# Lecture III: The Thermal History

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- treated by the module `thermodynamics.c`. So this lecture will refer mainly to the content of `input/thermodynamics.h`, `source/thermodynamics.c`, and to the structure referred as `th`:

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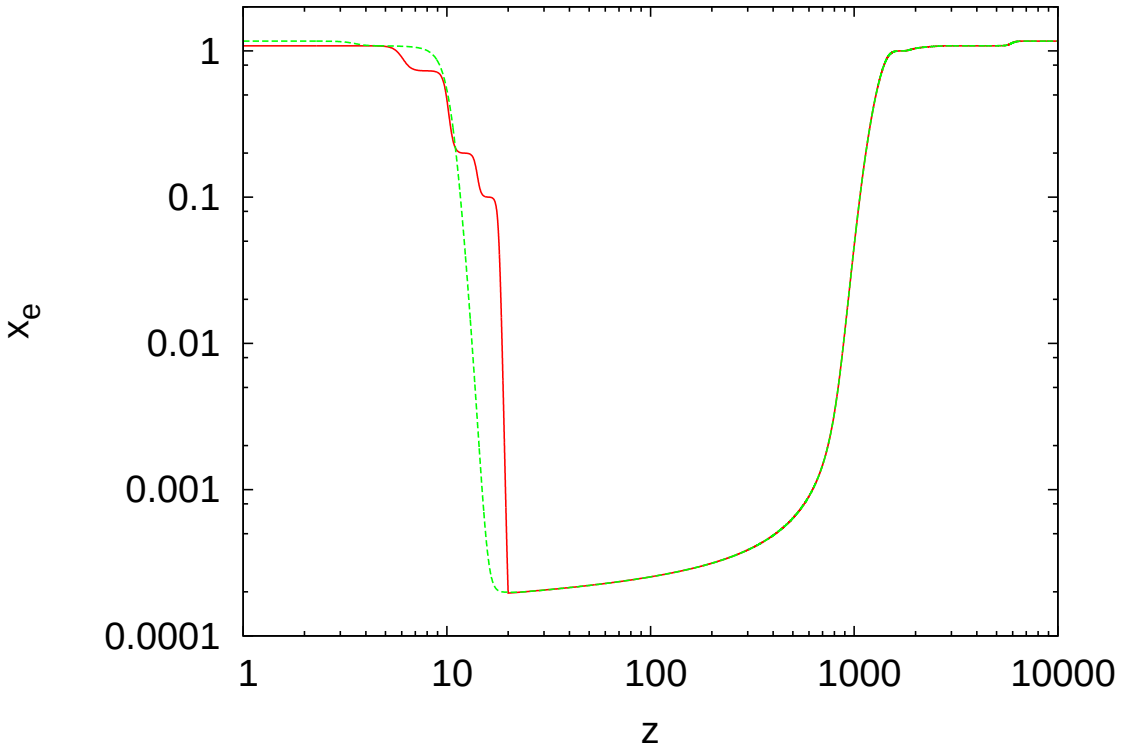
- the goal of this module is to solve the thermal history and store the results in a table. It should provide a function able to interpolate within this table at any value of redshift.
- other modules should be able to know all useful thermodynamical quantities at any given redshift.

# Thermodynamics

In this lecture we will address the following questions:

- what is assumed in CLASS about recombination and reionisation?
- how are they implemented?
- how to prepare plots of thermodynamical quantitites.



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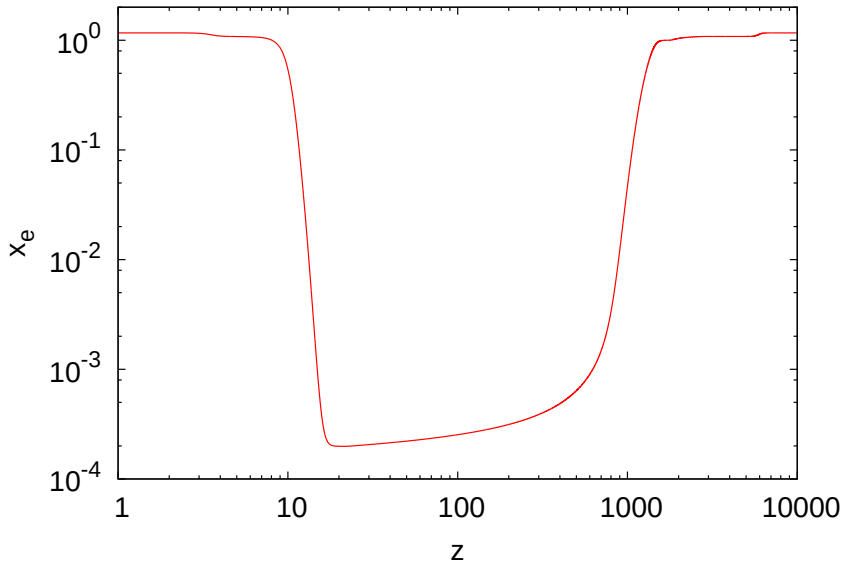
- free electron fraction $x_e = n_{\text{free electrons}}/n_p$ (not counting protons in He nuclei, so $x_e$ can be $> 1$)



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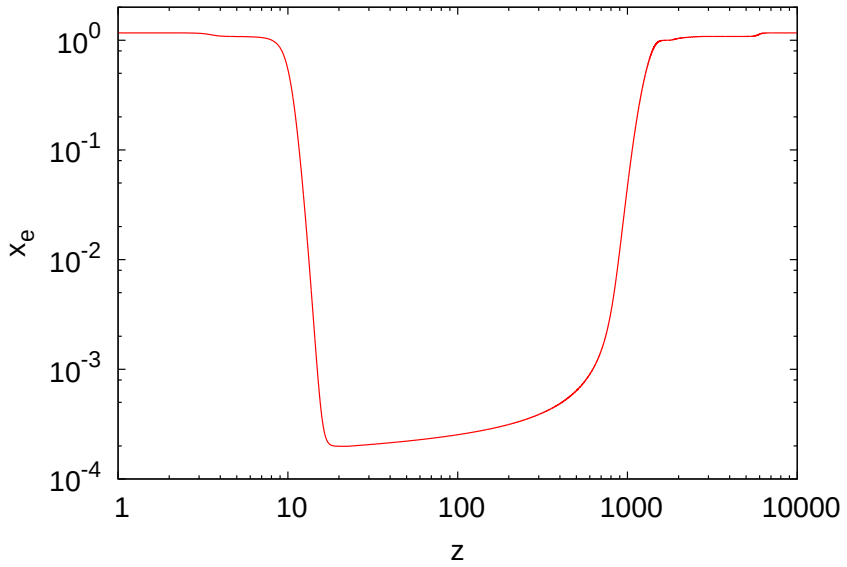


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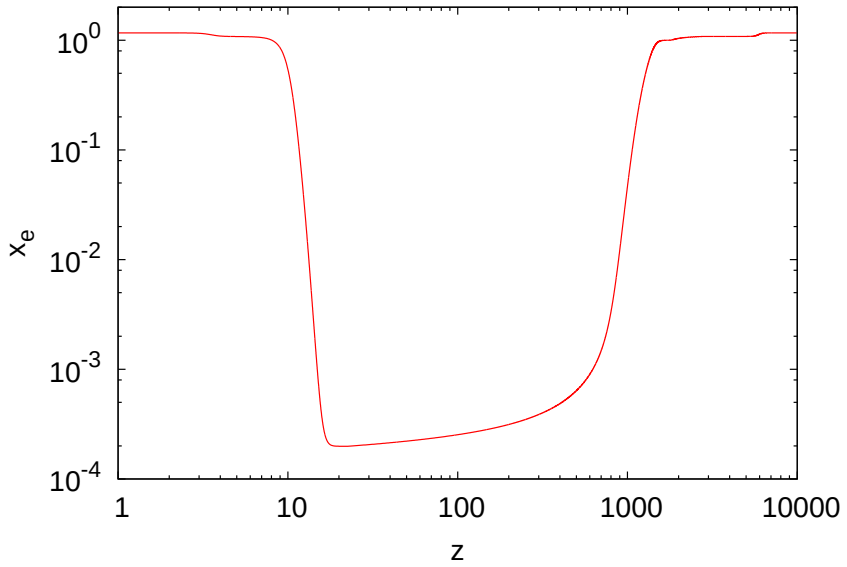
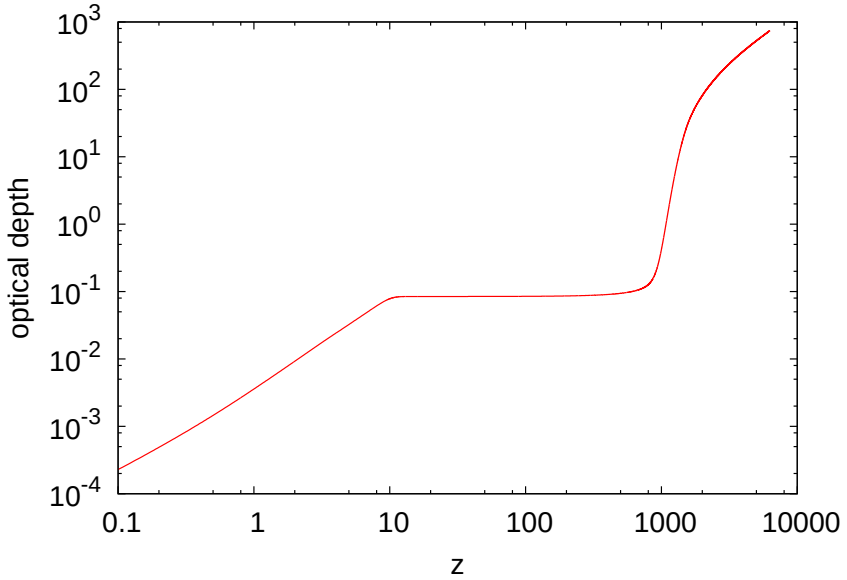



- observe the first/second He recombination, the H recombination (around $T \sim 0.3$ eV, not $13.6$ eV!), the H reionisation, the first He reionisation
- Thomson scattering rate $\kappa' = \sigma_T a n_p x_e$ : universe becomes transparent when $\kappa' < H$, i.e. at recombination

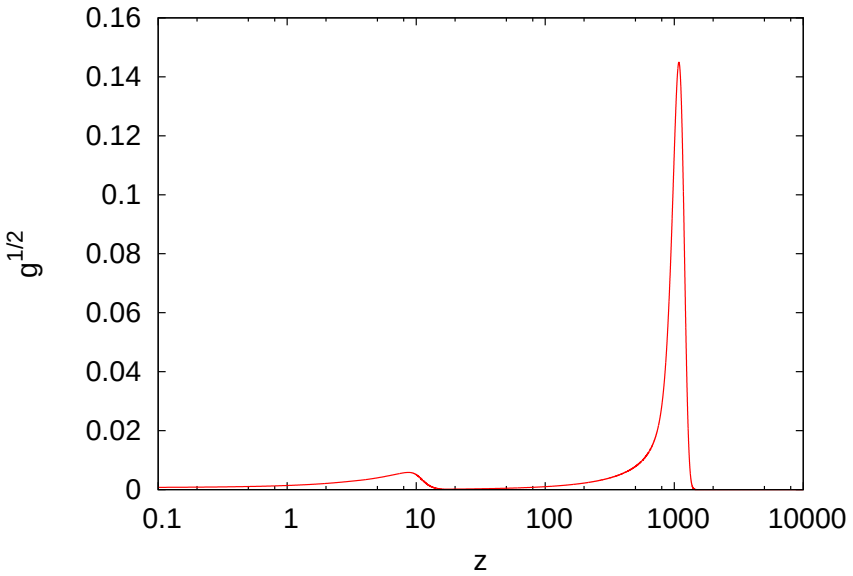
# Thermal history

- optical depth $\kappa(\tau) = \int_{\tau}^{\tau_0} \kappa' d\tau =$ depth of the cosmic fog



# Thermal history

- visibility function $g(\tau) = \kappa' e^{-\kappa}$ = probability that last interaction was at $\tau$



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- RECFAST integrates $\frac{d}{dz}\{x_{\rm H}, x_{\rm He}, T_b\}$. HyRec is more sophisticated.
- In both cases, CLASS needs to keep in memory an interpolation table for just $\{x_e(z), T_b(z)\}$.

# Recombination

- Recombination needs one more cosmological parameter: the primordial Helium fraction $Y_{\rm He}$.
- User can fix it to given value (e.g. `Y_He = 0.25`) or to `Y_He = BBN`. Then the value is infered from an interpolation table computed with a BBN code (Parthenope), for each given value of $N_{\rm eff}$, $\omega_b$ (assumes $\mu_{\nu_e} = 0$, easy to generalise).
- BBN interpolation table located in separate directory, in `bbn/bbn.dat`

# The module thermodynamics.c

External functions are:

- `thermodynamics_at_z(pba,pth,z,...,pvecthermo)`: interpolates in thermodynamics table (stored in `pth`) at a given $z$, returns a vector `pvecthermo`.
- `thermodynamics_init(ppr,pba,pth)`: computes thermodynamics table and stores it in `pth`.
- `thermodynamics_free(pth)`: free memory allocated in `pth`.

Let us now review the many tasks of `thermodynamics_init()`.

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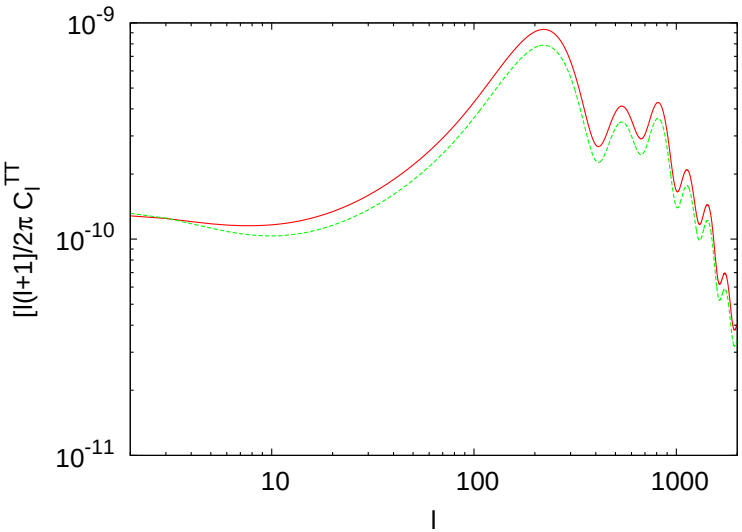
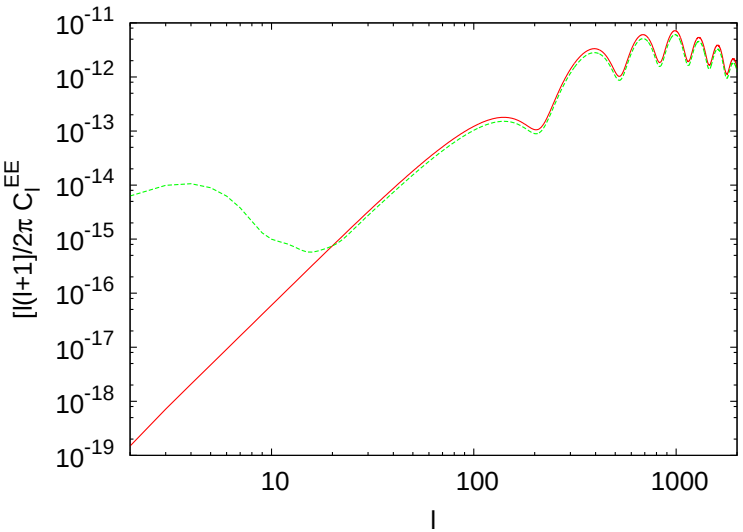
- `thermodynamics_merge_reco_and_reio()` fills the final interpolation table in `pth` using `preco` at high $z$ and `preio` at low $z$.
- derivation of a few more related quantities (see later).

# Reionization models

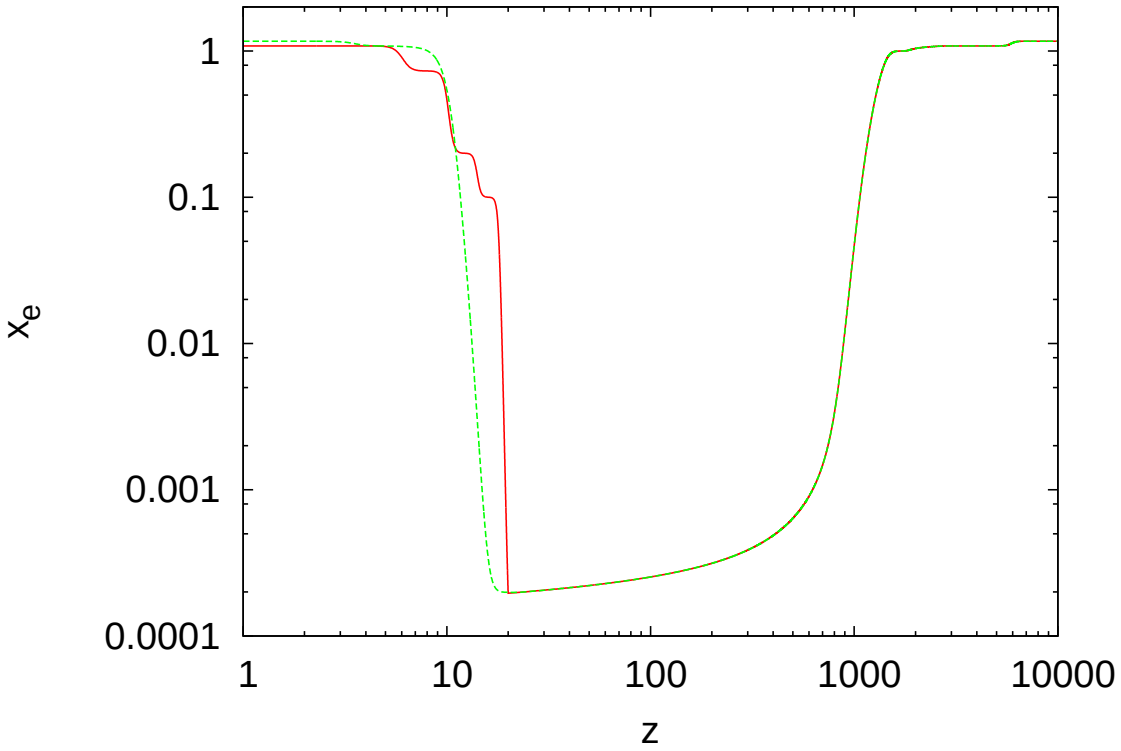
- reionisation very uncertain. Can be probed directly by looking at IGM (Lyman-$\alpha$, ...) but with large uncertainties.
- CMB probes mainly an integrated quantity, $\tau_{\rm reio} = \int_{\tau_*}^{\tau_0} \kappa' d\tau$, close to 0.08. Gives suppression of $C_l$'s at large $l$ due to rescattering.
- small-$l$ CMB (T and even better E) gives information on history (i.e. on $x_e(z)$, through $\kappa'(z)$).



# Reionization models

- if `reio_parametrization = reio_camb`, $x_e(z)$ has a $tanh$-shaped step, centered on $z_{\rm reio}$, and matched to the correct value corresponding to freeze-out after recombination. User free to pass either `z_reio = ...` or `tau_reio = ...`. Codes find the missing one automatically, stores it in `pth` (and indicates it in output if `thermodynamics_verbose` $> 0$).

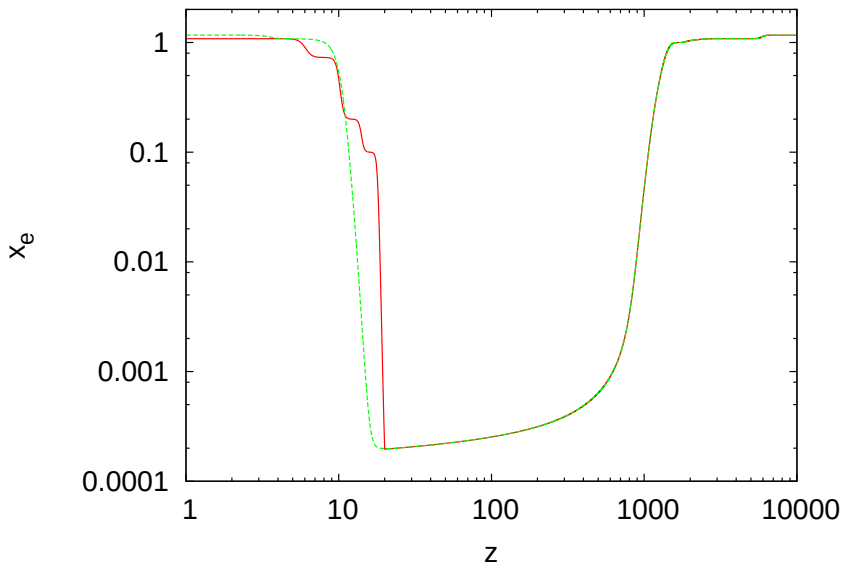


# Reionization models

- instead, if `reio_parametrization = reio_bins_tanh`, code assumes a binned reionisation history, with smooth $tanh$ steps between bin centers. User passes e.g.

```
binned_reio_num = 3
binned_reio_z = 8,12,16
binned_reio_xe = 0.7,0.2,0.1
binned_reio_step_sharpness = 0.3
```

- then `tau_reio` cannot be passed in input, but calculated, stored and given in output.



# Quantities stored in thermodynamics_table

The table `pth->thermodynamics_table[index_z*pth->th_size+pba->index_th]` has indices:

| | | |
|---|---|---|
| `index_th_xe` | ionization fraction | $x_e$ |
| `index_th_dkappa` | Thomson scattering rate | $\kappa'$ (units Mpc$^{-1}$) |
| `index_th_tau_d` | Baryon drag optical depth | $\int_\tau^{\tau_0} \frac{4\rho_\gamma}{3\rho_b}\kappa' d\tau$ |
| `index_th_exp_m_kappa` | exp. of (photon) optical depth | $e^{-\kappa}$ with $\kappa = \int_\tau^{\tau_0} \kappa' d\tau$ |
| `index_th_g` | visibility function | $g = \kappa' e^{-\kappa}$ |
| `index_th_Tb` | baryon temperature | $T_b$ given by RECFAST |
| `index_th_cb2` | squared baryon sound speed | $c_b^2 = \frac{k_B}{\mu} T_b \left(1 - \frac{1}{3}\frac{d\ln T_b}{d\ln a}\right)$ |
| `index_th_rate` | max. variation rate | (for sampling the sources) |

(plus extra indices for other derivatives: $\kappa''$, $\kappa'''$, $g'$, $g''$, $(c_b^2)'$, $(c_b^2)''$).

# Getting thermodynamical quantities from other modules

First allocate background and thermodynamics vectors:

```
double * pvecback;
double * pvecthermo;

class_alloc(pvecback,
            pba->bg_size_short*sizeof(double),
            ppt->error_message);

class_alloc(pvecthermo,
            pth->th_size*sizeof(double),
            ppt->error_message);
```

# Getting thermodynamical quantities from other modules

Then, fill them:

```
class_call(background_at_tau(pba,
                             tau,
                             pba->short_info,
                             ..., ...,
                             pvecback),
                             pba->error_message,
                             ppt->error_message);

class_call(thermodynamics_at_z(pba,
                               pth,
                               z,
                               ..., ...,
                               pvecback,
                               pvecthermo),
                               pth->error_message,
                               ppt->error_message);

if (pvecthermo[pth->index_th_dkappa] > H) {...}
```

(pvecback needed to extrapolate accurately for $z > z_{\text{recfast max.}}$)

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- These quantitites play a crucial role in choosing the sampling of the sources in $k$-space, because oscillation phase given by $\cos\left(2\pi\frac{d_s(z_{\rm rec})}{\lambda(z_{\rm rec})}\right)$. May give an estimate of $\theta_{\rm peak}=\frac{\pi}{l_{\rm peak}}=\frac{d_s(z_{\rm rec})}{d_a(z_{\rm rec})}$.

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- also finds the baryon drag time $z_d$ numerically, such that the baryon optical depth at $z_d$ is one.
- it stores the related quantities `pth->z_d`, `pth->tau_d`, `pth->ds_d`, `pth->rs_d` (the latter gives the phase of the BAOs in large scale structure).

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Two differences:

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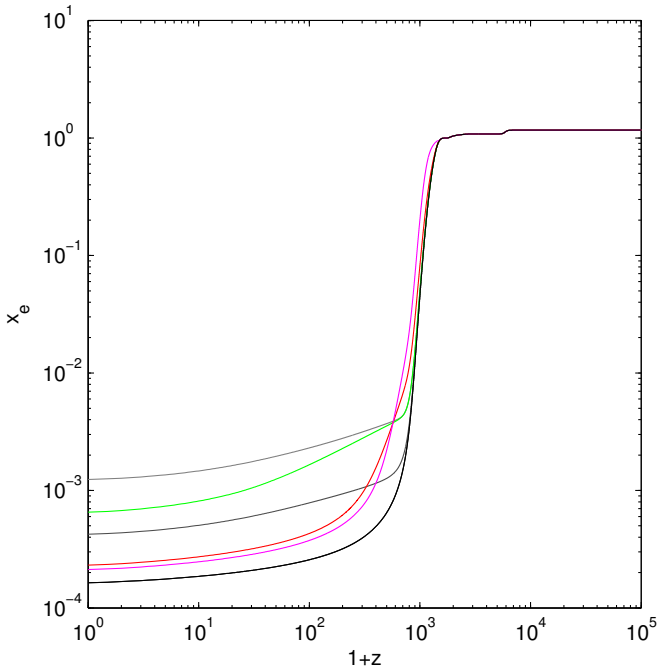
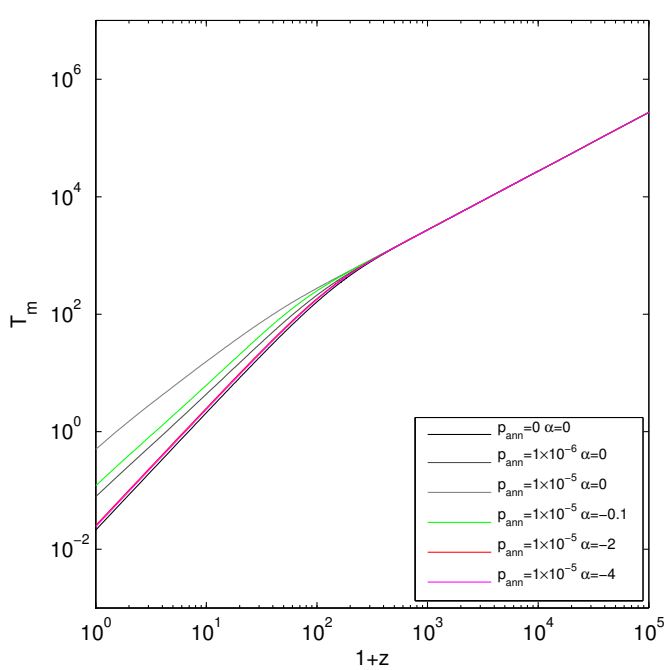
- RECFAST solution slightly smoothed around points where solution is not derivable. Just useful for testing the limit of high accuracy / small stepsize in RECFAST.
- several input parameters allow to play with a DM annihilation effect, as described in Giesen et al. 2012. Effect on $x_e$ and $T_b$, with signatures on CMB.



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Output easy to customise in `output.c`, by editing:
`output_one_line_of_thermodynamics(...)` for the quantities to plot in each line
`output_open_thermodynamics_file(...)` for the header (description of columns)
(another way would be to use `test_thermodynamics.c`)