The dark magnetism of the universe

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Outline

- The cosmic acceleration
- Electromagnetic dark energy
- Cosmic magnetic fields
- Conclusions
The cosmological constant problem

\[ S_g[\mathbf{g}_{\mu\nu}] = \int d^4x \sqrt{-g} \left( -\frac{1}{16\pi G} R + 2\Lambda \right) \]

\[ \Lambda \simeq (10^{-3}\text{eV})^4 \]
\[ G \simeq (10^{19}\text{GeV})^{-2} \]

Vacuum energy differs from the observed value?
(see M. Maggiore 1004.1782 [astro-ph.CO])

Coincidence problem
Dark energy models

- Scalar fields
  - Quintessence
  - K-essence
  - ...
- Chaplygin gas
- Voids
- Extradimensions: DGP
- Modifications of gravity at large scales

What about electromagnetism at large scales?

It has only been tested on scales up to 1.3 A.U.
Electromagnetic nature of dark energy
EM quantization in flat space

\[ S = \int d^4 x \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + A_\mu J^\mu \right) \]

\[ A_\mu \rightarrow A_\mu + \partial_\mu \Lambda \]
\[ \partial_\mu J^\mu = 0 \]

Gauge invariance

\[ \partial_\nu F^{\mu\nu} = J^\mu \]

Maxwell equations

- Conjugate momentum of the temporal component?
- Commutation relations?
- Photon propagator?
Coulomb gauge

\[ \partial_\mu A^\mu = 0 \]

\[ A_\mu \rightarrow A_\mu + \partial_\mu \theta \]

\[ \square \theta = 0 \]

\[ \square A^\mu = J^\mu \]

\[ A_0 = 0 \]

\[ \nabla \cdot \vec{A} = 0 \]

Covariant quantization

\[ S = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\xi}{2} (\partial_\mu A^\mu)^2 + A_\mu J^\mu \right] \]

\[ \partial_\nu F^{\mu\nu} + \xi \partial^\mu (\partial_\nu A^\nu) = J^\mu \]

\[ \square (\partial_\mu A^\mu) = 0 \]

\[ \partial_\mu A^\mu(+) |\phi\rangle = 0 \]

\[ \left[ a_0(\vec{k}) + a_{\parallel}(\vec{k}) \right] |\phi\rangle = 0 \]

\[ n_0(\vec{k}) = n_{\parallel}(\vec{k}) \]
EM quantization in an expanding universe

Covariant version of EM action in a curved spacetime

\[ S = \int d^4x \sqrt{-g} \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\zeta}{2} (\nabla_\mu A^\mu)^2 + A_\mu J^\mu \right] \]

Modified Maxwell equations

\[ \nabla_\nu F^{\mu\nu} + \zeta \nabla^\mu (\nabla_\nu A^\nu) = J^\mu \]

Free scalar field non-conformally coupled to gravity

\[ \Box (\nabla_\nu A^\nu) = 0 \]

Lorenz condition?
EM quantization in an expanding universe

Temporal and longitudinal modes coupled to each other and to gravity.

decoupled transverse modes

\[
\begin{align*}
\mathcal{A}''_{0k} - \left[ \frac{k^2}{\xi} - 2\mathcal{H}' + 4\mathcal{H}^2 \right] \mathcal{A}_{0k} - 2ik \left[ \frac{1 + \xi}{2\xi} \mathcal{A}'_{||k} - \mathcal{H}\mathcal{A}_{||k} \right] &= 0 \\
\mathcal{A}''_{||k} - k^2 \xi \mathcal{A}_{||k} - 2ik \xi \left[ \frac{1 + \xi}{2\xi} \mathcal{A}'_{0k} + \mathcal{H}\mathcal{A}_{0k} \right] &= 0 \\
\mathcal{A}''_{\perp k} + k^2 \mathcal{A}_{\perp k} &= 0
\end{align*}
\]
We consider an expanding universe with two asymptotically Minkowski regions: \( a(\eta) = 2 + \tanh(\eta / \eta_0) \)

\[
\begin{align*}
\partial_\mu A_\mu^{(+)} | \phi \rangle &= 0 \\
n_0^{\text{in}}(\vec{k}) &= n_\parallel^{\text{in}}(\vec{k}) = 0
\end{align*}
\]

\[
\begin{align*}
\partial_\mu A_\mu^{(+)} | \phi \rangle &\neq 0 \\
n_0^{\text{out}}(\vec{k}) &\neq n_\parallel^{\text{out}}(\vec{k})
\end{align*}
\]

Mixing of positive and negative frequency modes

Breakdown of Lorenz condition on super-Hubble scales

Lorenz condition restored on sub-Hubble scales
EM quantization without Lorenz condition

Fundamental action for EM

\[ S = \int d^4 x \sqrt{-g} \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\xi}{2} (\nabla_\mu A^\mu)^2 + A_\mu J^\mu \right] \]

- \( \nabla_\mu A^\mu \) is a propagating field.
- Although non-gauge invariant, it reduces to ordinary QED for transverse photons.
- \( \xi \) can be fixed so that the new polarization is canonically normalised.
EM quantization without Lorenz condition

Fundamental action for EM

\[ S = \int d^4x \sqrt{-g} \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\xi}{2} (\nabla_\mu A^\mu)^2 + A_\mu J^\mu \right] \]

Potential problems
- Modification of classical Maxwell equations
- Negative norm (energy) states
- Unobserved extra polarizations
- Conflicts with QED phenomenology
**EM quantization without Lorenz condition**

Fundamental action for EM

\[ S = \int d^4x \sqrt{-g} \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\xi}{2} (\nabla_\mu A^\mu)^2 + A_\mu J^\mu \right] \]

General solution \( A_\mu = A_\mu^{(1)} + A_\mu^{(2)} + A_\mu^{(s)} + \partial_\mu \theta \)

Residual gauge mode

- Photon
- New scalar state

\[ \Box \left( \nabla^\nu A_\nu^{(s)} \right) = 0 \]

\[ \nabla_\nu A_\nu^{(s)} \approx \frac{C}{a} e^{-i k \eta} \]

Cosmic magnetic fields

\[ \nabla_\nu A_\nu^{(s)} \approx \text{constant} \]

Dark energy
EM quantization without Lorenz condition

Fundamental action for EM

\[ S = \int d^4x \sqrt{-g} \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\xi}{2} (\nabla_\mu A^\mu)^2 + A_\mu J^\mu \right] \]

General solution \( A_\mu = A_\mu^{(1)} + A_\mu^{(2)} + A_\mu^{(s)} + \partial_\mu \theta \) Residual gauge mode

Photons New scalar state

The gauge-fixed QED effective action in the path-integral formalism in flat spacetime is:

\[ e^{iW} = \int [dA][dc][d\bar{c}][d\psi][d\bar{\psi}] e^{i \int d^4x \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\xi}{2} (\partial_\mu A^\mu)^2 + \partial_\mu \bar{c} \partial^\mu c + L_F \right)} \]

\[ \propto \int [dA][d\psi][d\bar{\psi}] e^{i \int d^4x \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\xi}{2} (\partial_\mu A^\mu)^2 + L_F \right)} \]

which coincides with the considered action for flat spacetime.
Cosmological evolution

Cosmological evolution

\[ \ddot{A}_0 + 3H\dot{A}_0 + 3H^2 A_0 = 0 \]
\[ \dddot{A} + H\ddot{A} = 0 \]

\[ \frac{d}{dt}(\nabla_\mu A^\mu) = \frac{d}{dt}(\dot{A}_0 + 3HA_0) = 0 \]

\[ \rho_{A_0} = \frac{\xi}{2} (\dot{A}_0 + 3HA_0)^2 = \text{constant} \]
\[ \rho_{\tilde{A}} = \frac{1}{2a^2}(\dot{A})^2 \propto \frac{1}{a^4} \]

Cosmological constant due to the presence of an absolute electric potential.

What is the predicted dark energy density?

What is the field amplitude generated during inflation?
Initial conditions during inflation

Power spectrum generated during inflation from quantum fluctuations

\[
\mathcal{P}_{\nabla \cdot A} = \frac{9H_I^4}{16\pi^2}
\]

\[
\rho_\Lambda \sim (10^{-3}\text{eV})^4
\]

\[
\rho_{\nabla \cdot A} = \frac{1}{6} \langle (\nabla_\mu A^\mu)^2 \rangle \simeq H_I^4 \simeq \left( \frac{M_I^2}{M_P} \right)^4
\]

\[
\rho_\Lambda \sim \left( \frac{M_{EW}^2}{M_P} \right)^4
\]

The cosmological constant value can be explained from physics at the EW scale

Arkani-Hamed et al. PRL 85 (2000) 4434

Electroweak scale

\[
M_I \sim 1\text{ TeV}
\]
Stability

Classical

The propagation speed of the scalar, vector and tensor modes: $\omega_k = v_k |\vec{k}|$

is equal to the speed of light so that there are no exponentially growing modes.

Quantum

The three physical states carry positive energy:

$$\rho = \left\langle T_{00}^{(2)} - \frac{1}{8\pi G} G_{00}^{(2)} \right\rangle$$

ensuring the absence of ghosts in the theory.
Local gravity tests

\[ S[g_{\mu\nu}, A_\mu] = \int d^4x \sqrt{-g} \left[ -\frac{1}{16\pi G} R + \omega RA_\mu A^\mu + \sigma R_{\mu\nu} A^\mu A^\nu + \lambda (\nabla_\mu A^\mu)^2 + \epsilon F_{\mu\nu} F^{\mu\nu} \right] \]

\[ \gamma = \frac{1 + 4\omega A^2 (1 + \frac{2\omega + \sigma}{\epsilon})}{1 - 4\omega A^2 (1 - \frac{4\omega}{\epsilon})} \]

\[ \beta = \frac{1}{4} (3 + \gamma) + \frac{1}{2} \Theta \left[ 1 + \frac{\gamma(\gamma - 2)}{G} \right] \]

\[ \alpha_1 = 4(1 - \gamma) [1 + 2\epsilon \Delta] + 16\omega A^2 \Delta a \]

\[ \alpha_2 = 3(1 - \gamma) \left[ 1 + \frac{4}{3} \epsilon \Delta \right] + 8\omega A^2 \Delta a - 2 \frac{bA^2}{G} \]

PPN parameters exactly the same as those of GR for any value of \( A_0 \), so it has the same small scales behavior.
Perturbations

Same background as LCDM, but it fluctuates

CMB and LSS

Compatible as long as the initial perturbation is not too large, implying a reduction in the inflation scale by a factor of 15.
# Potential problems

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Generation of cosmic magnetic fields
Cosmic magnetic fields

- Astrophysical mechanisms: Difficulties to explain intergalactic magnetic fields.

- Inflation-based models: generate super-Hubble modes that are severely constrained by BBN.

- Phase transitions: strongly constrained by causality. Very blue power spectrum leading to weak magnetic fields on large scales.

- Second order perturbations: very weak magnetic fields.

Effective electromagnetic current

\[ \nabla_\nu F^{\mu \nu} + \xi \nabla^\mu (\nabla_\nu A^\nu) = J^\mu \]

Even if the primordial plasma is electrically neutral, the universe acquires an effective stochastic distribution of charge density given by

\[ \rho_g = -\xi \partial_0 (\nabla_\mu A^\mu) \]
Spectrum of effective electric charge

\[ P_{\nabla A}(k) = \frac{9H^4_0}{16\pi^2} \left( \frac{k}{k_0} \right)^{-4\epsilon} \]

\[ \Box (\nabla_v A^v) = 0 \]

\[ P_\rho(k) = \begin{cases} 
0, & k < H_0 \\
\frac{\Omega_M^2 H_0^2 H^4_{k0}}{16\pi^2} \left( \frac{k}{k_0} \right)^{-4\epsilon-2}, & H_0 < k < k_{eq} \\
2\Omega_M H_0^2 H^4_{k0} \frac{1}{16\pi^2 (1+z_{eq})} \left( \frac{k}{k_0} \right)^{-4\epsilon}, & k > k_{eq} \end{cases} \]

- **Super-Hubble modes**
- **Modes entering in the matter era**
- **Modes entering in the radiation era**
Cosmic magnetic fields

Ohm’s law \( J^\mu - u^\mu u_\nu J^\nu = \sigma F^{\mu\nu} u_\nu \)

\( F^{\mu\nu} u_\nu = \frac{e^{\mu\nu\rho\sigma}}{\sqrt{g}} B_{\rho\sigma} u_\nu = J^\mu \nabla \cdot A u_\mu \)

Infinity conductivity limit

\( E_\mu = 0 \)

\( \omega \cdot B = \rho_g^0 \)

\[ \langle B_i(\vec{k})B_j^*(\vec{h})\rangle = \frac{(2\pi)^3}{2} \left( \frac{\delta_{ij} - \hat{k}_i \hat{k}_j}{\Omega} \right) \delta(\vec{k} - \vec{h}) Bk^n \]

\[ \langle \omega_i(\vec{k})\omega_j^*(\vec{h})\rangle = \frac{(2\pi)^3}{2} \left( \frac{\delta_{ij} - \hat{k}_i \hat{k}_j}{\Omega} \right) \delta(\vec{k} - \vec{h}) \Omega k^m \]
Cosmic magnetic fields

Amplitude of the magnetic field at a scale $\lambda$

$$B^2_\lambda \lesssim \frac{4\pi \rho^2_\lambda G(\lambda, n) G(\lambda, m)}{\omega^2_\lambda S(\lambda, n, m)}.$$

CMB constraints on the vorticity

$$\omega^2_\lambda \lesssim 10^{-10} \frac{z^2_{rec} G(\lambda, m)}{8l^3(l + 1) R(l, m)}.$$

Upper limits on vorticity impose “lower” limits on $B$
Cosmic magnetic fields

\[ B_\lambda (G) = \begin{cases} \lambda = 3000 \, h^{-1} \, \text{Mpc} & m = 0 \\ \lambda = 0.1 \, h^{-1} \, \text{Mpc} & m = -3 \end{cases} \]

\[ B_\lambda (G) = \begin{cases} \lambda = 3000 \, h^{-1} \, \text{Mpc} & m = -3 \\ \lambda = 0.1 \, h^{-1} \, \text{Mpc} & m = -5 \end{cases} \]
Conclusions

- EM field can be consistently quantized with three physical states without the need of Lorenz condition.
- Quantum fluctuations of the new state during an inflationary epoch at the electroweak scale give rise to an effective cosmological constant on large scales with the correct value.
- The model satisfies all the viability conditions and it is in agreement with CMB and LSS measurements.
- Relatively strong cosmic magnetic fields can be generated on large scales.