Non-Cold Dark Matter in CLASS

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What we mean by NCDM

- **Non-Cold**: Can be assumed massless initially.
- **Dark**: Non-interacting for CMB relevant epoch.
- Arbitrary **distribution function**.
- Examples:

Massive Neutrinos

Warm Dark Matter

Sterile Neutrinos

...

The Boltzmann equation

- The fundamental equation in cosmology:

$$\mathcal{L}[f_i(\vec{x},\vec{p})] = C[f_i, f_j]\ (=0)$$

- First order perturbation theory: $f_i = f_0(1+\Psi)$
- Boltzmann equation for Ψ:

$$\frac{\partial\Psi}{\partial\tau} + \mathrm{i}\frac{qk}{\epsilon}(\hat{k}\cdot\hat{n})\Psi + \frac{d\ln \mathrm{f}_0}{d\ln \mathrm{q}}\left[\dot{\eta} - \frac{\dot{h}+6\dot{\eta}}{2}(\hat{k}\cdot\hat{n})^2\right] = 0,$$

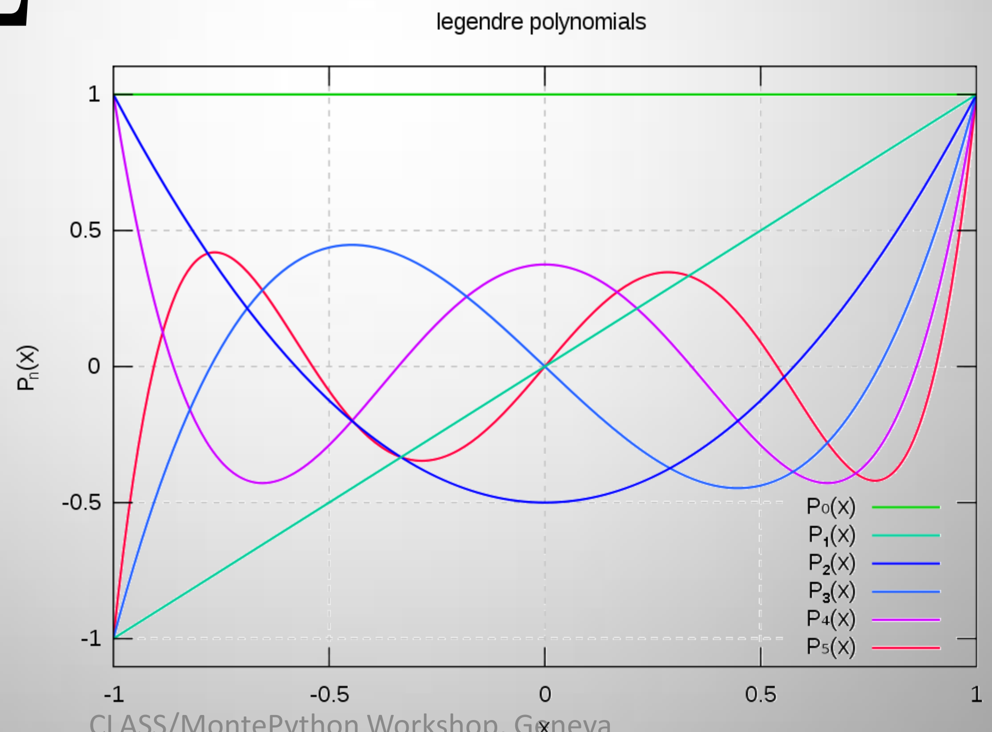
$$q \equiv \frac{p}{T_{\mathrm{ncdm}}}, \qquad \epsilon \equiv \frac{\sqrt{p^2+m^2}}{T_{\mathrm{ncdm}}}.$$

$$\boldsymbol{\hat{k}\cdot\hat{n} = \cos\theta \Rightarrow}$$
no ϕ-dependence

Legendre expansion

Expand Ψ in Legendre multipoles:

$$\Psi = \sum_{l}^{\infty} (-\mathbb{i})^l (2l+1) \Psi_l P_l(\hat{k} \cdot \hat{n}) .$$



$$\Psi = \sum (-\mathbb{i})^l (2l+1)\Psi_l P_l(\hat{k}\cdot\hat{n}).$$

The Boltzmann hierarchy:

$$\dot{\Psi}_0 = -\frac{qk}{\epsilon}\Psi_1 + \frac{\dot{h}}{6}\frac{d\ln \mathrm{f}_0}{d\ln \mathrm{q}},$$

$$\dot{\Psi}_1 = \frac{qk}{3\epsilon}(\Psi_0 - 2\Psi_2),$$

$$\dot{\Psi}_2 = \frac{qk}{5\epsilon}(2\Psi_1 - 3\Psi_3) - \left(\frac{\dot{h}}{15} + \frac{2\dot{\eta}}{5}\right)\frac{d\ln \mathrm{f}_0}{d\ln \mathrm{q}},$$

$$\dot{\Psi}_l = \frac{qk}{(2l+1)\epsilon}(l\Psi_{l-1} - (l+1)\Psi_{l+1}), \qquad l \geq 3.$$

$\delta T^{\mu\nu}$ contribution

NCDM couples through integrated quantities:

$$\delta\rho_{\mathrm{ncdm}} = 4\pi T_{\mathrm{ncdm}}^4 \int q^2 dq f_0 \epsilon \Psi_0,$$

$$\delta p_{\mathrm{ncdm}} = \frac{4\pi}{3} T_{\mathrm{ncdm}}^4 \int q^2 dq f_0 \frac{q^2}{\epsilon} \Psi_0,$$

$$(\bar{\rho} + \bar{p})\theta_{\mathrm{ncdm}} = 4\pi k T_{\mathrm{ncdm}}^4 \int q^2 dq f_0 q \Psi_1,$$

$$(\bar{\rho} + \bar{p})\sigma_{\mathrm{ncdm}} = \frac{8\pi}{3} T_{\mathrm{ncdm}}^4 \int q^2 dq f_0 \frac{q^2}{\epsilon} \Psi_2.$$

$$\int_0^\infty dq f_0\, I(q, \ldots) \rightarrow \sum_{i=1}^{N} w_i\, I(q_i, \ldots)$$

Novelties in the CLASS implementation

See CLASS IV paper, arXiv:1104.2935

- Advanced scheme for selecting w_i and q_i
- Imperfect fluid approximations when perturbation goes sub-Hubble

$$\dot\sigma_{\mathrm{MB}} = -3\left(\frac{1}{\tau}+\frac{\dot a}{a}\left[\frac{2}{3}-c_g^2-\frac{1}{3}\frac{\mathfrak{p}}{p}\right]\right)\sigma+\frac{4}{3}\frac{c_{\mathrm{vis}}^2}{1+w}\left[2\theta+\dot h+6\dot\eta\right], \quad c_{\mathrm{vis}}^2 = 3wc_g^2,$$

$$\dot\sigma_{\mathrm{Hu}} = -3\frac{\dot a}{a}\frac{c_g^2}{w}\sigma+\frac{4}{3}\frac{c_{\mathrm{vis}}^2}{1+w}\left[2\theta+\dot h+6\dot\eta\right], \qquad c_{\mathrm{vis}}^2 = w,$$

$$\dot\sigma_{\mathrm{CLASS}} = -3\left(\frac{1}{\tau}+\frac{\dot a}{a}\left[\frac{2}{3}-c_g^2-\frac{1}{3}\frac{\mathfrak{p}}{p}\right]\right)\sigma+\frac{4}{3}\frac{c_{\mathrm{vis}}^2}{1+w}\left[2\theta+\dot h\right], \qquad c_{\mathrm{vis}}^2 = 3wc_g^2.$$

How to use NCDM in CLASS:

```
explanatory.ini:
	N_ncdm = 0
	m_ncdm = 0.04, 0.04, 0.04
	Omega_ncdm =
	#T_ncdm = 0.715985
	deg_ncdm =
	ksi_ncdm =
	#ncdm_psd_parameters = s_12,s23,s13
	ncdm_psd_parameters = 0.3 ,0.5, 0.05
	use_ncdm_psd_files = 0
	ncdm_psd_filenames = psd_FD_single.dat
```

$$f(q) = \frac{1}{(2\pi)^3}\left[\frac{1}{e^{q-ksi\_ncdm}+1} + \frac{1}{e^{q+ksi\_ncdm}+1}\right]$$

- $T_{\mathrm{ncdm}}^{\mathrm{CLASS}} = \left(\frac{4}{11}\right)^{\frac{1}{3}}$ ← Instantaneous decoupling
- $T_{\mathrm{ncdm}}^{\mathrm{CLASS}} = 0.715985$ ← Fudged to satisfy

```
massiveNu.ini:
      N_ncdm = 1
      m_ncdm = 0.04
      T_ncdm = 0.715985
      deg_ncdm = 3
```

$$\omega_\nu = \frac{\sum m_\nu}{93.14\mathrm{eV}}$$

```
-> ncdm species i=1 sampled with 11 (resp. 5) points for purpose of background (resp. perturbation) integration. In the relativistic limit it gives N_eff = 3.03747
 -> non-cold dark matter species with i=1 has m_i = 4.000000e-02 eV (so m_i / omega_i =3.104339e+01 eV)
```

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$$\mathrm{f(q)} = \frac{1}{(2\pi)^3}\left[\frac{1}{e^{q-ksi\_ncdm}+1}+\frac{1}{e^{q+ksi\_ncdm}+1}\right]$$

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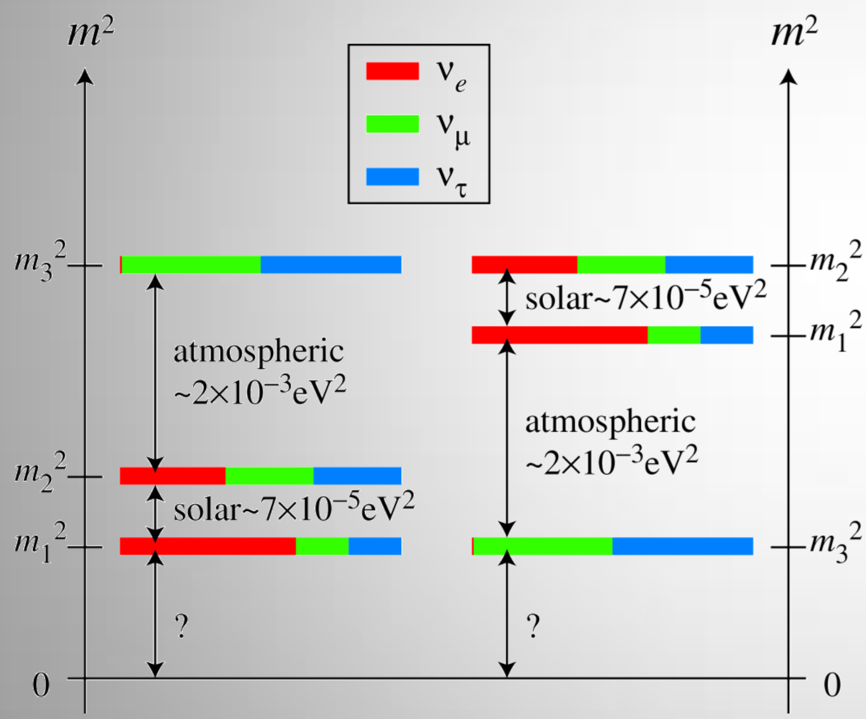
```

$$\mathrm{f(q)} = \frac{1}{(2\pi)^3}\left[\frac{1}{e^{q-ksi\_ncdm}+1} + \frac{1}{e^{q+ksi\_ncdm}+1}\right]$$

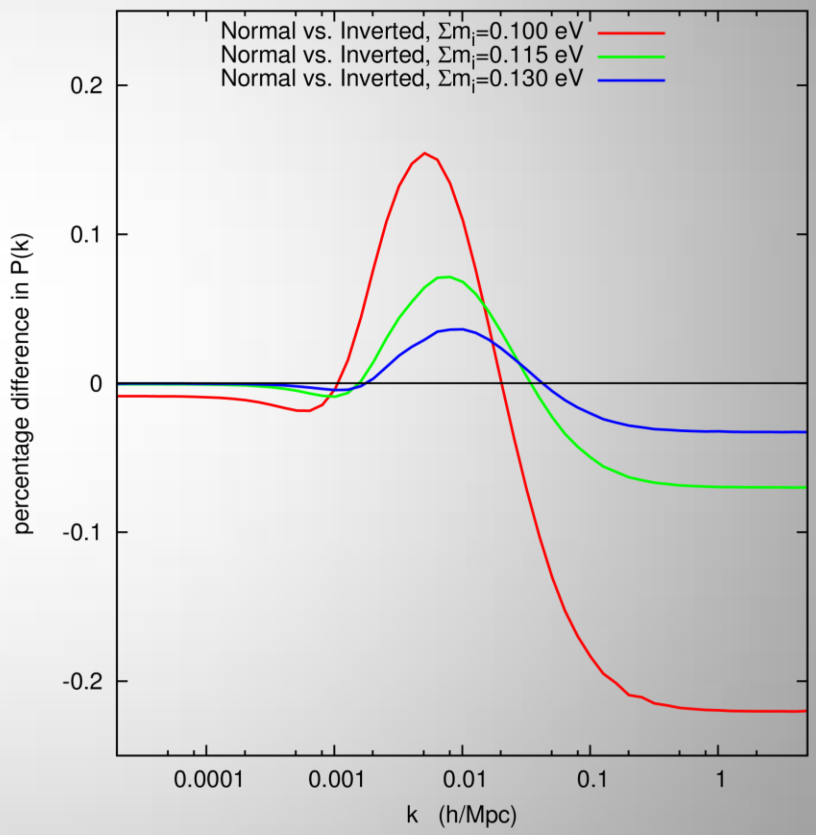
background_ncdm_distribution()

```
if (pba->got_files[n_ncdm]==_TRUE_) {
  (...);
}
else{
*f0 = 1.0/pow(2*_PI_,3)*
  (1./(exp(q-ksi)+1.) +1./(exp(q+ksi)+1.));
  if (_FALSE_) {
    /* extract values from the list */
    double square_s12=param[0];
    double square_s23=param[1];
    double square_s13=param[2];
    /* loop over flavor eigenstates and compute psd */
    for(i=0;i<3;i++)
      *f0 = (...);
  } /* end of region not used, but shown as an example*/
}
```

Exercise 1: Normal vs Inverted hierarchy



From arXiv:1301.1340



Exercise 1: Hints

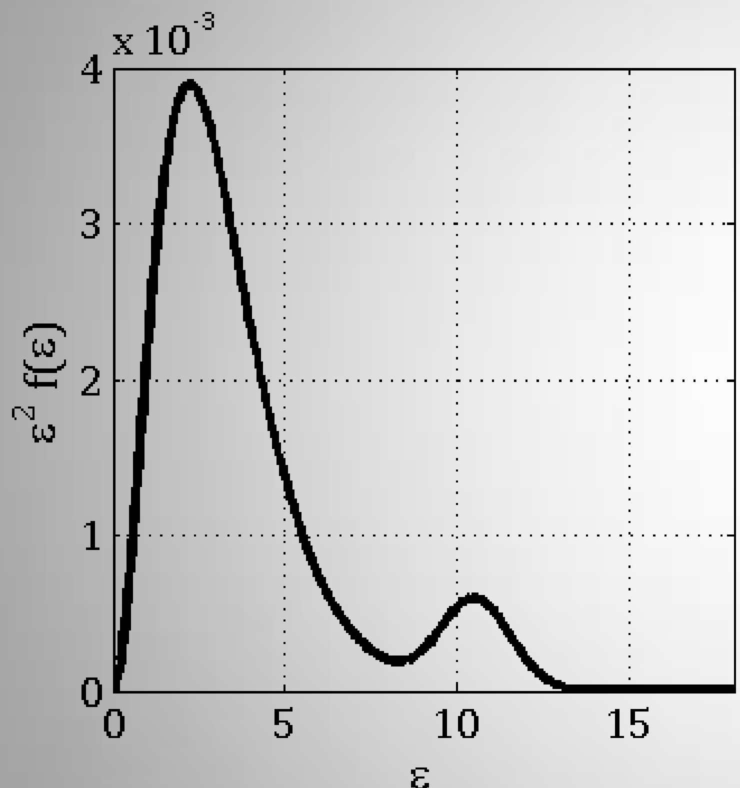
```
NH.ini:
      N_ncdm = 2
      m_ncdm = 0.0035, 0.0482
      deg_ncdm = 2.0, 1.0
      root = output/NH100_
      ncdmfa = 3
```

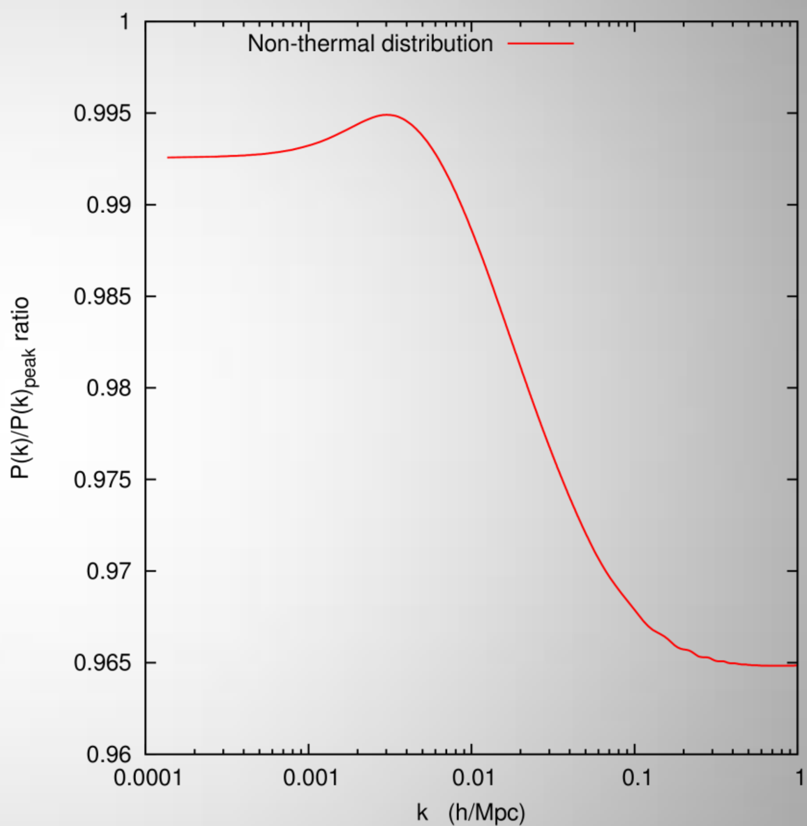
$$2\mathrm{m}_1 + \left(\mathrm{m}_1 + \sqrt{2 \cdot 10^{-3}\mathrm{eV}^2}\right) = \sum m_\nu \quad \text{(NH)}$$

$$m_3 + 2\left(m_3 + \sqrt{2 \cdot 10^{-3}\mathrm{eV}^2}\right) = \sum m_\nu \quad \text{(IH)}$$

```
$] python CPU NH100_pk.dat IH100_pk.dat -i -x
```

Exercise 2: Decay peak





$$f(\epsilon) = \frac{2}{(2\pi)^3}\left[\frac{1}{e^{\epsilon}+1} + \frac{A\pi^2}{\epsilon^2\sigma\sqrt{2\pi}}\exp\left(\frac{(\epsilon-\epsilon_c)^2}{2\sigma^2}\right)\right]$$

Exercise 2: Hints

```
decaypeak.ini:
      N_ncdm = 1
      m_ncdm = 1.0
      deg_ncdm = 3.0
      #ncdm_psd_parameters = A, sigma, eps_c
      ncdm_psd_parameters = 0.018, 1.0, 10.5
      root = output/peak1ev_
```

```
$] python CPU peak1ev_pk.dat nopeak_pk.dat -i -x
```