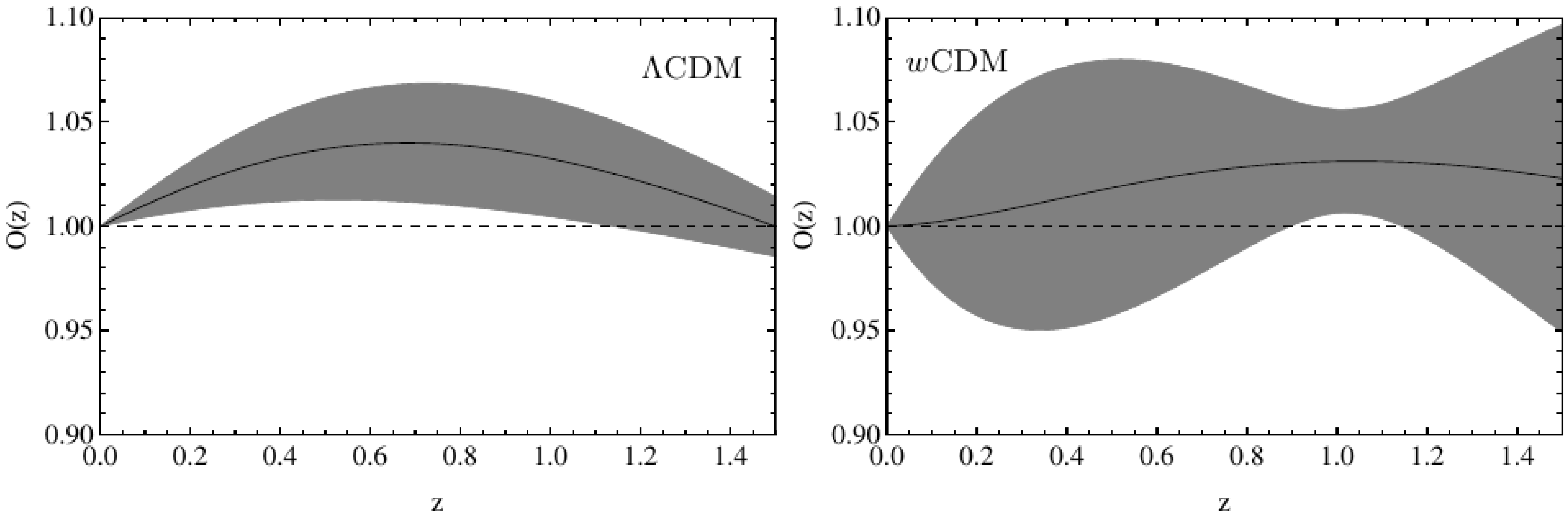
LTB



Results: Modeling with ΛCDM & wCDM

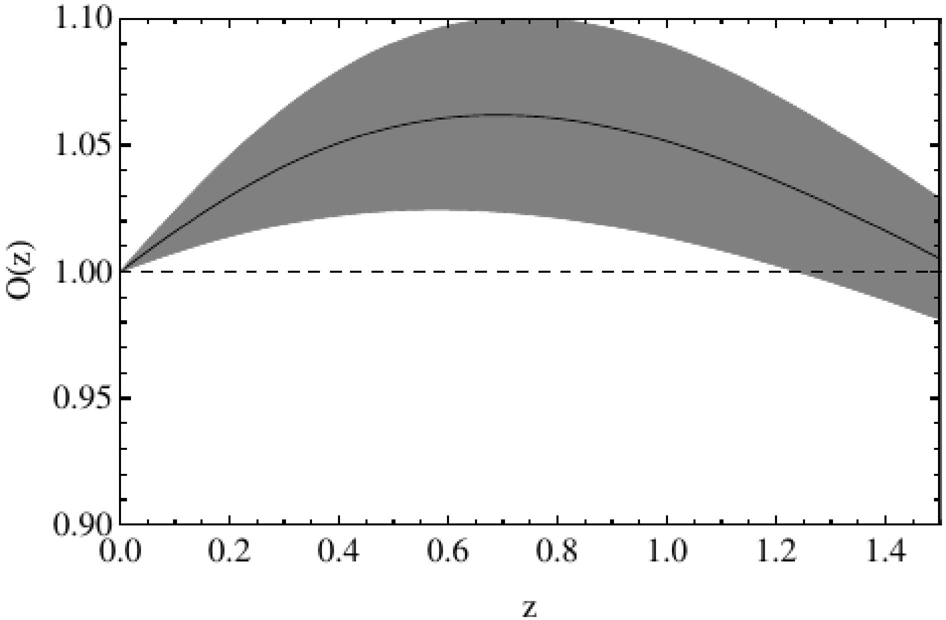
Real data: fit with ΛCDM (left) and wCDM (right)

$G_{\rm eff}/G_N = 1$

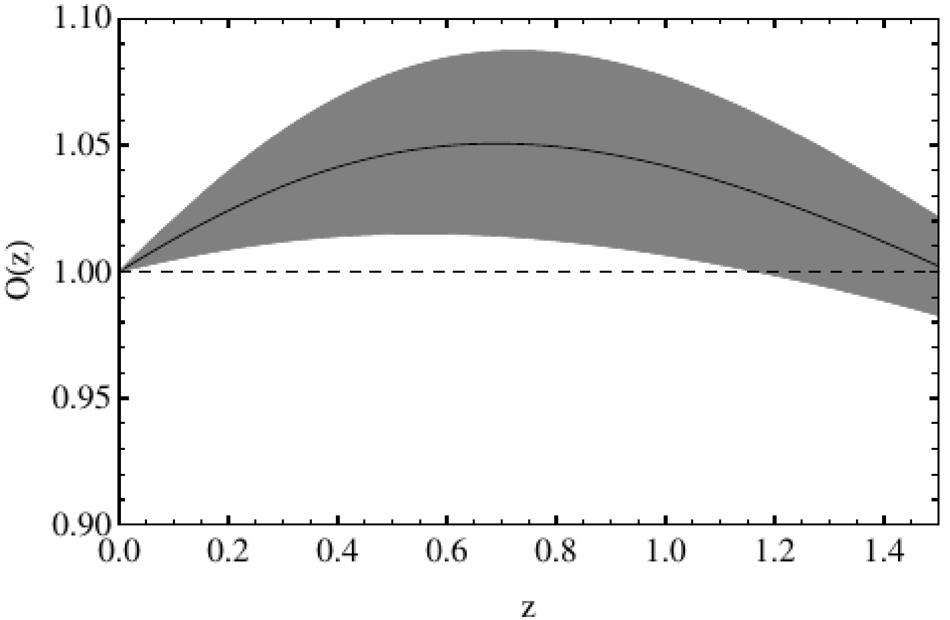


Results: Modeling with f(R)

Real data: fit with f(R) SnIa (left) and H(z) (right)



SnIa and $f\sigma_8$

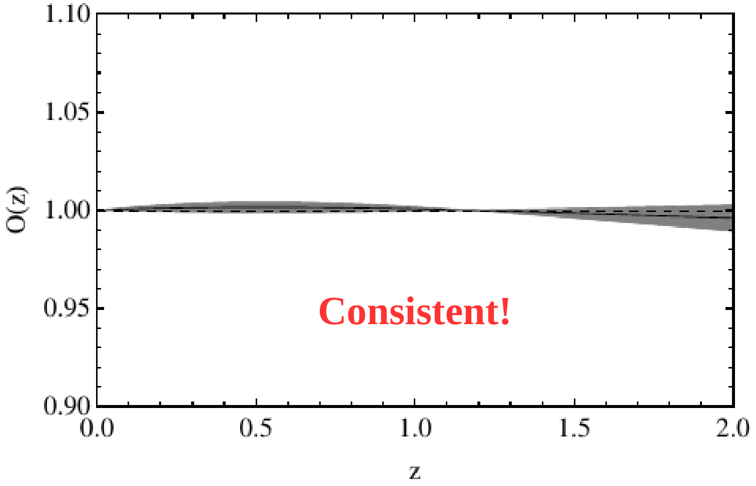


$H(z)$ and $f\sigma_8$

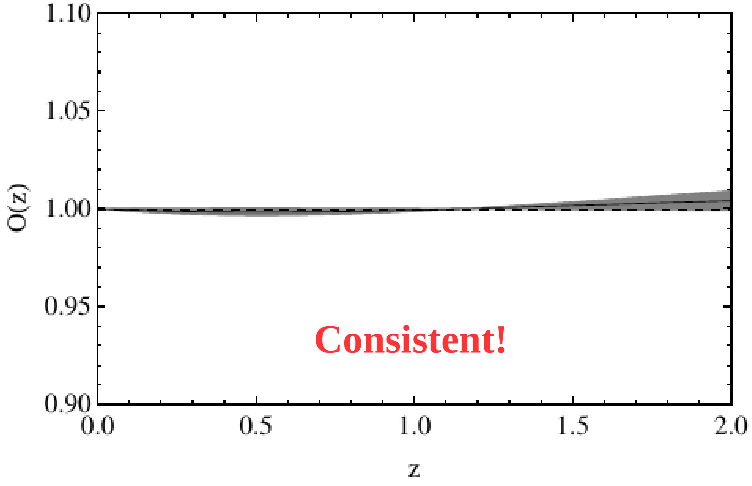
Results: Modeling with ΛCDM

$G_{\rm eff}/G_N = 1$

ΛCDM mock:



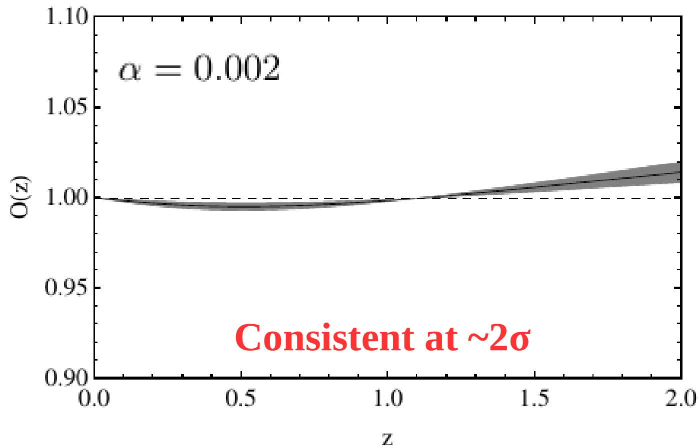
DE perturbations mock:

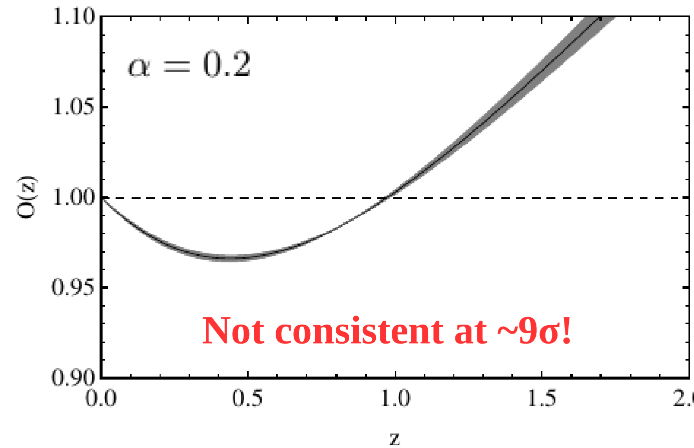


Results: Modeling with ΛCDM

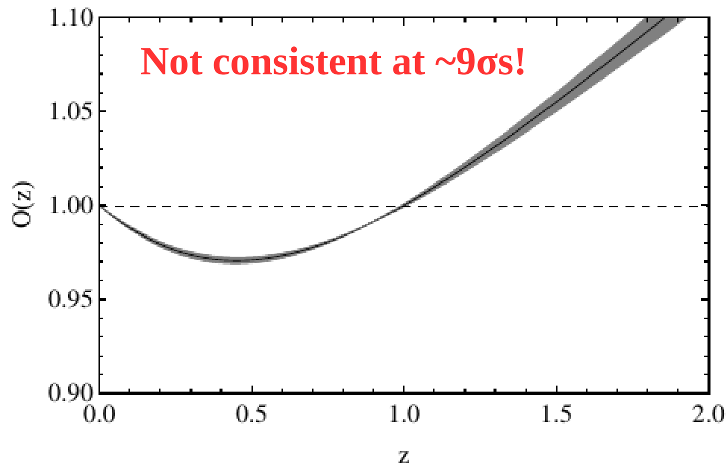
$G_{\rm eff}/G_N = 1$

f(R) mock:





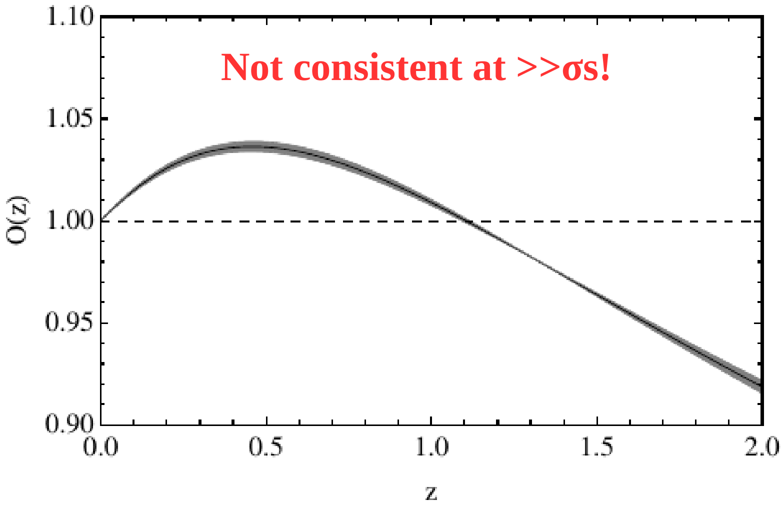
f(G) mock:



Results: Modeling with ΛCDM

$G_{\rm eff}/G_N = 1$

LTB mock:



RESULTS:

The null test (with ΛCDM) can: detect f(R), f(G), LTB
confirm ΛCDM
cannot discriminate DE perturbations

Conclusions - Outlook

Introduced the null test O(z)

It can successfully detect MoG & LTB at many σs, but not DE perturbations

(effect too small??)

Results of real data are in good agreement with ΛCDM (~2σ)

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In this part I will try to explain several key issues in data analysis and statistics with the use of explicit examples and numerical codes. Most of the following material is intended for master and fledgling PhD students who want to understand the basics of data analysis with a focus on cosmology and want to enter the world of research. However, some of the examples might be a bit more advanced...

Prerequisites:

1) Study Chapter 15 of Numerical Recipes regarding data-fitting, minimization, MCMC, statistics etc **[1]**, see also **[2]**.
2) Download the Mathematica codes found below and that illustrate several key issues, like minimization and basic statistical analysis, contours, MCMC, Fourier analysis, parallelization (CPU/GPU) etc.
3) Get CAMB from **here** and follow the instructions in the **Readme** to compile and install it. Gfortran 4.5+ is highly recommended.
4) Run the codes and try to understand what's going on and most importantly *why*.

Numerical codes: (right-click on "Download" and hit "Save as")

1) Statistical Significance and Sigmas. **Download**.
2) Stuff about covariance matrices. **Download**.
3) Data fitting, contours, error bars etc. **Download**.
4) Markov Chain Monte Carlo (MCMC). **Download**.
5) Bootstrap Monte Carlo. **Download**.
6) The Jack-knife **[3]**. **Download**.
7) Genetic Algorithms **[4]**. **Download**.
8) A Mathematica Interface for CosmoMC, go **here**.
9a) Fitting the SnIa data (standard) **[5]** **Download**.
9b) Fitting the SnIa data (**ultra-fast**) **[5]** **Download**.
10) Joint SnIa, CMB, BAO and growth-rate likelihood! (**ultra-fast**) **Download**.
11) Parallelization CPU/GPU (coming soon).
12) The CMB power spectrum and the cosmological parameters; the correlation function (no RSD) **Download**.

Note 1: Mathematica 8+ is recommended, but probably older versions will work as well.

Note 2: The Genetic Algorithms code might have some memory issues under Mathematica 9, in some systems.

Other cool stuff:

1) The sound Doppler effect visualized in Mathematica and a measurement of g, **here**.
2) How NDSolve works (also illustrates Dynamic and StepMonitor). **Download**
3) How ContourPlot works (also illustrates Dynamic and EvaluationMonitor). **Download**.
4) Slides of a talk I gave at the Discovery Center in Copenhagen (March 2011) on how the ancient Greek astronomers measured the distance to the Sun. **Download**.